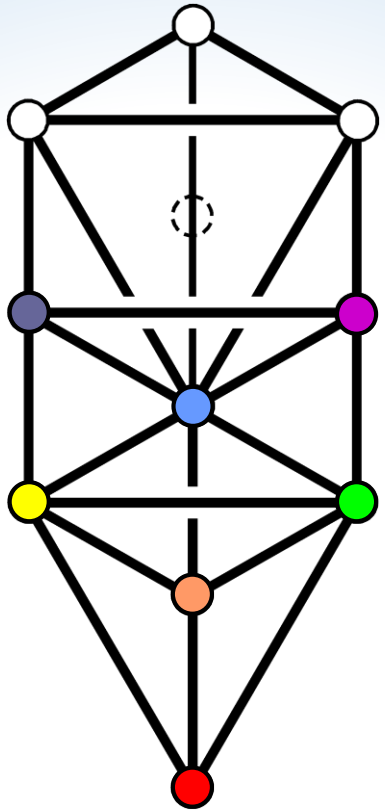
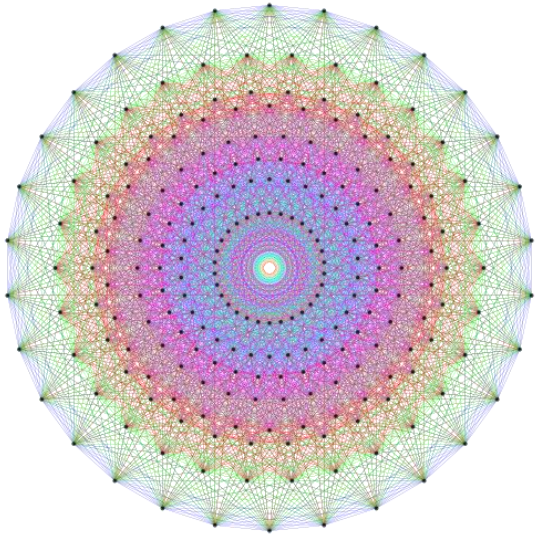


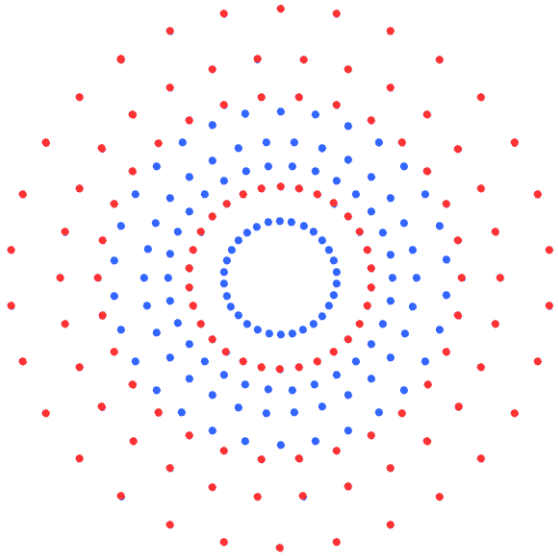
The sacred-geometrical basis of the $E_8 \times E_8$ heterotic superstring symmetry group

(For more details, see "4-d sacred geometries/Polychorons & Gosset polytope" at: www.smphillips.mysite.com)



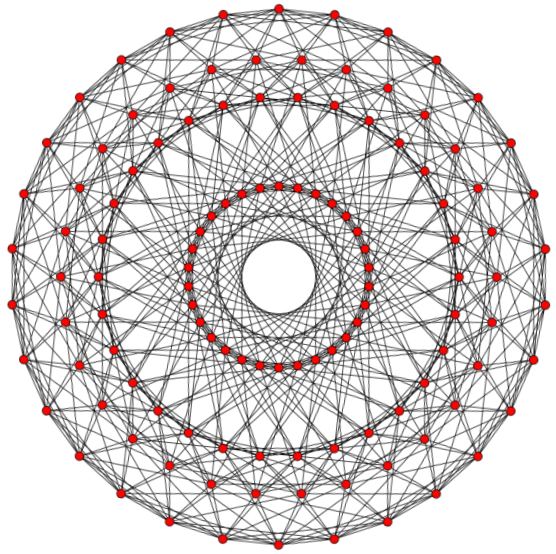
E_8 Coxeter plane projection of the 240 vertices of the 4_{21} polytope.

=



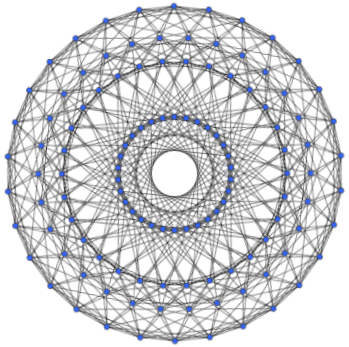
Four triacontagons with 120 red vertices and a smaller copy of these with 120 blue vertices.

=

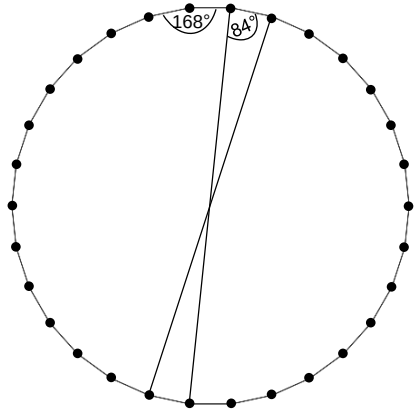


The 120 vertices of the 600-cell form four triacontagons.

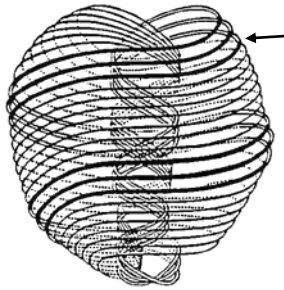
+



The 120 vertices of a smaller 600-cell form four triacontagons.

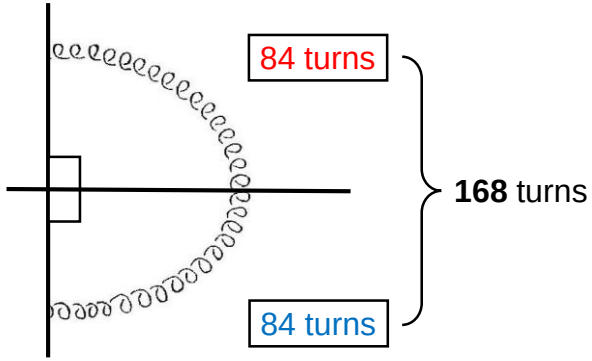


The interior angle of a triacontagon is **168**°. Each base angle of an isosceles sector is 84°.

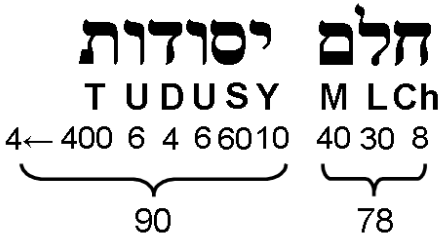


UPA

Each of the half-revolutions of a helical whorl of the UPA comprises **168** circular turns.



Each quarter-revolution of a whorl of the UPA comprises 84 turns.



= **168**

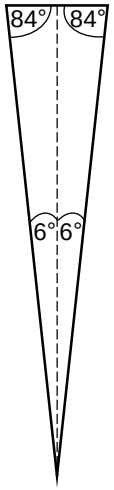
The gematria number value of *Cholem Yesodeth*, the Mundane Chakra of Malkuth, is **168**. This is both the size of the interior angle of each triacontagon in the Coxeter plane projection of the 4_{21} polytope and the number of turns in every half-revolution of a whorl of the UPA.

The Kabbalistic connection between the UPA and the 4_{21} polytope

$3 + 5 + 7 + \ldots + 81 = 1680$

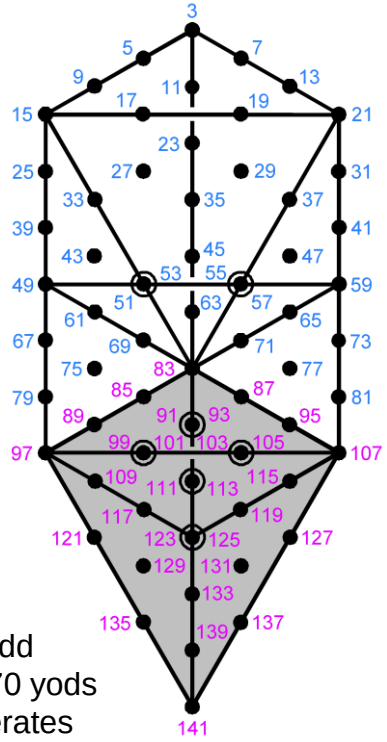
$83 + 85 + \mathbf{87} + \ldots + 141 = 2 \times 1680$

$3 + 5 + 7 + \ldots + 141 = 3 \times 1680 = 5040 =$



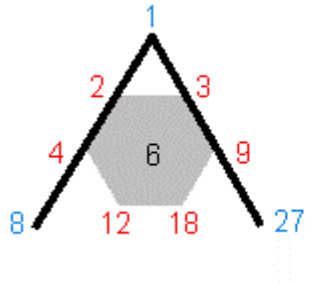
A sector of the triacontagon

$6° + 84° = 90°$

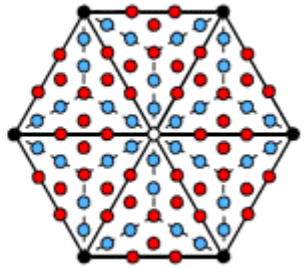


Assigning the first 70 odd integers after 1 to the 70 yods in the Tree of Life generates the number 5040 as their sum.

$90 =$



$= \mathbf{36} + \mathbf{48} + 6 =$ 84



36 (●)

48 (●)

84

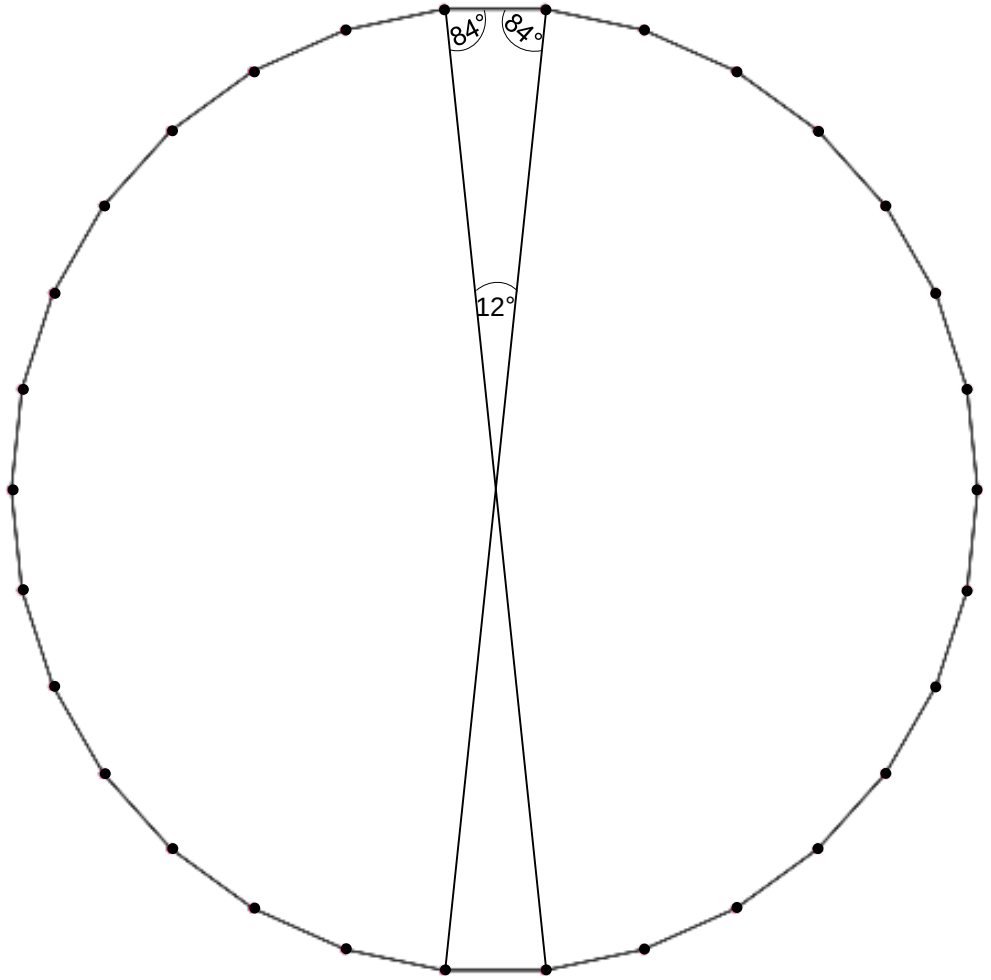
6 (●)

