ARTICLE 52

How EHYEH, YAH & YAHWEH Prescribe the Root Structure & Dimension of E_8

by

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Abstract

A systematic analysis of the angles subtended by pairs of simple roots of all Lie groups is carried out. The number value ***21*** *of EHYEH, the Godname of Kether, is the number of pairs of orthogonal roots of* E_8 *and the number of angles between the seven simple roots of* E_7*, its largest exceptional subgroup. The number value* ***15*** *of YAH, the older version of YAHWEH, the Godname of Chokmah, is the number of pairs of orthogonal roots of* E_7 *and the number of angles between the six simple roots of* E_6*, which is the next largest exceptional subgroup of* E_8*. None of the exceptional Lie groups has a dimension that is a Godname number. Allowing dimensions of groups to be 10×Godname number, the Godname numbers* ***21****,* ***15*** *&* ***36*** *define dimensions of subgroups of* E_8*. Only* ***21*** *&* ***15*** *define groups of less rank than* E_8*, all of which are its subgroups. Only* ***21*** *&* ***15*** *define groups of rank less than or equal to that of* E_8 *whose numbers of orthogonal pairs of simple roots are Godname numbers. The letter values of EYHEH & YAH denote the numbers of right angles between sets of simple root vectors of* E_6*,* E_7 *&* E_8*. The integers 1-****15*** *forming a cross pattée array sum to 4960. This is the number of space-time components of the* ***496*** *gauge fields of* $E_8 \times E_8$ *& SO(32), which are the gauge symmetry groups of the heterotic superstring. A cross pattée array of integers 2-****26*** *add up to 24800, showing how YAHWEH determines the dimension* ***248*** *of* E_8*. The Godnames ELOHIM, EL, YAHWEH SABAOTH & ELOHIM SABAOTH also determine this number. ELOHIM with number value* ***50*** *prescribes* ***496*** *because this number is the arithmetic mean of the first* ***50*** *triangular numbers after 3.*

	SEPHIRAH	GODNAME	ARCHANGEL	ORDER OF ANGELS	MUNDANE CHAKRA
1	Kether (Crown) **620**	EHYEH (I am) **21**	Metatron (Angel of the Presence) **314**	Chaioth ha Qadesh (Holy Living Creatures) **833**	Rashith ha Gilgalim First Swirlings. (Primum Mobile) **636**
2	Chokmah (Wisdom) **73**	YAHWEH, YAH (The Lord) **26**, **15**	Raziel (Herald of the Deity) **248**	Auphanim (Wheels) **187**	Masloth (The Sphere of the Zodiac) **140**
3	Binah (Understanding) **67**	ELOHIM (God in multiplicity) **50**	Tzaphkiel (Contemplation of God) **311**	Aralim (Thrones) **282**	Shabathai Rest. (Saturn) **317**
	Daath (Knowledge) **474**				
4	Chesed (Mercy) **72**	EL (God) **31**	Tzadkiel (Benevolence of God) **62**	Chasmalim (Shining Ones) **428**	Tzadekh Righteousness. (Jupiter) **194**
5	Geburah (Severity) **216**	ELOHA (The Almighty) **36**	Samael (Severity of God) **131**	Seraphim (Fiery Serpents) **630**	Madim Vehement Strength. (Mars) **95**
6	Tiphareth (Beauty) **1081**	YAHWEH ELOHIM (God the Creator) **76**	Michael (Like unto God) **101**	Malachim (Kings) **140**	Shemesh The Solar Light. (Sun) **640**
7	Netzach (Victory) **148**	YAHWEH SABAOTH (Lord of Hosts) **129**	Haniel (Grace of God) **97**	Tarshishim or Elohim **1260**	Nogah Glittering Splendour. (Venus) **64**
8	Hod (Glory) **15**	ELOHIM SABAOTH (God of Hosts) **153**	Raphael (Divine Physician) **311**	Beni Elohim (Sons of God) **112**	Kokab The Stellar Light. (Mercury) **48**
9	Yesod (Foundation) **80**	SHADDAI EL CHAI (Almighty Living God) **49**, **363**	Gabriel (Strong Man of God) **246**	Cherubim (The Strong) **272**	Levanah The Lunar Flame. (Moon) **87**
10	Malkuth (Kingdom) **496**	ADONAI MELEKH (The Lord and King) **65**, **155**	Sandalphon (Manifest Messiah) **280**	Ashim (Souls of Fire) **351**	Cholem Yesodoth The Breaker of the Foundations. The Elements. (Earth) **168**

Table 1. Gematria number values of the 10 Sephiroth in the four Worlds.