Plato's Lambda Tetractys

Type B hexagon

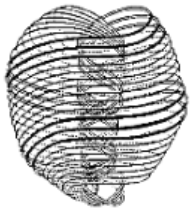
The 6:84 division of angles in the two right-angled triangles making up each sector of the triacontagon conforms to the pattern of numbers in Plato's Lambda Tetractys and to the distribution of yods in the Type B hexagon.

Sum of interior angles = 30×**168**° = 5040° = 3×1680° = 7!°.
Sum of interior angles of 8 triacontagons in the E_8 Coxeter plane projection of the 4_{21} polytope = 8×7!° = 8!°.

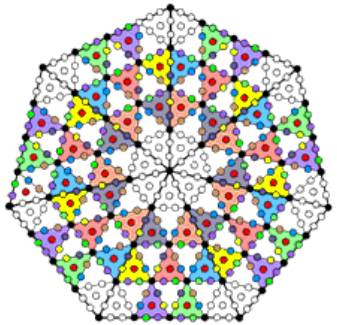


Triacontagon

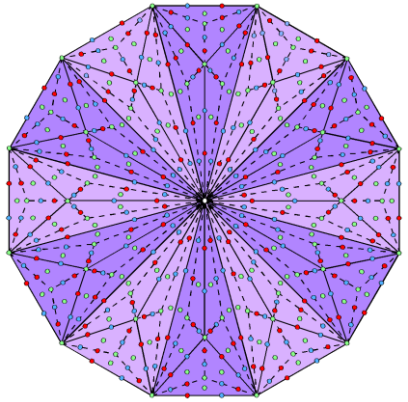
A whorl is a helix with 1680 turns. It makes 5 revolutions around the spin axis of the UPA.



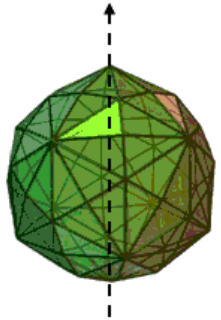
The three major whorls of the UPA/subquark superstring have (3×1680=5040) turns spread along their (3×10=30) half-revolutions, each with **168** turns.



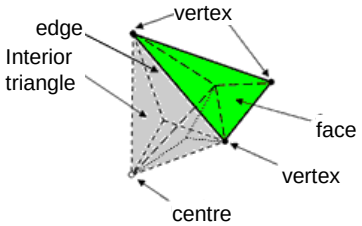
504 yods surround the centre of the heptagon.



504 yods (3 sets of **168** yods) surround the centre of the Type C dodecagon.



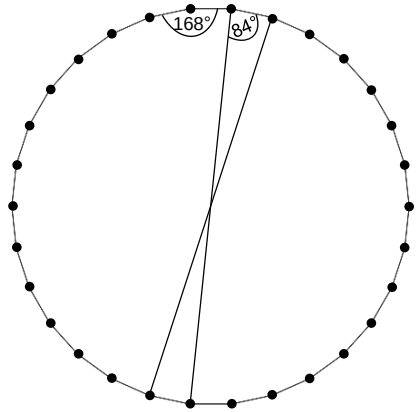
5040 geometrical elements surround the axis of the disdyakis triacontahedron.



120 Type A triangles with 180 edges in the faces.

(180 + 120×3 = 540) Type A triangles in the interior.

	Corners	Sides	Triangles	Total
Faces	60 vertices	180	120×3 = 360	240
	120	120×3 = 360		840
Subtotal	180	540	360	1080
Interior	180	60	180×3 = 540	1320
		180×3 = 540		
Subtotal	180	600	540	
Interior	360	120	360×3 = 1080	2640
		360×3 = 1080		
Subtotal	360	1200		
Total	720	2340	1980	5040



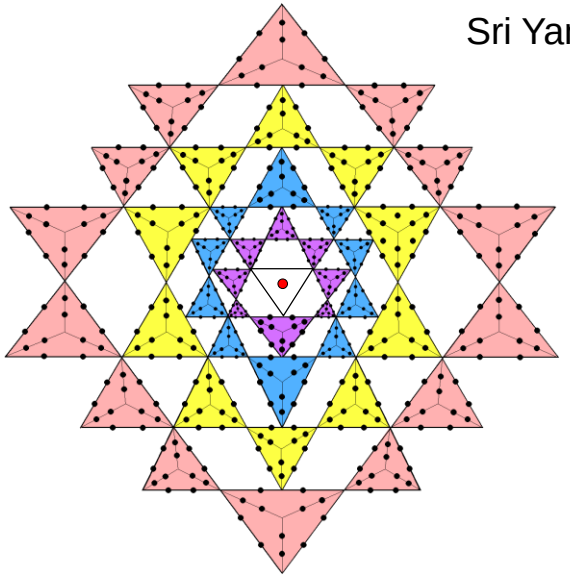
Triacontagon

The triacontagon is the Petrie polygon of the 4_{21} polytope with E_8 symmetry

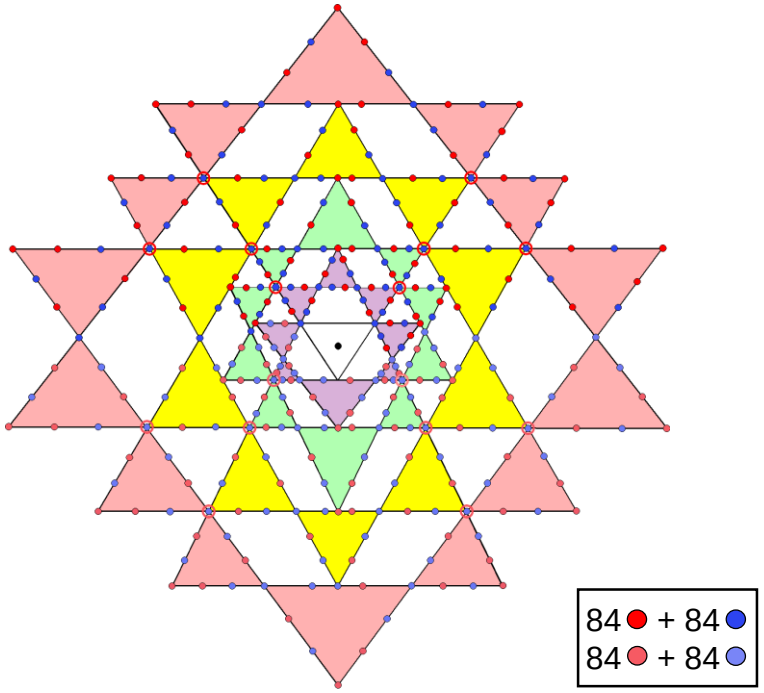
Sum of interior angles of the triacontagon = 30×**168** = 5040°. This factorisation is identical to that of the 5040 circular turns of the three major whorls of the UPA, each one making 10 half-revolutions of **168** turns. The 30 vertices of the triacontagon are analogous to the 30 half-revolutions of the major whorls. Each interior angle **168**° is the sum of two angles of 84°.

Circles denote corners of triangles that are directly above corners of triangles in the next lower layer.

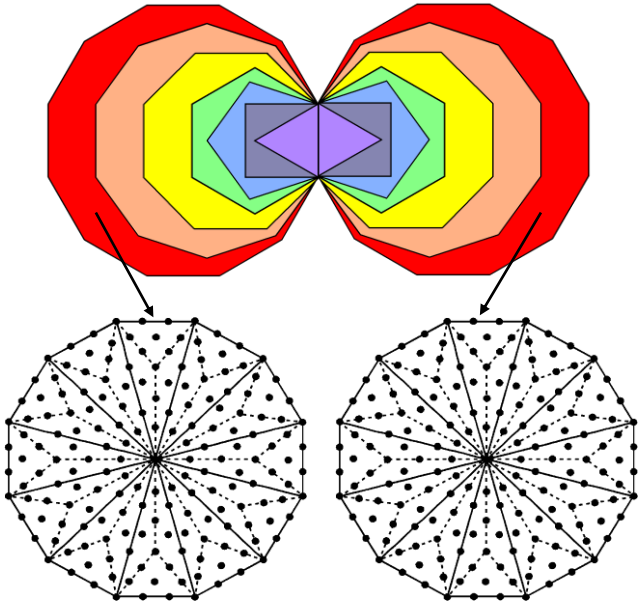
Sri Yantra



504 hexagonal yods on the 252 sides of the 126 tetractyses in the 42 Type A triangles of the Sri Yantra.

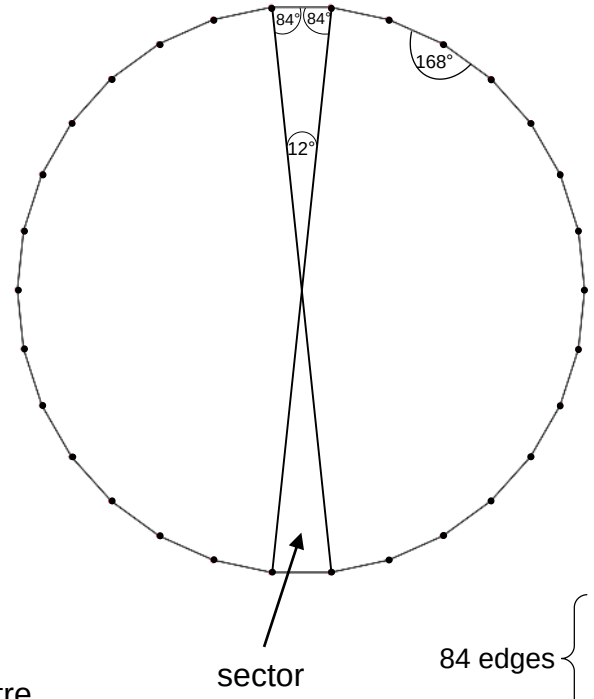


168 yods line the 126 sides of the 63 tetractyses in the **21** Type A triangles of each half of the 3-d Sri Yantra. They consist of two sets of 84 yods.

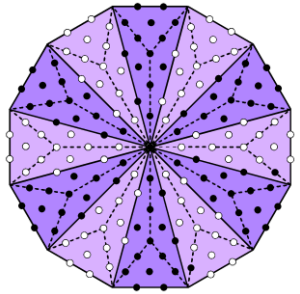


The (7+7) enfolded regular polygons of the inner form of the Tree of Life.

(180+180=360) yods surround the centres of the two Type B dodecagons. They may be interpreted as symbolising the 360 degrees of a full rotation.

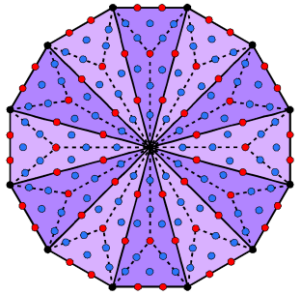


The triacontagon has 30 corners. Its interior angle is **168**°. The two base angles of an isosceles sector are 84° and the angle at its apex is 12°. The fact that the interior angle is (apart from a factor of 10) the *same* number that Charles Leadbeater counted in the turns of a helical whorl is powerful evidence that the UPA is an $E_8 \times E_8$ heterotic superstring because it is implausible that a number with such paranormal provenance could appear by chance in the basic geometry of a symmetry group that is associated with superstrings.



12 corners
84 ●
84 ○

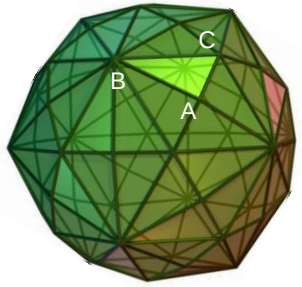
The 180 yods surrounding the centre of the Type B dodecagon consist of its 12 corners and **168** yods. Six sectors have 84 black yods and six sectors have 84 white yods.



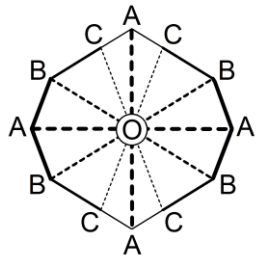
60 ●
108 ●
168

The **168** yods surrounding the centre of the Type B dodecagon that are not its corners consist of 60 red yods making up the Type A dodecagon and 108 blue yods. This division corresponds to the fact that the triacontagon is the largest regular polygon whose interior angle (**168**°) is the sum of the interior angles of smaller polygons: **168**° = 60° (triangle) + 108° (pentagon).

84 edges
12 edges
84 edges



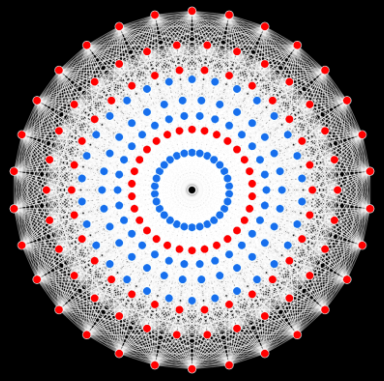
Disdyakis triacontahedron



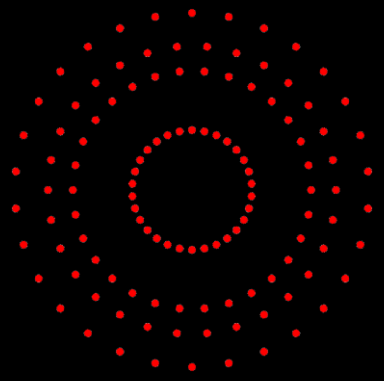
Central 12-gon

As the polyhedral counterpart of the inner Tree of Life, the disdyakis triacontahedron has 180 edges. They comprise 12 edges forming the central 12-gon and 84 edges both above and below it. They are the counterpart of the 12° vertex angle and the two 84° base angles of each sector of a triacontagon, the Petrie polygon of the 4_{21} polytope.

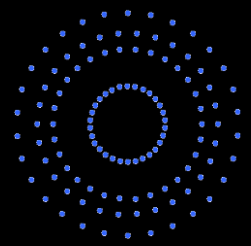
As the Petrie polygon of the 4_{21} polytope, the triacontagon conforms to the holistic 12:84:84 pattern shown by the Type B dodecagon and the disdyakis triacontahedron



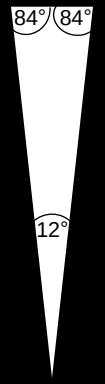
E_8 Coxeter plane projection of the 4_{21} polytope



600-cell



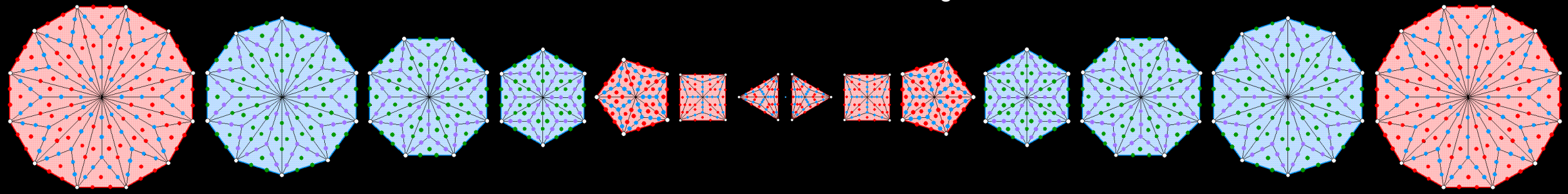
600-cell



Sector of triacontagon

600-cell/600-cell

Sum of base angles	Sum of vertex angles
168°	12°
168°	12°
168°	12°
168°	12°
Total = 720°	



Number of yods in the Type B n-gon = **15**n + 1.
Number of yods surrounding the centres of 7 Type B polygons with **48** corners = **15×48** = 720.
The composition of yods is:

(4+3) polygons

(24+24) ●

168 ● + **168** ●

168 ● + **168** ●

Total = 360 + 360 = 720

(4+3) polygons

(24+24) ●

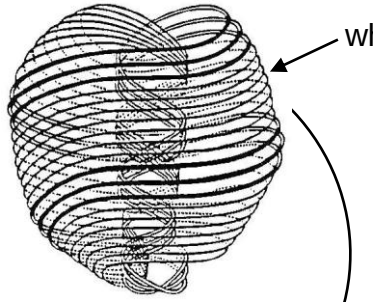
168 ● + **168** ●

168 ● + **168** ●

Total = 360 + 360 = 720

Correspondences

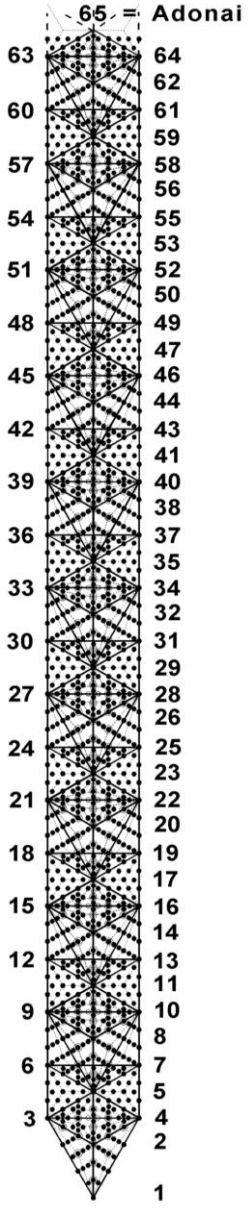
48 corners of each set of 7 polygons	→	sum (**48**°) of vertex angles of one sector in each of the 4 triacontagons making up each 600-cell.
4 sets of **168** yods in each set of 7 polygons	→	sum (4×**168**) of base angles of a sector in each of the 4 triacontagons.
720 yods in each set of 7 polygons	→	sum (720°) of angles in a sector of each of the 4 triacontagons in each 600-cell.



Each of the 10 whorls of the UPA spirals 5 times around its axis

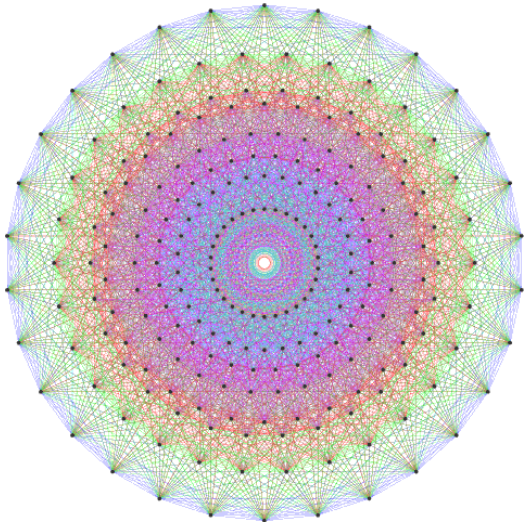
Each helical whorl of the UPA has 1680 circular turns (**168** turns in each half-revolution)

Below the top of the 10th Tree of Life are 1680 yods. Each whorl of the UPA (subquark state of the $E_8 \times E_8$ heterotic superstring) is the microscopic manifestation of these 10 Trees. Every yod denotes a turn of a helical whorl. ADONAI, the Godname of Malkuth, prescribes this structural parameter because its number value **65** is the number of Sephirothic emanations up to the top of the 10th Tree, below which are 1680 yods.

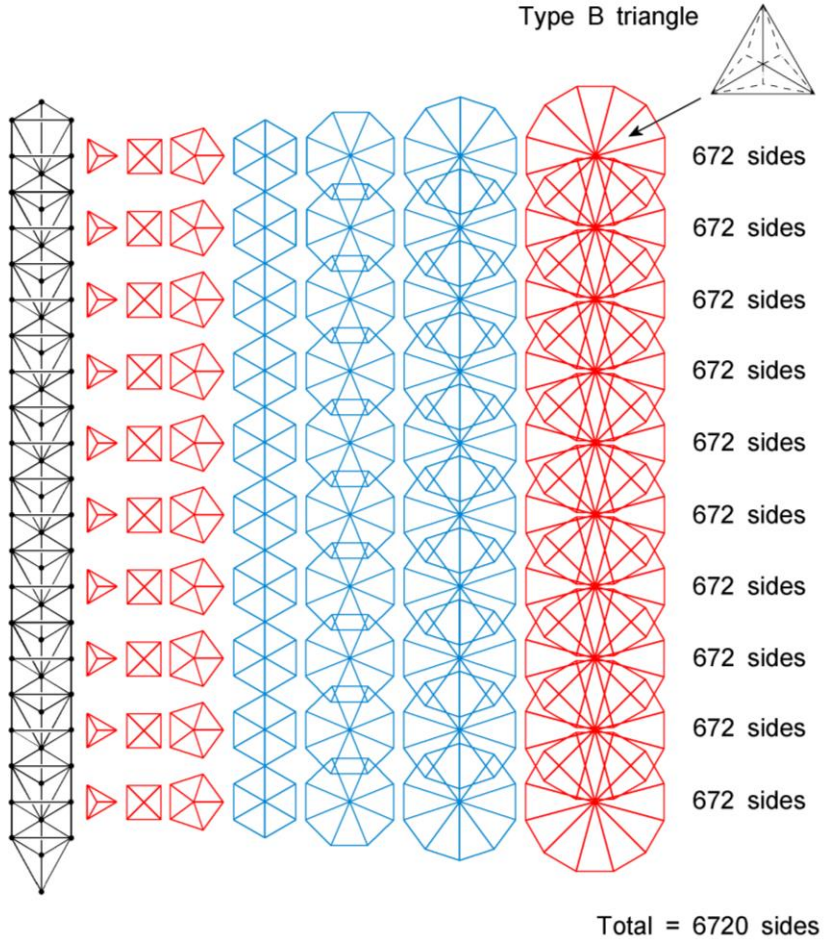


1680 ●

A Type C polygon has 14 sides of 9 triangles per sector.
The 7 separate polygons of the inner Tree of Life have **48** sectors.
The 7 separate, Type C polygons have (**48**×9=432) triangles with (**48**×14=672) sides.