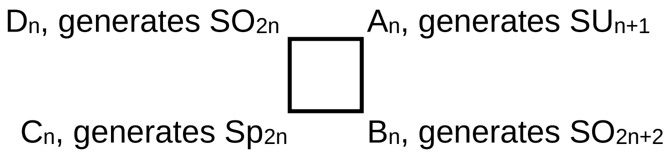
The Sephiroth exist in the four Worlds of Atziluth, Beriah, Yetzirah and Assiyah. Corresponding to them are the Godnames, Archangels, Order of Angels and Mundane Chakras (their physical manifestation, traditionally symbolised by celestial bodies). This table gives their number values obtained by the ancient practice of gematria, wherein a number is assigned to each letter of the alphabet, thereby giving to a word a number value that is the sum of the numbers of its letters.

(All numbers in this table referred to in the article are written in ***boldface****).*

1. Numbers of EHYEH & YAH prescribe E_8

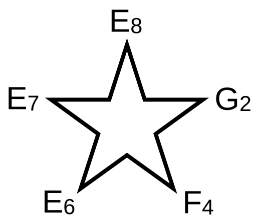
The gematria number values of the ancient Hebrew Godnames have hitherto been found to prescribe the dimensions of E_8 and $E_8 \times E_8$ or SO(32) either arithmetically or in the geometrical context of the yod population of overlapping trees or the yod/corner populations of enfolded polygons generated by the Tree of Life. In this article, we shall demonstrate how the Godname numbers of Kether and Chokmah select E_8 and its two exceptional subgroups E_7 and E_6 from the infinite number of Lie groups. Because of its importance in proving rigorously that Godnames prescribe superstring physics, the following discussion must be technical. It is therefore intended primarily for mathematicians and physicists. For this reason, readers unfamiliar with group theory are advised to turn to the simplified summary on page 10.

The mathematician E. Cartan showed in 1894 that there are four* infinite series of simple Lie algebras:

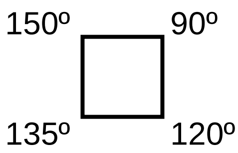


n = 1, 2, 3, 4…

and five "exceptional" Lie algebras:



The structure of a simple Lie algebra or group G is defined completely by a set of (rank G)-dimensional vectors called "simple roots," which span the "root space" of G, a (rank G)-dimensional Euclidean space. The "Dynkin diagram" specifies the set of simple roots α_i (i = 1, 2, 3... rank G) of G. Each simple root is denoted by a dot in 2-d space. In the case of a G having α_i's with two different lengths $|\alpha_i|$, the longer roots are denoted by open dots "○" and the shorter roots by filled-in dots "●" (no simple group has roots with three or more different lengths). There are *four* possible angles θ_{ij} between any pair of simple roots α_i and α_j in root space:

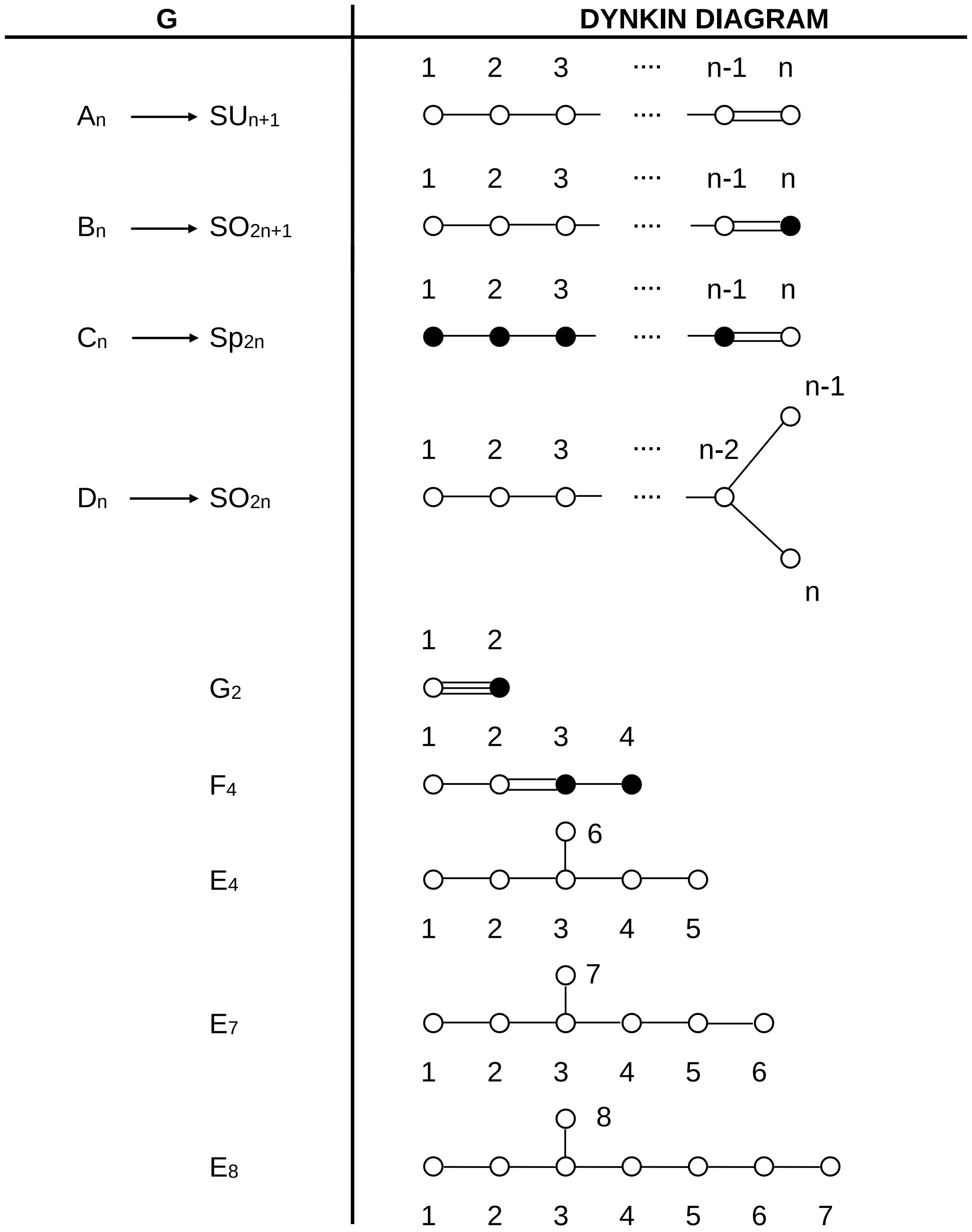


The angle between a pair of simple roots is denoted in the Dynkin diagram of G by lines connecting corresponding dots. The following convention is used:

α_i	α_j	θ_{ij}	$\lvert\alpha_i\rvert / \lvert\alpha_j\rvert$
○	○	90°	no constraint
○—	—○	120°	1
○═	═●	135°	2
○≡	≡●	150°	3

* This is an example of the Tetrad Principle discussed in Article 1.

Shown below is an exhaustive list of the Dynkin diagrams of all simple Lie algebras G:



Suppose that G has N simple roots. There are $\binom{N}{2}$ = ½N(N-1) different pairs of root vectors (α_i, α_j) enclosing between them the angle θ_{ij}. Therefore, the mutual orientation between the N root vectors in root space is specified completely by $\binom{N}{2}$ angles. This number is the sum of the number of different pairs of orthogonal roots unjoined by a line in the Dynkin diagram of G and the number of pairs joined by one or more lines, which enclose any of the angles 120º, 135º or 150º. In the Dynkin diagram of every Lie algebra, there are no isolated roots: every simple root is joined to at least one other.

This means that there are $\binom{N-1}{2}$ =½(N-1) (N-2) right angles and $\binom{N-1}{1}$ = (N-1) angles which are either 120º, 135º, 150º or mixtures thereof [N.B. $\binom{N}{2} = \binom{N-1}{2} + \binom{N-1}{1}$]. Tabulated below is the number of angles enclosed by all pairs of simple roots defining all possible Lie algebras:

G	N	Θ_{ij} = 90º	120º	135º	150º	NUMBER OF ANGLES = $\binom{N}{2}$
SU_{n+1}	n	$\binom{n-1}{2}$ = ½(n-1)(n-2)	n-1	0	0	½n(n-1)
SO_{2n+1}	n	$\binom{n-1}{2}$	n-2	1	0	½n(n-1)
Sp_{2n}	n	$\binom{n-1}{2}$	n-2	1	0	½n(n-1)
SO_{2n}	n	$\binom{n-1}{2}$	n-1	0	0	½n(n-1)
G_2	2	0	0	0	1	1
F_4	4	3	2	1	0	6
E_6	6	10	5	0	0	**15**
E_7	7	**15**	6	0	0	**21**
E_8	8	**21**	7	0	0	28

The Godname number **21** of Kether is the number of pairs of *orthogonal* roots of the superstring symmetry group E_8 and the number of angles between the seven simple roots of E_7. The Godname number **15** of Chokmah is the number of pairs of orthogonal roots of E_7 and the number of angles between the six simple roots of E_6, a subgroup of E_8 favoured by many string theorists as the probable product of symmetry breaking of E_8.