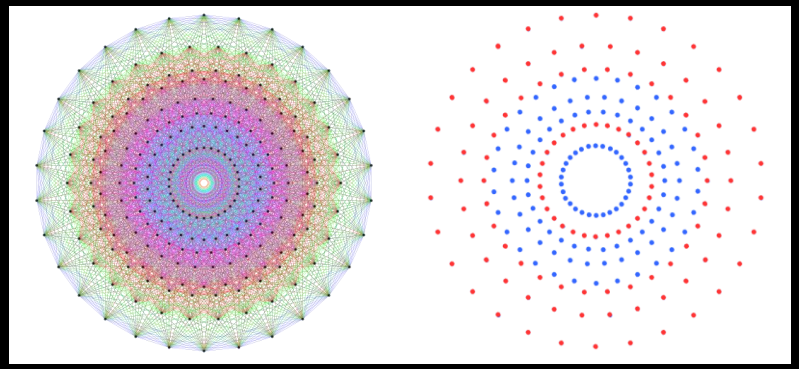


Coxeter plane projection of the 240 vertices of the 4_{21} polytope with 6720 edges. It contains eight circular rings (triacontagons with 30 vertices).



The inner form of 10 Trees of Life. The 4_{21} polytope representing the 240 roots of E_8 is its microscopic manifestation. Each side of a triangle corresponds to an edge of the 4_{21} polytope.

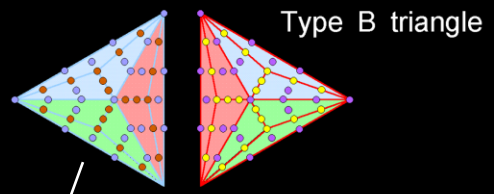
The Tree of Life connection between the UPA and the 4_{21} polytope

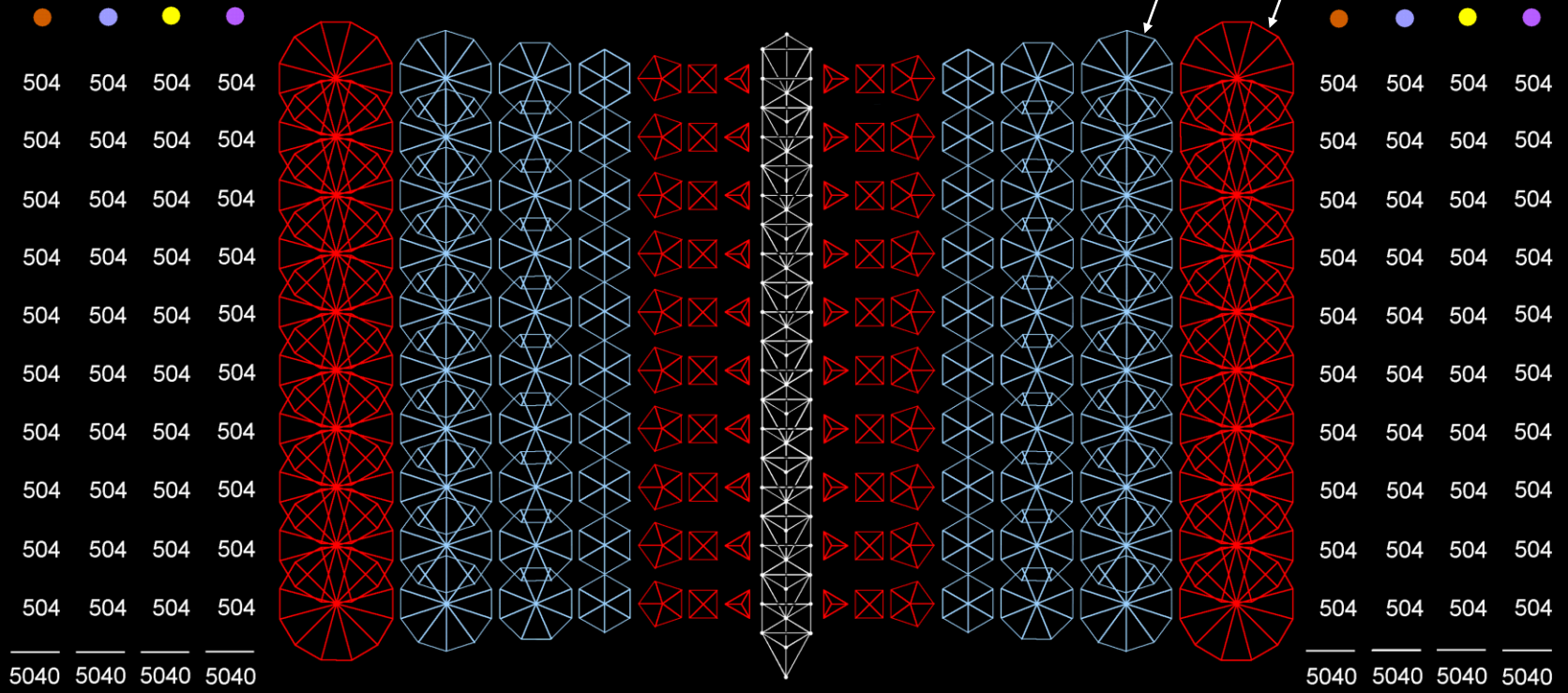


4_{21} polytope (4+4=8) triacontagons

Interior angle of triacontagon = **168**°.
Sum of 30 interior angles = 30×**168°** = 5040° = 7!°
4-d Coxeter plane projection of the 4_{21} polytope consists of 4 triacontagons with 120 red vertices and 4 smaller triacontagons with 120 blue vertices.
Sum of 240 interior angles of 8 triacontagons = 8×7! = 8!° = 40320°.

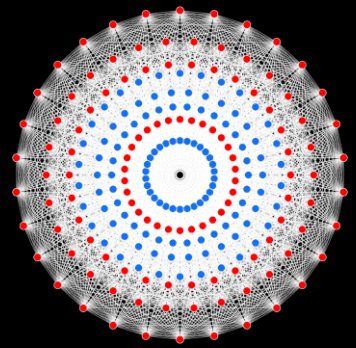
A Type C polygon has (**21**+**21**=42) yods in 9 tetractyses per Type B sector.
7 separate polygons have (24+24=**48**) sectors.
(9x**48**=432) tetractyses with (**48**x42=4x504) yods surround the centres of the (4+3=7) separate, Type C polygons.
(2x432=864) tetractyses with (2x4x504) yods surround the centres of the (7+7) separate, Type C polygons.
(10x864=8640) tetractyses with (2x4x5040) yods surround the centres of the (70+70) separate, Type C polygons.
As 5040 = 7!, total number of yods surrounding centres of the (70+70) separate Type C polygons = 8x7! = 8!.



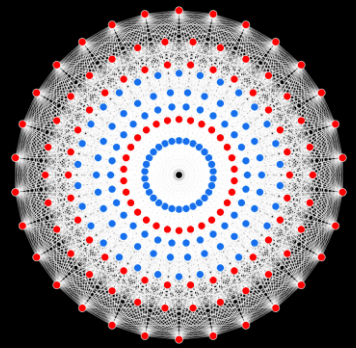


The inner form of the Tree of Life consists of 4 red polygons with 24 sectors and 3 light blue polygons with 24 sectors. **21**● yods and **21**● yods in each sector surround the centre of each red polygon. **21**● yods and **21**● yods in each sector surround the centre of each light blue polygon. The 4 red polygons have (24×**21**=504)● & 504●; the 3 light blue polygons have 504● yods & 504● yods. The (40+30=70) polygons have 4 sets of 5040 (=7!) yods. The (70+70) polygons have 2×4 sets of 7! yods, i.e., 8! yods. This is the sum in degrees of the 240 interior angles of the 8 triacontagons. *Each yod denotes a degree*. The 4 sets of 7! yods in the 70 polygons making up one half of the inner form of 10 Trees of Life correspond to the 4 red triacontagons with 7!° as the sum of their interior angles; the 4 sets of 7! yods in the 70 polygons that make up their mirror-image half correspond to the 4 blue triacontagons. The two sets of yods in a polygon and the two sets of polygons with 24 vertices create the counterpart of (2×2=4) triacontagons. Each half of the inner form of 10 Trees corresponds to 4 triacontagons.

The 4_{21} polytope has 6720 edges.
The 4 rings of 120 red dots are the 120 vertices of 4 triacontagons. Similarly for the 4 rings of blue dots. The triacontagon is the Petrie polygon of the 600-cell.



4_{21} polytope



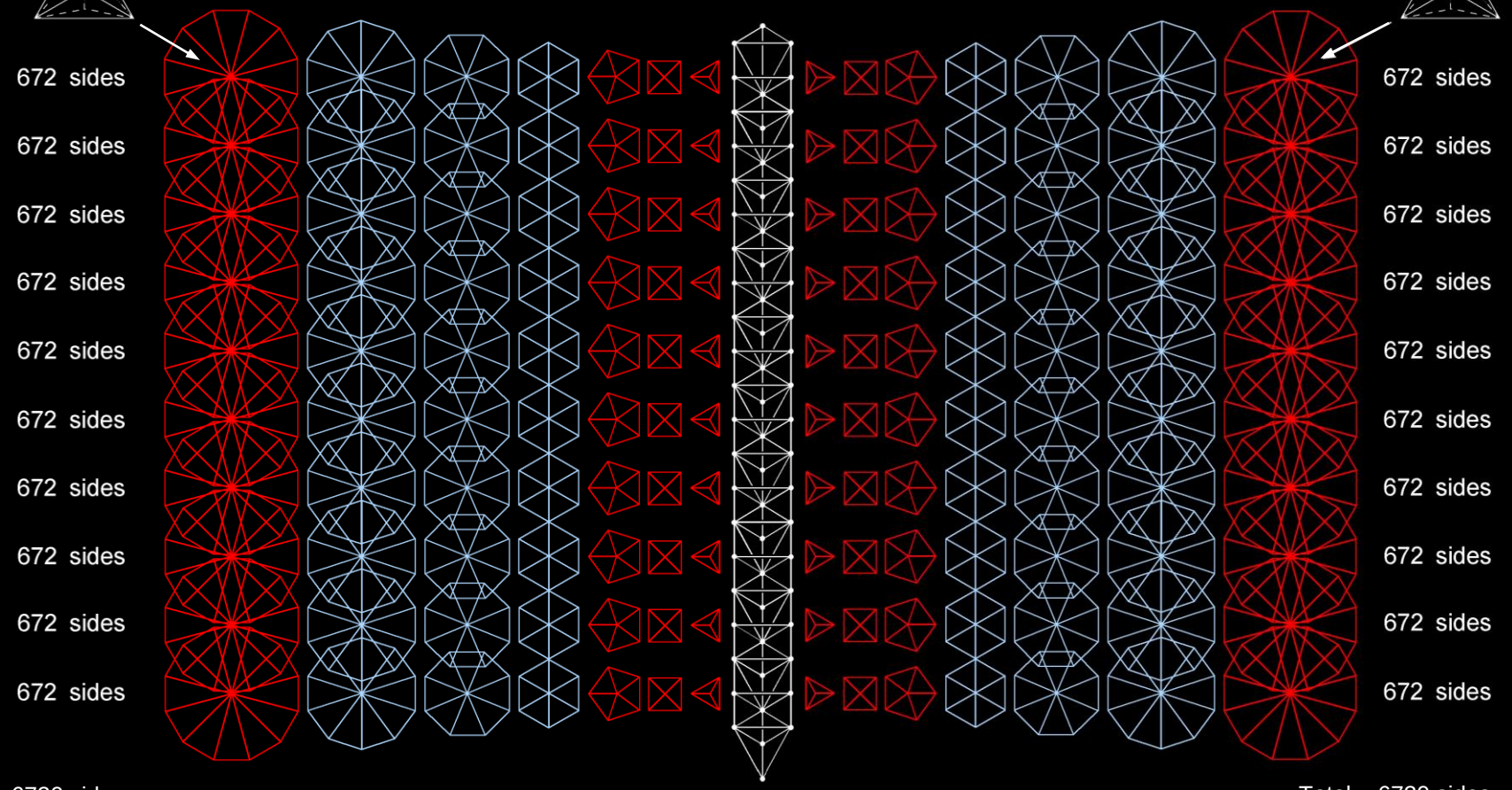
4_{21} polytope

The (70+70) Type C polygons in the inner form of 10 Trees of Life have (6720+6720) sides of triangles that correspond to the (6720+6720) edges of two 4_{21} polytopes representing the (240+240) vertices of $E_8 \times E_8$