Turning to the non-exceptional groups:

1. for n = 6, number of angles between the simple roots of SU_7; SO_{12}, SO_{13} & Sp_{12} = **15**;
2. for n = 7, number of right angles between the simple roots of SU_8, SO_{15}, SO_{14} & Sp_{14} = **15**;
3. for n = 8, number of right angles between the simple roots of SU_9, SO_{17}, SO_{16} & Sp_{16} = **21**.

Since the superstring symmetry group is E_8, any other groups picked out by the Godnames of Kether or Chokmah must be subgroups of E_8. These are: SU_9, SO_{16} (n = 8), SU_8 (n = 7) & SU_7, SO_{13}, SO_{12} (n = 6).

The dimension d_G of G is the number of its generators. For what Lie algebra is $d_G = N_{GOD}$, where N_{GOD} is a Godname number? The Lie algebras have the following dimensions:

ALGEBRA	**d_G**
A_n	n(n+2)
B_n	n(2n+1)
C_n	n(2n+1)
D_n	n(2n-1)
G_2	14
F_4	52
E_6	78
E_7	133
E_8	**248**

None of the exceptional Lie algebras has a dimension that is a Godname number. Below are tabulated all values of n covering the range **15**≤N_{GOD}≤**543**, where **15** is the smallest and **543** is the largest Godname number:

n	**$A_n \rightarrow SU_{n+1}$ d_G = n(n+2)**	**$B_n \rightarrow SO_{2n+1}$, $C_n \rightarrow Sp_{2n}$ n(2n+1)**	**$D_n \rightarrow SO_{2n}$ n(2n-1)**
1	3	3	1
2	8	10	6
3	**15**	**21**	**15**
4	24	**36**	28
5	35	55	45
6	**48**	78	66
7	63	105	91
8	**80**	136	120
9	99	171	**153**
10	120	210	190
11	143	253	231
12	**168**	300	276
13	195	**351**	325
14	224	406	378

15	255	465	435
16	288	52	**496**
17	323		
18	360		
19	399		
20	440		
21	483		
22	528		

n = 16 determines the anomaly-free group SO_{32} with dimension **496**. It is amusing that the dimension of SU_{n+1} exceeds the largest Godname number **543** for n>22, the number of Paths in the Tree of Life. B_3, (SO_7) and C_3 (Sp_6) have dimension **21**, A_3 (SU_4) and D_3 (SO_6) have dimension **15**, B_4 (SO_9) and C_4 (Sp_8) have dimension **36** and D_9 (SO_{18}) has dimension **153**. Allowing the further kind of prescription:

$$
\begin{array}{cccc}
 & & N_{GOD} & \\
 & N_{GOD} & N_{GOD} & \\
d_G = & N_{GOD} & N_{GOD} & N_{GOD} \\
N_{GOD} & N_{GOD} & N_{GOD} & N_{GOD},
\end{array}
$$

i.e., **15**≤d_G≤5430, then B_{10} ($SO_{\mathbf{21}}$) and C_{10} (Sp_{20}) have the dimension:

$$
\begin{array}{cccc}
 & & \mathbf{21} & \\
 & \mathbf{21} & \mathbf{21} & \\
d_G = & \mathbf{21} & \mathbf{21} & \mathbf{21} \\
\mathbf{21} & \mathbf{21} & \mathbf{21} & \mathbf{21}
\end{array}
$$

and SU_{19} has the dimension:

$$
\begin{array}{cccc}
 & & \mathbf{36} & \\
 & \mathbf{36} & \mathbf{36} & \\
d_G = & \mathbf{36} & \mathbf{36} & \mathbf{36} \\
\mathbf{36} & \mathbf{36} & \mathbf{36} & \mathbf{36}
\end{array}
$$

(there are no other possibilities). The groups with rank less than or equal to the rank 8 of the superstring group E_8 are SO_7 and Sp_6 (with dimension **21**), SU_4 and SO_6 (with dimension **15**), and SO_9 and Sp_8 (with dimension **36**). Since

$$E_8 \supset SO_{16} \supset SO_9 \supset SO_7 \supset SU_4,\ E_8 \supset E_6 \supset Sp_6 \supset SU_2 \times Sp_6,\ \text{and}\ SU_4 \approx SO_6,$$

the Godname numbers **21**, **15** and **36** define dimensions of subgroups of E_8.