Type B triangle

A Type C polygon has 9 triangles with 14 sides per Type B, triangular sector.
7 separate polygons have **48** sectors.
The 7 separate Type C polygons have (9×**48**=432) triangles with (**48**×14=672) sides.

Type B triangle

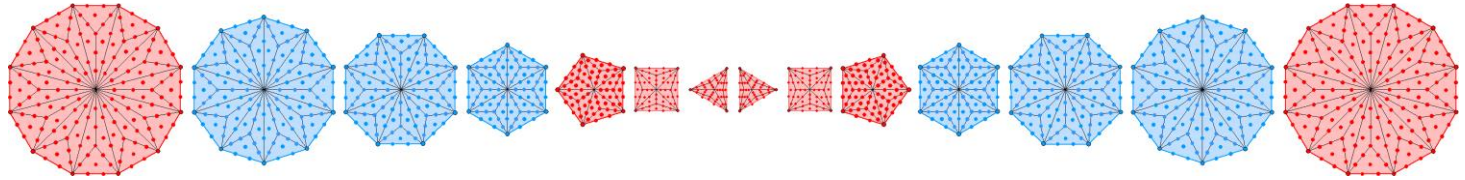


Total = 6720 sides

Total = 6720 sides

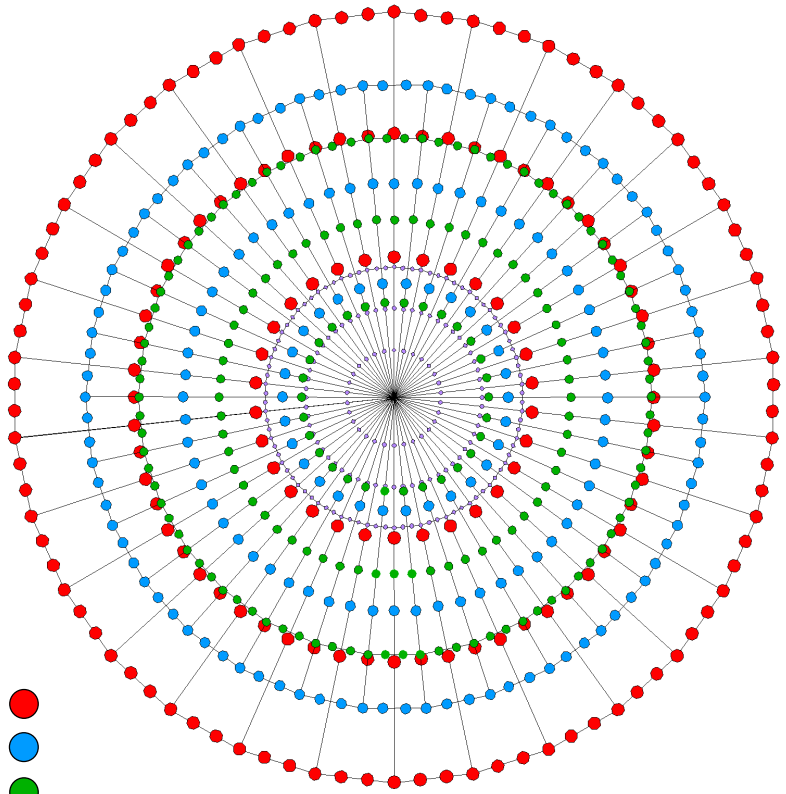
Surrounding the centres of the 7 polygons are:

720 { 360 360



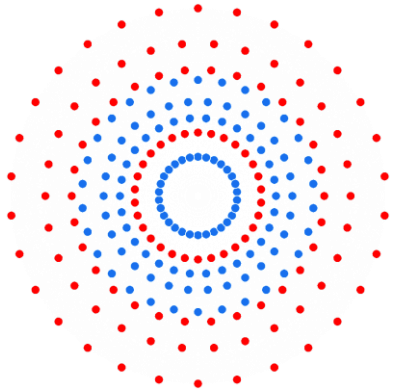
Surrounding the centres of the 7 polygons are:

720 { 360 360

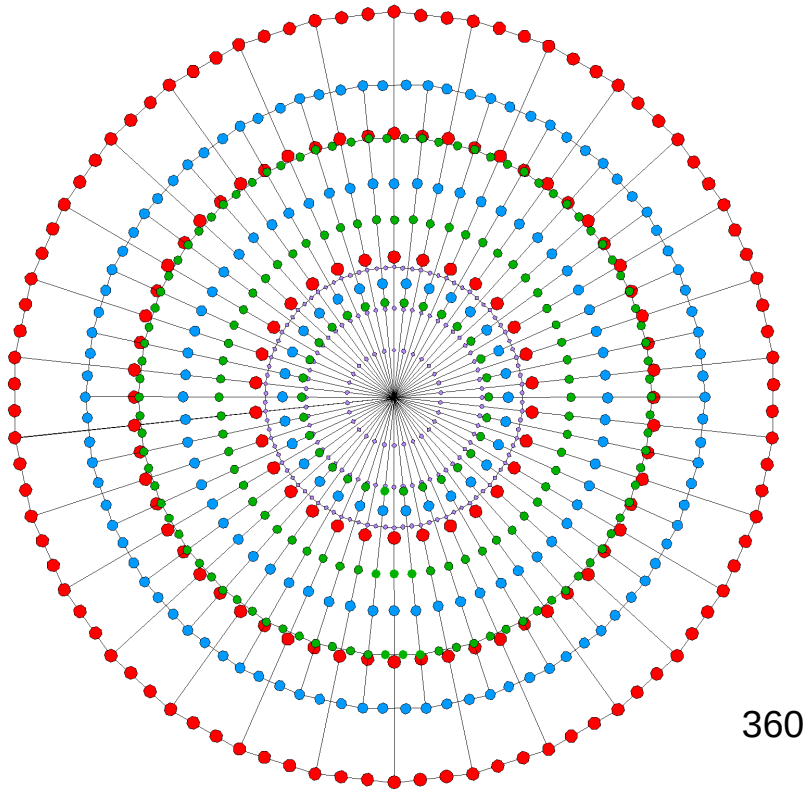


360 { 180 180

360 { 180 180



(4+4=8) triacontagons in the E_8 Coxeter plane of the 4_{21} polytope



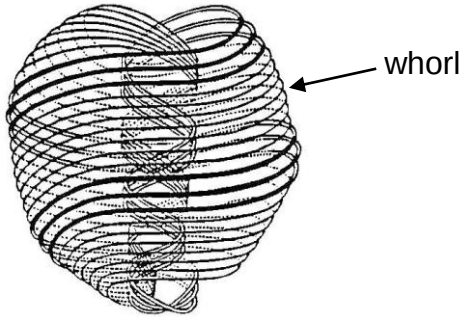
360 { 180 180

360 { 180 180

720 yods in the 4 Type A triacontagons in a 600-cell

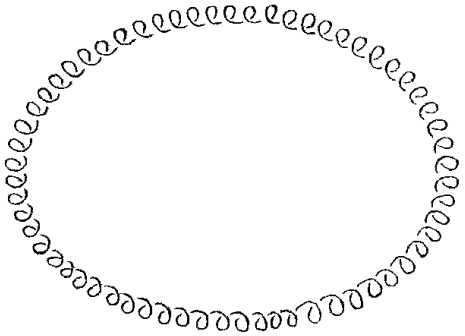
720 yods in the 4 Type A triacontagons in a 600-cell

180 yods surround the centre of a Type A triacontagon. (4×180=720) yods surround the centres of the 4 Type triacontagons (red, blue, green & violet yods) in the Coxeter plane projection of a 600-cell. (2×720=1440) yods surround the centres of the 2×4 triacontagons in the Coxeter plane projection of a compound of two 600-cells whose 240 vertices are the vertices of the Coxeter plane projection of the 240 vertices of the 4_{21} polytope. Each half of the inner Tree of Life with 720 yods in 7 Type B polygons denotes the four Type A triacontagons in a 600-cell with 720 yods surrounding their centres. The complete inner form of the Tree of Life expresses the projection of the 4_{21} polytope in the E_8 Coxeter plane.



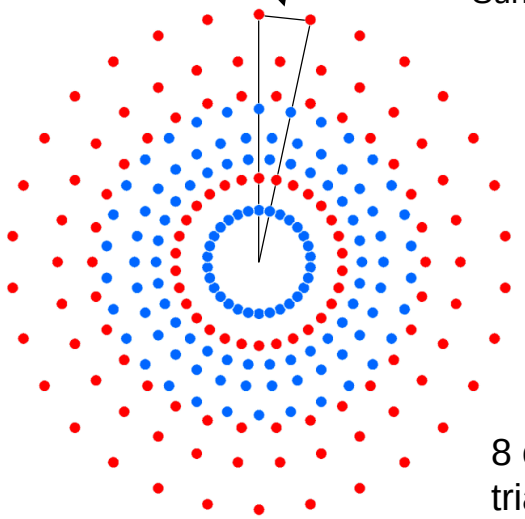
UPA

The UPA is the ground state of the subquark $E_8 \times E_8$ heterotic superstring. It consists of 10 helical whorls, each of which spirals 5 times around its axis (2½ times in its outer half, 2½ times in its narrower, inner half).



A whorl comprises 1680 circular turns (336 turns per revolution, **168** turns per half-revolution). Each of the 5 revolutions of all 10 whorls contains 3360 turns (1680 per half-revolution).

Type A triangle



8 concentric triacontagons

The 10 whorls of the UPA "carry" the 240 gauge charges associated with the 240 roots of E_8. Each of the 240 vertices of the 8 triacontagons that are the Petrie polygons of the 240 vertices of the 4_{21} polytope denotes a root. When the triacontagons are Type B, the number of yods other than vertices that surround their 240 sectors = 3360 (1680 in the 4 red triacontagons & 1680 in the 4 blue triacontagons). The 120 red vertices are the Coxeter plane projection of the 120 vertices of a 600-cell; the 120 blue vertices are the Coxeter plane projections of the 120 vertices of a smaller, concentric 600-cell. The 1680 yods in the 4 red Type B triacontagons symbolise the 1680 turns in an outer half-revolution of the 10 whorls. The 1680 yods in the 4 blue Type B triacontagons symbolise the 1680 turns in an inner half-revolution of the 10 whorls. The two 600-cells determine the outer and inner halves of the UPA. The 4:4 division of the 8 triacontagons exhibits the basic **168**:**168** division that is characteristic of holistic systems. The number **168** is the number of degrees in the interior angle of the triacontagon!

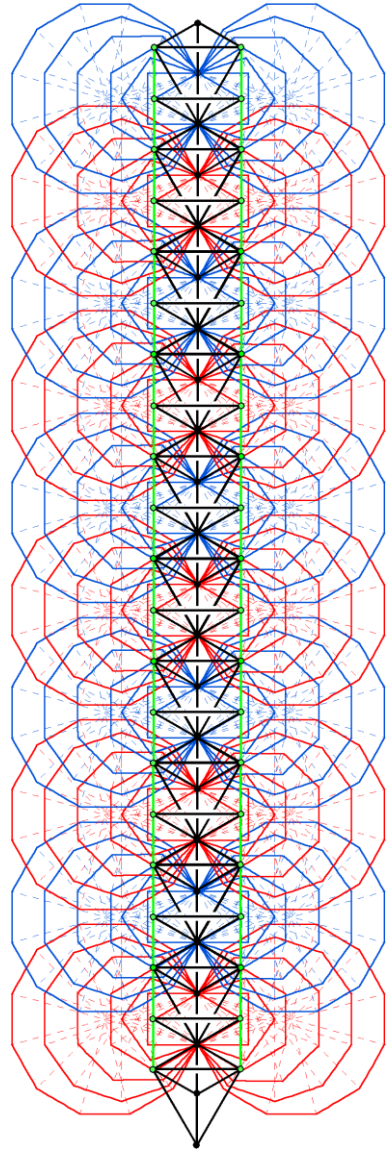
Number of yods surrounding the centre of the Type B n-gon = **15**n.

Surrounding the centres of the 8 Type B triacontagons are:

4 red triacontagons	**4 blue triacontagons**
(4×30=120) vertices	(4×30=120) vertices
(4×30×14=1680) yods	(4×30×14=1680) yods

(1680+1680=3360) yods surround the centres of the 8 triacontagons

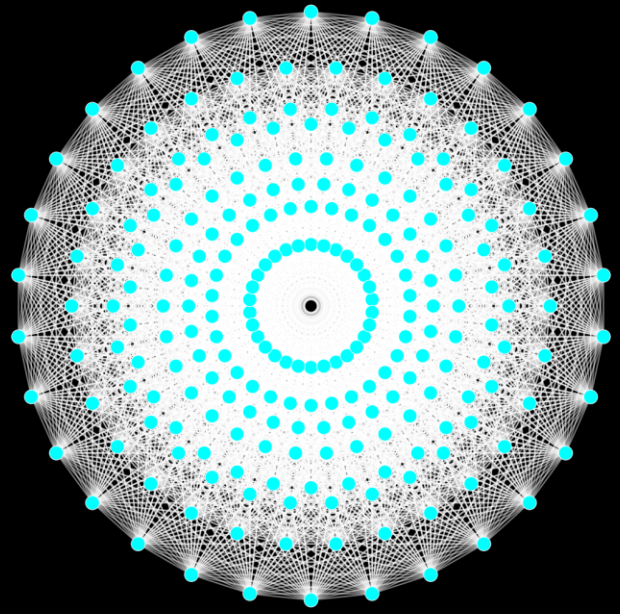
The (7+7) enfolded, Type A polygons have 349 corners, sides & triangles. Number of geometrical elements in the (7n+7n) polygons enfolded in n Trees of Life = 347n + 2. Number of geometrical elements outside the n root edges of these polygons = 344n + 2. The 6 green corners & 4 green sides in the two side pillars of a single Tree of Life are shared with its inner form. Number of shared geometrical elements in n Trees of Life = 8n + 2. Number of geometrical elements in the inner form of n Trees that are unshared with its outer form = 336n. The inner form of 10 Trees has 3360 *unshared* geometrical elements (1680 in each set of 70 polygons).



The inner & outer forms of 10 Trees of Life

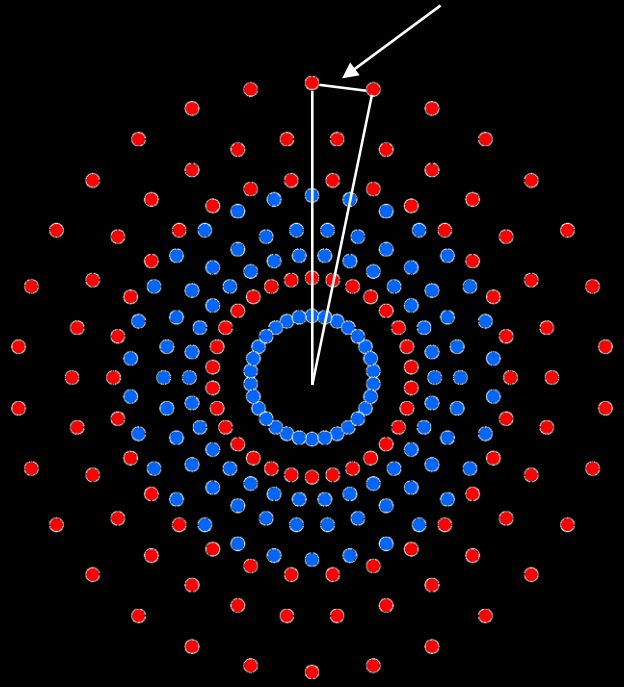
A Type B triangle has 7 corners, **15** sides & 9 triangles, i.e., **31** geometrical elements. **31** is the number value of EL (“God”), the Godname of Chesed. A triacontagon has 30 sectors with **31** corners.

A Type C polygon has 14 sides of 9 triangles per Type B sector. The 7 separate polygons of the inner Tree of Life have **48** sectors. The 7 separate Type C polygons have (**48**×9=432) triangles with (**48**×14=672) sides. The 70 separate Type C polygons have 4320 triangles with 6720 sides.

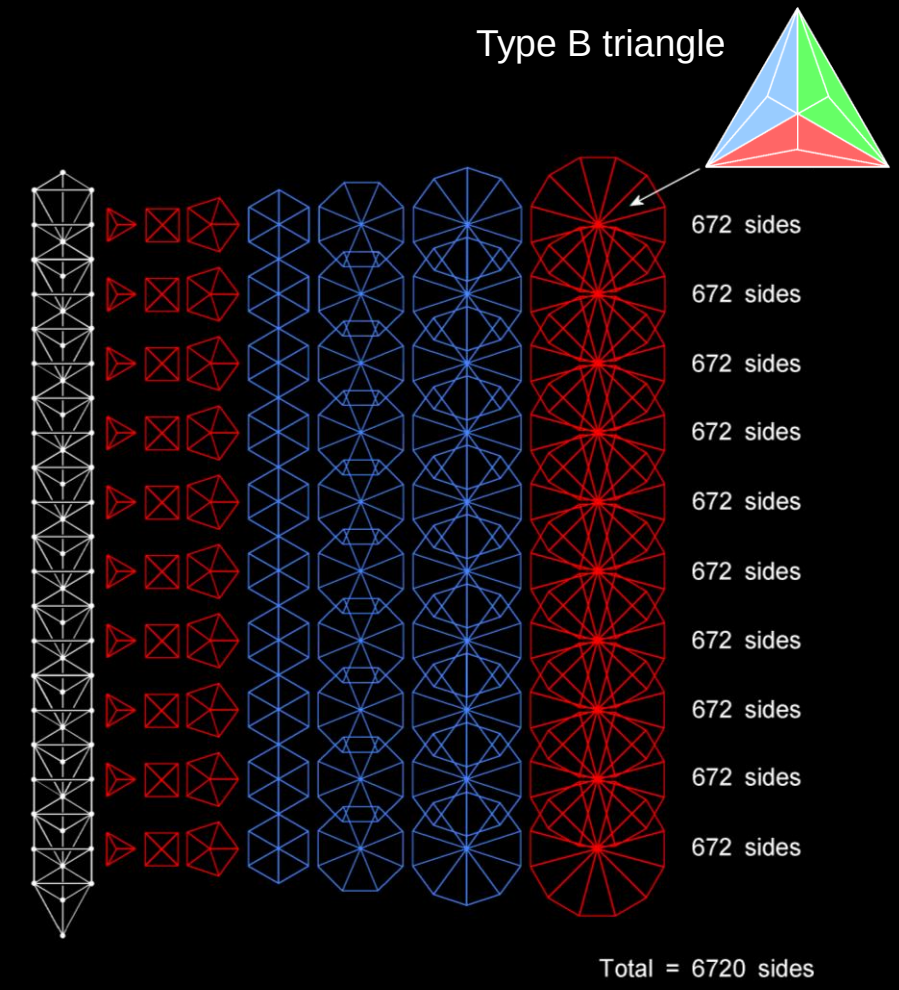


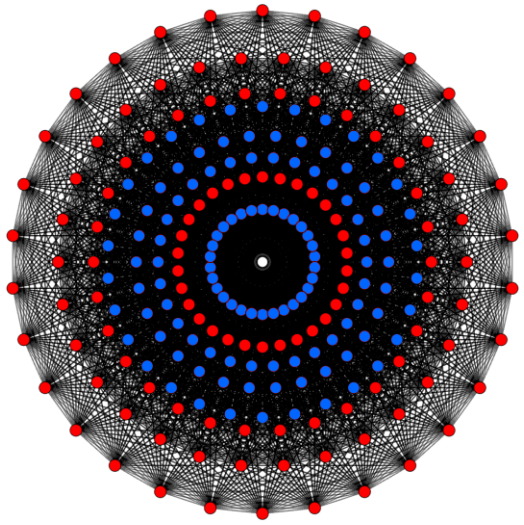
The 4_{21} polytope has 6720 edges

The 4_{21} polytope with the symmetry of E_8 has as many edges (6720) as there are geometrical elements in the 8 Type C triacontagons and sides of triangles in the 70 Type C polygons making up half of the inner form of 10 Trees of Life. The 70 polygons in the other half correspond to the second E_8 group in $E_8 \times E_8$.

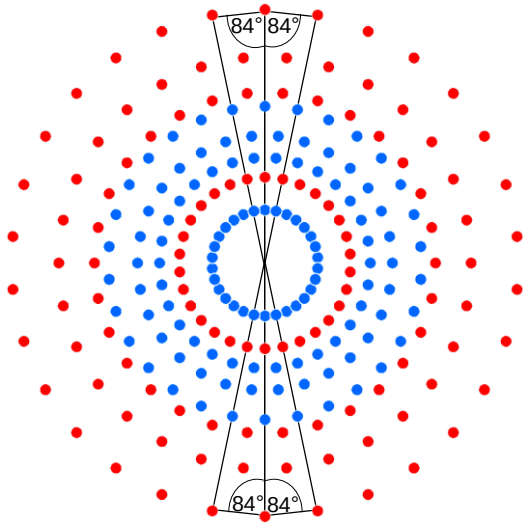


The 8 concentric triacontagons are the Petrie polygons of the 4_{21} polytope. Their 240 vertices consist of the 120 red vertices of a 600-cell and the 120 blue vertices of a smaller 600-cell. Surrounding the centre of these Type C triacontagons are 240 sectors comprising (240×28=6720) geometrical elements. The 120 Type B sectors of the 4 red triacontagons contain 3360 geometrical elements; they correspond to the 3360 sides of triangles in the 40 red polygons in the inner form of 10 Trees of Life. The 120 Type B sectors of the 4 blue triacontagons also contain 3360 geometrical elements; they correspond to the 3360 sides of triangles in the 30 blue polygons.