For what Lie algebras is the total number of angles between pairs of root vectors equal to N_{GOD}, and what are their dimensions? Below are tabulated values of n determining N_{GOD} and the corresponding dimension of the Lie algebra:

N_{GOD} d_G =	n	SU_{n+1} n(n+2)	SO_{2n+1}, Sp_{2n} n(2n+1)	SO_{2n} n(2n-1)	G_2	F_4	E_6	E_7	E_8
21	7	63	105	91	–	–	–	133	–
15 (**26**)	6 (–)	**48** (–)	78 (–)	66 (–)	–	–	78	–	–
50	–	–	–	–	–	–	–	–	–
31	–	–	–	–	–	–	–	–	–
36	9	99	171	**153**	–	–	–	–	–
76	–	–	–	–	–	–	–	–	–
129	–	–	–	–	–	–	–	–	–
153	18	360	666	**630**	–	–	–	–	–
49 (**363**)	– (–)	– (–)	– (–)	– (–)	–	–	–	–	–
65 (**155**)	– (–)	– (–)	– (–)	– (–)	–	–	–	–	–

Godnames numbers of four Sephiroth each define four, non-exceptional Lie algebras:

n = 7: $\binom{7}{2}$ = **21**

SO_{14}(91) SU_8(63)
Sp_{14}(105) $SO_{\mathbf{15}}$(105)

n = 6: $\binom{6}{2}$ = **15**

SO_{12}(66) SU_7(**48**)
Sp_{12}(78) SO_{13}(78)

(number in brackets is dimension of Lie algebra)

n = 9: $\binom{9}{2}$ = **36**

SO_{18}(**153**) SU_{10}(99)
Sp_{18}(171) SO_{19}(171)

n = 18: $\binom{18}{2}$ = **153**

$SO_{\mathbf{36}}$(**630**) SU_{19}(360)
$Sp_{\mathbf{36}}$(666) SO_{37}(666)

21 also defines E_7 and **15** defines E_6. The Lie algebras corresponding to **153** have rank 18, which exceeds the rank 8 of E_8 and the rank 16 of $E_8 \times E_8$ and SO_{32}. They are therefore disallowed. The groups corresponding to **36** are also forbidden because they have rank 9. Only the Godname numbers **21** and **15** define groups of lesser rank than E_8, all of which are its subgroups.

For what Lie algebras of rank N is the number of right angles between pairs of simple roots $\binom{N-1}{2}$ = N_{GOD}, and what are their dimensions? Below are tabulated values of n determining N_{GOD} and the corresponding dimension d_G of the Lie algebra:

N_{GOD} **d_G =**	**n**	**SU_{n+1}** **n(n+2)**	**SO_{2n+1}, Sp_{2n}** **n(2n+1)**	**SO_{2n}** **n(2n-1)**	**G_2**	**F_4**	**E_6**	**E_7**	**E_8**
21	8	**80**	136	120	–	–	–	–	**248**
15 (**26**)	7 (–)	63 (–)	105 (–)	91 (–)	–	–	–	133	–
50	–	–	–	–	–	–	–	–	–
31	–	–	–	–	–	–	–	–	–
36	10	120	210	190	–	–	–	–	–
76	–	–	–	–	–	–	–	–	–
129	–	–	–	–	–	–	–	–	–
153	19	399	741	703	–	–	–	–	–
49 (**363**)	– (–)	– (–)	– (–)	– (–)	–	–	–	–	–
65 (**155**)	– (–)	– (–)	– (–)	– (–)	–	–	–	–	–

The Godname numbers of four Sephiroth (the same as above) define four non-exceptional Lie algebras:

n = 8: $\binom{7}{2}$ = **21**

SO_{16}(120) SU_9(**80**)
Sp_{16}(136) SO_{17}(136)

n = 7: $\binom{6}{2}$ = **15**

SO_{14}(91) SU_8(63)
Sp_{14}(105) SO_{15}(105)

(number in brackets is dimension of Lie algebra)

n = 10: $\binom{9}{2}$ = **36**

SO_{20}(190) SU_{11}(120)
Sp_{20}(210) $SO_{\mathbf{21}}$(210)

n = 19: $\binom{18}{2}$ = **153**

SO_{38}(703) SU_{20}(399)
Sp_{38}(741) SO_{39}(741)