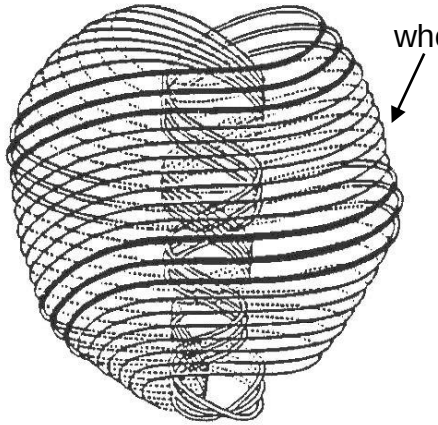




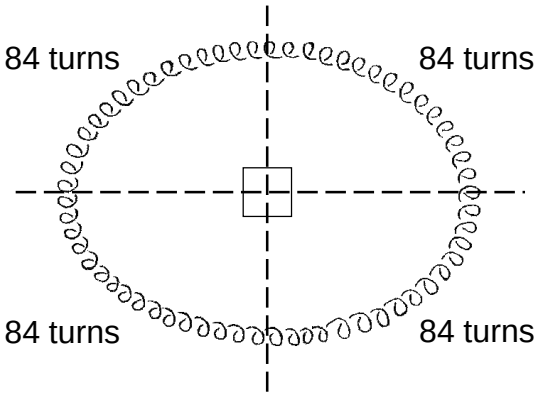
E_8 Coxeter plane projection of the 240 vertices of the 4_{21} polytope. They form the 240 corners of (4+4=8) concentric triacontagons.



The interior angle of a triacontagon is 168°. Each base angle of an isosceles sector is 84°.



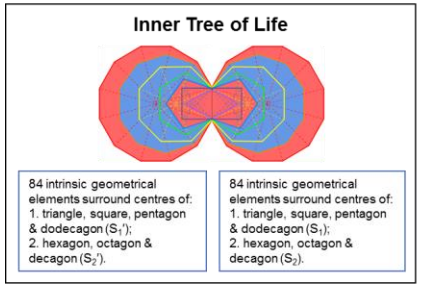
Each of the 10 helical whorls of the UPA revolves 5 times around its axis and contains 1680 circular turns.



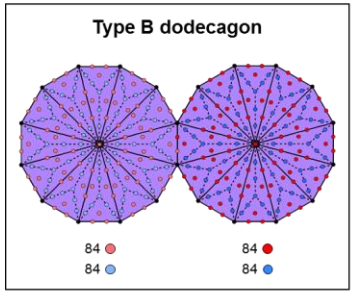
Each ¼-revolution of a whorl contains 84 circular turns.

How 8 sacred geometries embody the 84:84:84:84 division

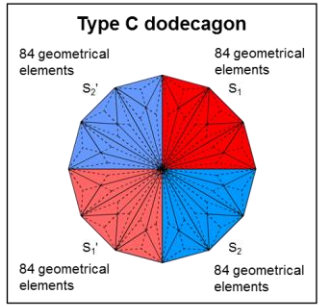
1



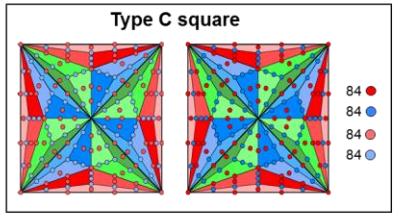
2



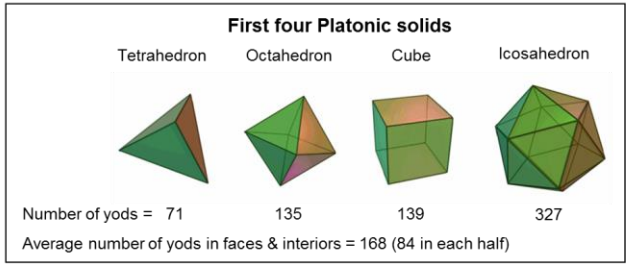
3



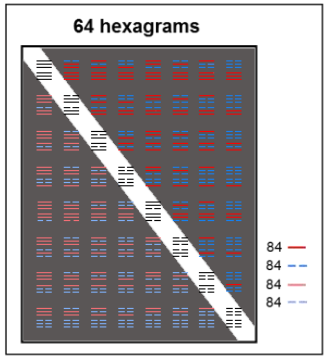
4



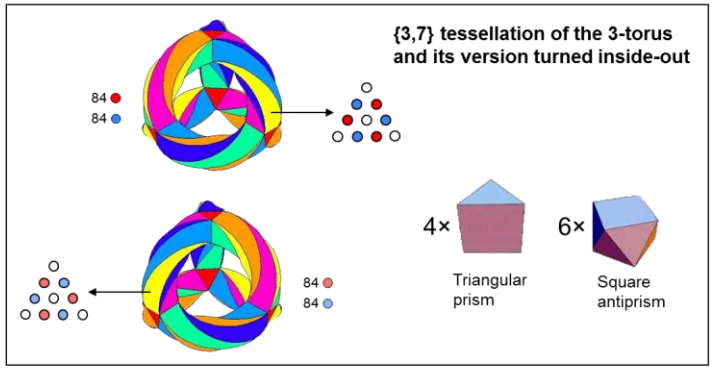
5



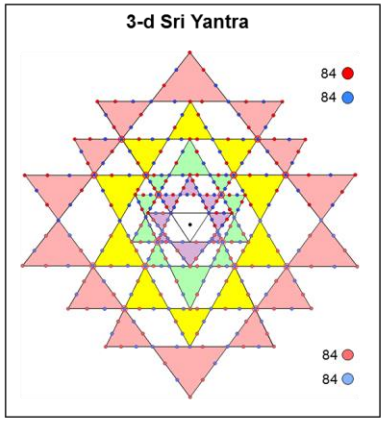
6



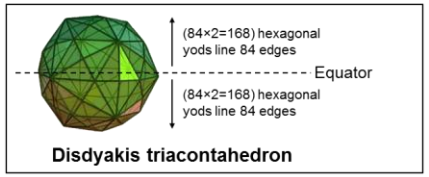
7



8

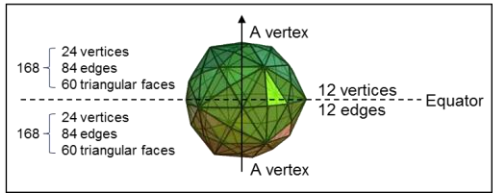


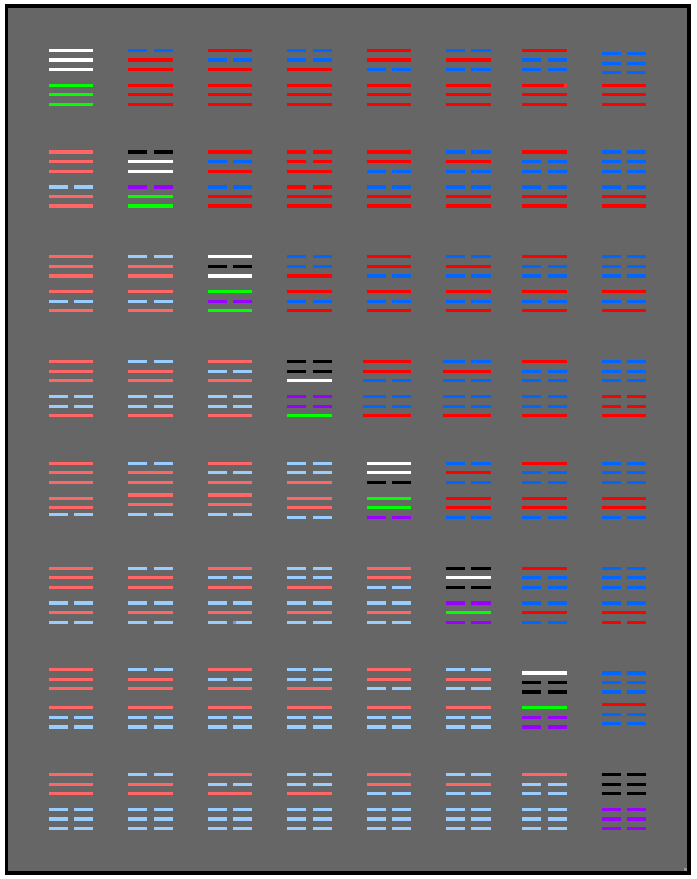
9



See Article 64 for discussion of these sacred geometries.

10





64 hexagrams with 384 lines/broken lines correspond to:
1. pair of sectors & their reflections in one triacontagon with interior & vertex angles adding to 384°:
168+12+12+12+12+**168** = 384.
2. pairs of sectors in 8 triacontagons with (2×2×8=32) vertex angles of 12° adding to 384°:
8×(12+12+12+12) = 384.

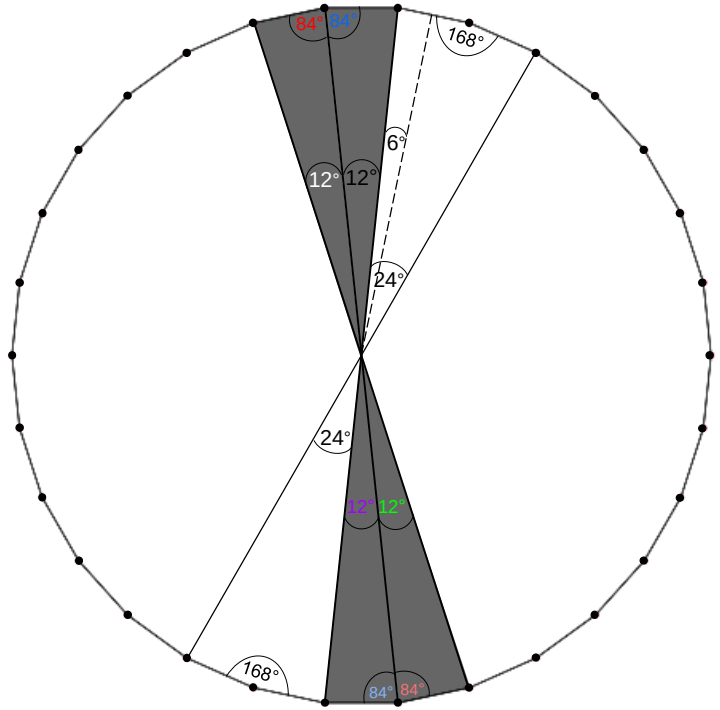
Vertex angles only

A hexagram with 6 lines/broken lines corresponds to a right angled half-sector with a vertex angle of 6°. The 1st 4 hexagrams in a row correspond to 2 adjacent sectors. The 2nd set of 4 hexagrams corresponds to their reflections. 8 hexagrams with (24+24=**48**) lines/broken lines correspond to the 2 adjacent sectors & their reflections with vertex angles adding to **48**°. The 8 rows of 8 hexagrams with (8×**48**=384) lines & broken lines correspond to the 8 sets of 8 half-sectors in the 8 triacontagons with a total vertex angle of 384°. The two sets of 4 triacontagons correspond to the two diagonal halves of the 8×8 array of hexagrams.