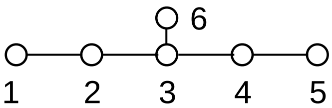
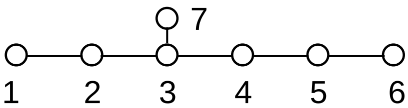
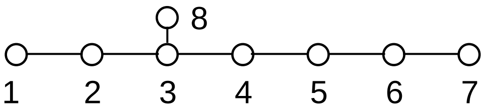
Comments

1. **21** also defines E_8 and **15** defines E_7, as found earlier. SO_{17} is excluded because SO_{16} is a maximal subgroup of E_8. Since E_8 contains the others as subgroups, the group selected by **21** with the largest dimension is E_8.
2. **15** selects the rank-7 groups SU_8, SO_{15}, Sp_{14}, SO_{14} and E_7.
3. **36** selects only rank-10 groups, which are forbidden, having higher rank than E_8.

Similarly, the rank-19 groups selected by **153** are disallowed.

It is concluded that only **21** and **15** define groups of rank less than or equal to that of E_8 whose numbers of orthogonal pairs of simple roots are equal to Godname numbers.

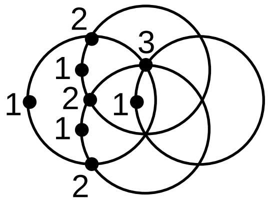
We shall now prove that the Godnames EHYEH and YAH have letter values which denote the numbers of right angles between sets of simple root vectors of E_6, E_7 and E_8. Below are tabulated the Dynkin diagrams of these groups, their simple roots and the labelled simple roots that are perpendicular to them:

GROUP	DYNKIN DIAGRAM	ROOT	LOWER NUMBERED ORTHOGONAL ROOTS	
E_6:		6	A = (5,4,2,1)	10 ≡ Y
		5 4 3	B = {3,2,1 / 2,1 / 1}	
E_7:		7 6	C = (6,A) D = (4,3,2,1)	5 ≡ H
		5 4 3	B	TOTAL = 10 + 5 ≡ YH
E_8:		8	{7 / C}	1 ≡ A 5 ≡ H
		7	(5,4,3,2,1)	5 ≡ H
		6 5 4 3	D + B	10 ≡ I
				TOTAL = 1 + 5 + 10 + 5 ≡ AHIH

Notice that:

1. The number **15** of YAH (Hebrew: YH) is the number of right angles between the seven simple roots of E_7. The value 10 of the letter Y is the number of right angles between the root vectors of E_6 and the value 5 of letter H is the *extra* number of right angles between the simple root vectors of E_7.
2. The number **21** of EHYEH (Hebrew: AHIH) is the number of right angles between the eight simple root vectors of E_8. The value 1 of the letter A denotes the right angle between roots 8 & 7, the value 5 of the first letter H is the number of right angles between simple root 8 and simple roots 1, 2, 4, 5 & 6, and the value 5 of the second letter H is the number of right angles between simple root 7 and roots 1, 2, 3, 4 & 5. Comparing AHIH with YH and remembering that the Hebrew letters for I and Y are the same, we see that AHIH specifies the superstring symmetry group E_8, YH specifies E_7 and I (or Y) specifies E_6.

According to the Dynkin diagram of E_8, any simple root vector is orientated at an angle of 120º to one, two or three other simple root vectors. Compare the eight simple root vectors of E_8 with one of the sets of eight generators of the (7+7) polygons enfolded in the Tree of Life:



Each generator lies on one, two or three circles indicated by the integers 1, 2, & 3 above as the endpoint of a vertical or horizontal diameter. The simple root vector 3 is unique in being orientated at 120º to three other simple root vectors. Similarly, the generators located at Daath or Tiphareth in the Tree of Life are unique in being the points of intersection of three circles. The following correspondence emerges between the (7+1) simple root vectors of E_8 and the seven Sephiroth + Daath of the Tree of Life:

ROOT		**SEPHIRAH**
8	⟶	(Daath)
1	⟶	Chesed
2	⟶	Geburah
3	⟶	Tiphareth
4	⟶	Netzach
5	⟶	Hod
6	⟶	Yesod
7	⟶	Malkuth

Notice that the unique root vector 3 corresponds to Tiphareth, which uniquely occupies the centre of the Tree of Life both in a geometrical and in a metaphysical sense. This analogy between the generators of the polygons enfolded in the Tree of Life and the Dynkin diagram of the superstring group E_8 should come as no surprise because, as we have already seen, the group mathematics of superstrings reflects the geometrical properties of the Divine Image, the cosmic paradigm of Creation.