Yang	Yin
84 —	84 – –
12 —	12 – –
12 —	12 – –
84 —	84 – –

168 + 24 = 192
24 + **168** = 192
192 + 192 = 384

Total = 192 Total = 192



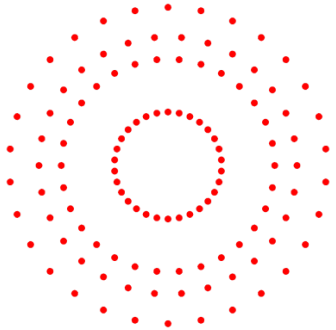
84° → 84 —	84° → 84 – –
12° → 12 —	12° → 12 – –
12° → 12 —	12° → 12 – –
84° → 84 —	84° → 84 – –

Half-sector with 6° vertex angle → Two trigrams with 6 lines/broken lines

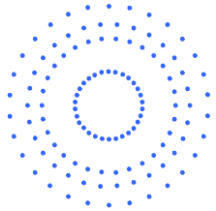
8 half-sectors in 2 pairs of sectors → row of 8 hexagrams
64 half-sectors in 8 triacontagons → **64** hexagrams in 8 rows

The angles in a pair of sectors of the triacontagon and its reflection conform to the

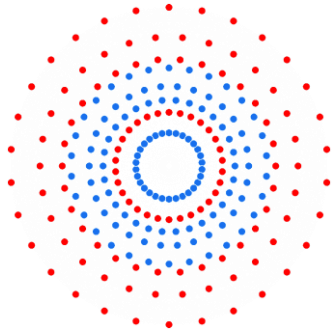
24:168:24:168 pattern of the 192 lines & 192 broken lines of the 64 hexagrams



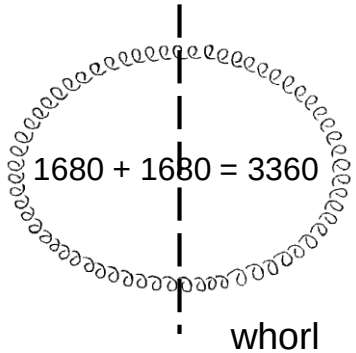
4 triacontagons are the projection in the H_4 Coxeter plane of the 120 vertices of the 600-cell. (4×420=1680) yods other than vertices surround their shared centre when they are Type B.



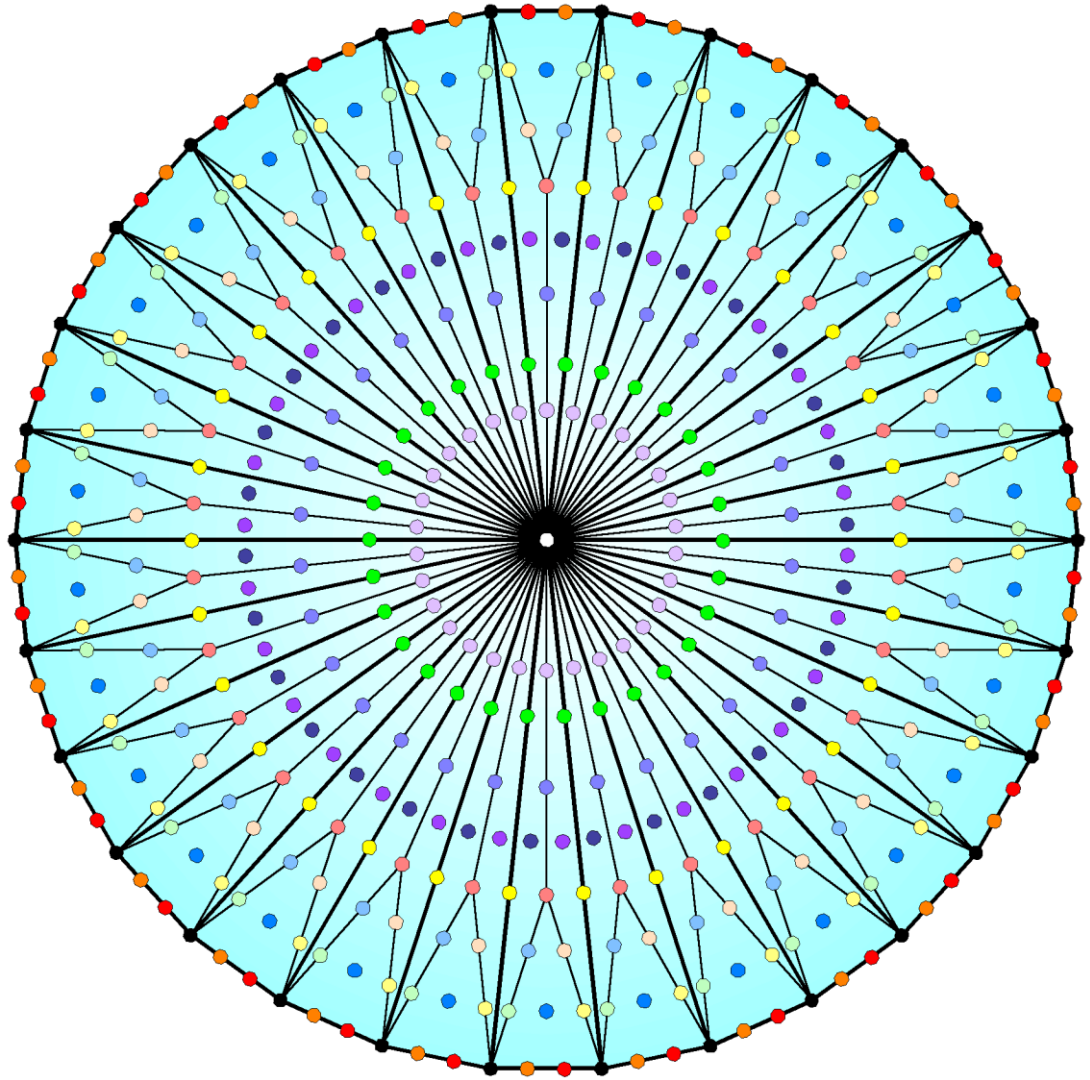
4 triacontagons are the projection in the H_4 Coxeter plane of the 120 vertices of the 600-cell. (4×420=1680) yods other than vertices surround their shared centre when they are Type B.



8 triacontagons are the projection in the E_8 Coxeter plane of the 240 vertices of the 4_{21} polytope. When Type B, (1680+1680=3360) yods other than vertices surround their shared centre.

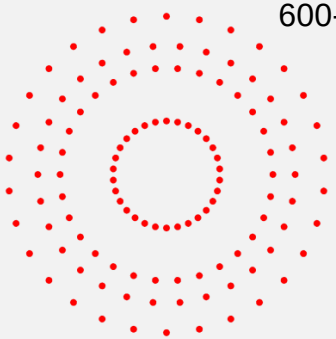


A half-revolution of the 10 helical whorls of the UPA comprises 1680 turns. One revolution comprises 3360 turns.
The 4 Type B triacontagons in each 600-cell correspond to an outer/inner half-revolution of the 10 whorls. The 1680 turns in a *single* whorl are circularly polarised waves composed of 3360 plane-polarised oscillations. The pattern of division of the whole also applies to its parts because they are wholes in themselves.



420 yods other than vertices surround the centre of the Type B triacontagon

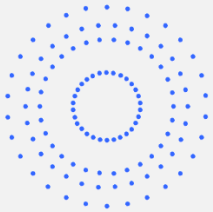
840 turns → 840 corners & sides in 4 Type B triacontagons → 600-cell
1680 turns → 1680 corners & turns in 8 Type B triacontagons → 4_{21} polytope



600-cell

4 triacontagons have:
Type A: 360 corners & sides of 120 sectors;
Type B: 840 additional geometrical elements.

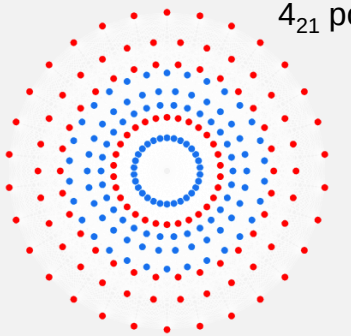
4 Type B triacontagons have 840 corners & sides and 360 triangles.



600-cell

4 triacontagons have:
Type A: 360 corners & sides of 120 sectors;
Type B: 840 additional geometrical elements.

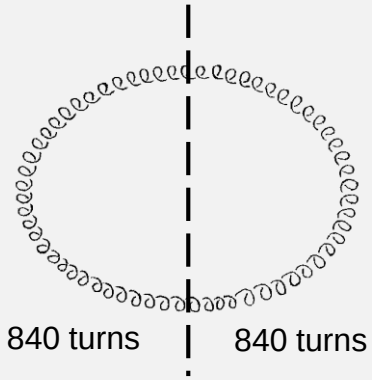
4 Type B triacontagons have 840 corners & sides and 360 triangles.



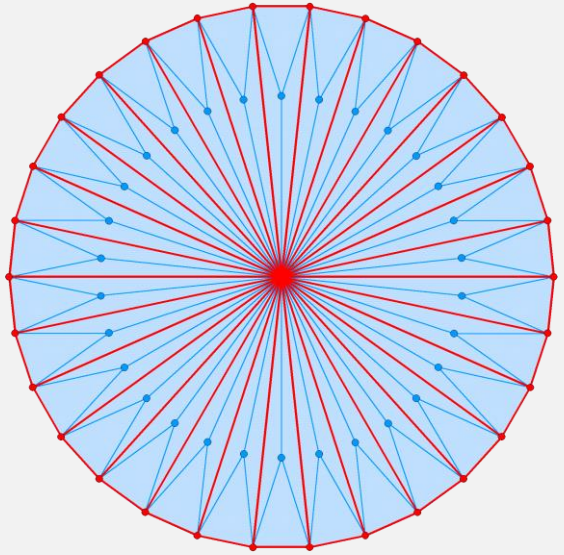
4_{21} polytope

8 triacontagons have:
Type A: 720 corners & sides of 240 sectors.
Type B: (840+840=1680) additional geometrical elements.

8 Type B triacontagons have (840+840=1680) corners & sides and (360+360=720) triangles, i.e., 2400 geometrical elements.



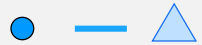
whorl



Type A

90

Type B (additional)

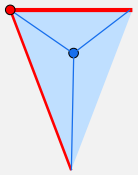


210

Total = 300

A Type A triacontagon has 90 corners & sides and 30 triangles.
A Type B triacontagon has 210 corners & sides and 90 triangles.

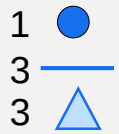
Sector



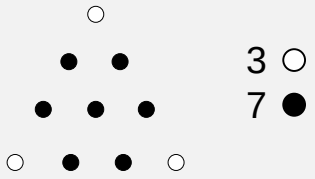
Type A

Total = 3

Type B (additional)



Total = 7



The Type B sector has 3 geometrical elements of the Type A sector & 7 more.

2400 geometrical elements in 240 Type B sectors = 240×(3+7)
= 720 (Type A) + 1680 (additional)
The Type A/Type B distinction generates the 3:7 pattern of the tetractys and the 720:1680 division in triacontagons that is characteristic of holistic systems. It manifests in the 240 roots of E_8 as the **72** roots of E_6 and the remaining **168** roots.