SUMMARY

Physicists use the mathematical language of group theory to describe the symmetries displayed by the four forces known to act between subatomic particles. Postulating that these symmetries exist at every point in space-time requires the existence of so-called gauge bosons. These are quantum particles, the exchange of which between particles generates the force whose symmetry is described by the group in question. The number of gauge bosons associated with a gauge symmetry group describing a given kind of force is its dimension. Superstring theory predicts that the gauge symmetry group that accounts for all the forces (other than gravity) acting between superstrings must have a dimension of **496**. The groups having this dimension are SO(32) and $E_8 \times E_8$, where E_8 is the so-called exceptional group with the largest dimension (**248**). A group is defined by its roots, the number of which is equal to its dimension. Each root can be expressed in terms of a set of so-called 'simple roots,' which are depicted schematically by the Dynkin diagram of the group. The simple roots characterizing a group can be represented by finite straight lines that point in various directions in a mathematical space. The Dynkin diagram specifies both the lengths of these lines and the angles between them. Only four angles are possible: 90º, 120º, 135º & 150º. The number of right angles between the lines in this space representing the simple roots of E_8 is **21**, as is the total number of angles between the lines representing the simple roots of E_7 (a group whose symmetries are part of the larger set of symmetries of E_8). The number of right angles between the lines representing the simple roots of E_7 is **15**, as is the total number of angles between the simple roots of E_6 (a group that belongs to E_8 as well). Hence, the Godname numbers of Kether (**21**) and Chokmah (**15**) prescribe the very gauge symmetry group (and two of its subgroups) that accounts for the unified force between superstrings.

2. YAH & YAHWEH prescribe 248 & 496

The dimension **248** of E_8 and the dimension **496** of $E_8 \times E_8$ and SO(32) are determined in a purely *arithmetic* way by the two Godname numbers of Chokmah: YAH = **15** and YAHWEH = **26**. It is instructive to analyse these arithmetic prescriptions in some detail because their existence is no fortuitous coincidence but, instead, arises from and reflects the beautiful, geometrical properties of the inner form of the Tree of Life.

The sum of the squares of the first **15** integers is

$$1240 = 1^2 + 2^2 + 3^2 + \ldots + \mathbf{15}^2.$$

This is a triangular array of integers 1–**15**. Therefore, 2480 (=2×1240) is the sum of a 16×16 square array of these integers shown in Fig. 1 with a diagonal of zeros

2480 =

1	2	3	4	5	6	7	8	9	10	11	12	13	14	**15**	0
2	3	4	5	6	7	8	9	10	11	12	13	14	**15**	0	**15**
3	4	5	6	7	8	9	10	11	12	13	14	**15**	0	15	14
4	5	6	7	8	9	10	11	12	13	14	**15**	0	**15**	14	13
5	6	7	8	9	10	11	12	13	14	**15**	0	**15**	14	13	12
6	7	8	9	10	11	12	13	14	**15**	0	**15**	14	13	12	11
7	8	9	10	11	12	13	14	**15**	0	**15**	14	13	12	11	10
8	9	10	11	12	13	14	**15**	0	**15**	14	13	12	11	10	9
9	10	11	12	13	14	**15**	0	**15**	14	13	12	11	10	9	8
10	11	12	13	14	**15**	0	**15**	14	13	12	11	10	9	8	7
11	12	13	14	**15**	0	**15**	14	13	12	11	10	9	8	7	6
12	13	14	**15**	0	**15**	14	13	12	11	10	9	8	7	6	5
13	14	**15**	0	**15**	14	13	12	11	10	9	8	7	6	5	4
14	**15**	0	**15**	14	13	12	11	10	9	8	7	6	5	4	3
15	0	**15**	14	13	12	11	10	9	8	7	6	5	4	3	2
0	**15**	14	13	12	11	10	9	8	7	6	5	4	3	2	1

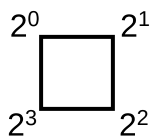
Fig. 1

separating the two triangular arrays. It contains 256 (=4^4) integers comprising a diagonal row of 16 (=4^2) zeros and

$$4^4 - 4^2 = 240 = \begin{array}{c} 24 \\ 24 \quad 24 \\ 24 \quad 24 \quad 24 \\ 24 \quad 24 \quad 24 \quad 24 \end{array} \qquad (24 = 1\times2\times3\times4)$$

non-zero integers 1, 2, 3... **15**, where

15 =



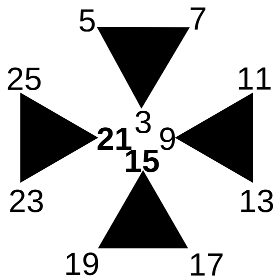
is the number of YAH. Remarkably, the square representation of the dimension of E_8 also encodes as non-zero integers its number of non-zero roots. The Pythagorean Tetrad prescribes this arithmetic representation of the superstring parameter **248**.

The identity:

$$4960 = 2\times 2480 = 4(1^2 + 2^2 + 3^2 + \ldots + \mathbf{15}^2)$$

is represented in Fig. 2 by a cross pattée array of integers 1, 2, 3... **15**. The cross is made up of 480 integers, of which

168 =



integers form its *boundary*. Once again, the number **168** of the Mundane Chakra of

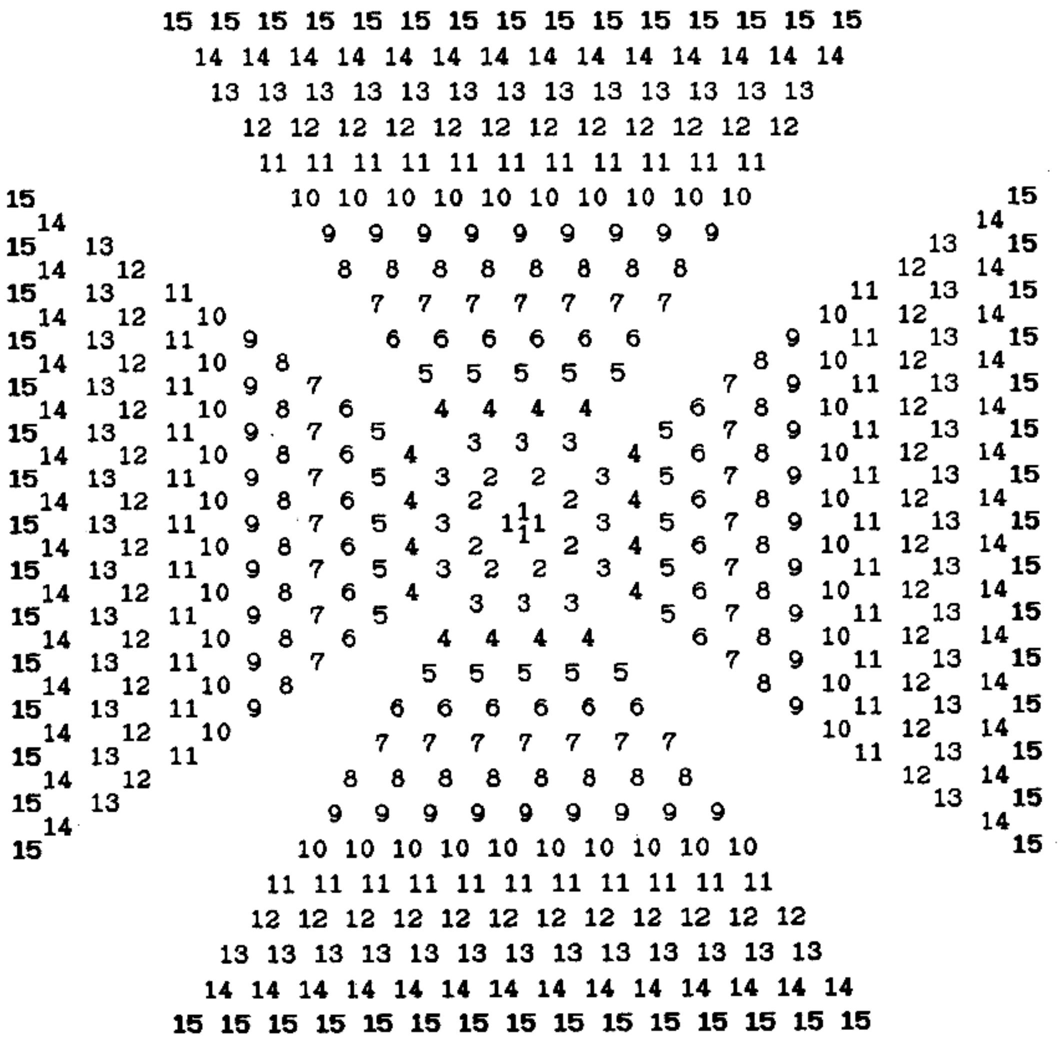


Fig. 2. Integers 1-**15** in a cross pattée array sum to 4960.

4960 =

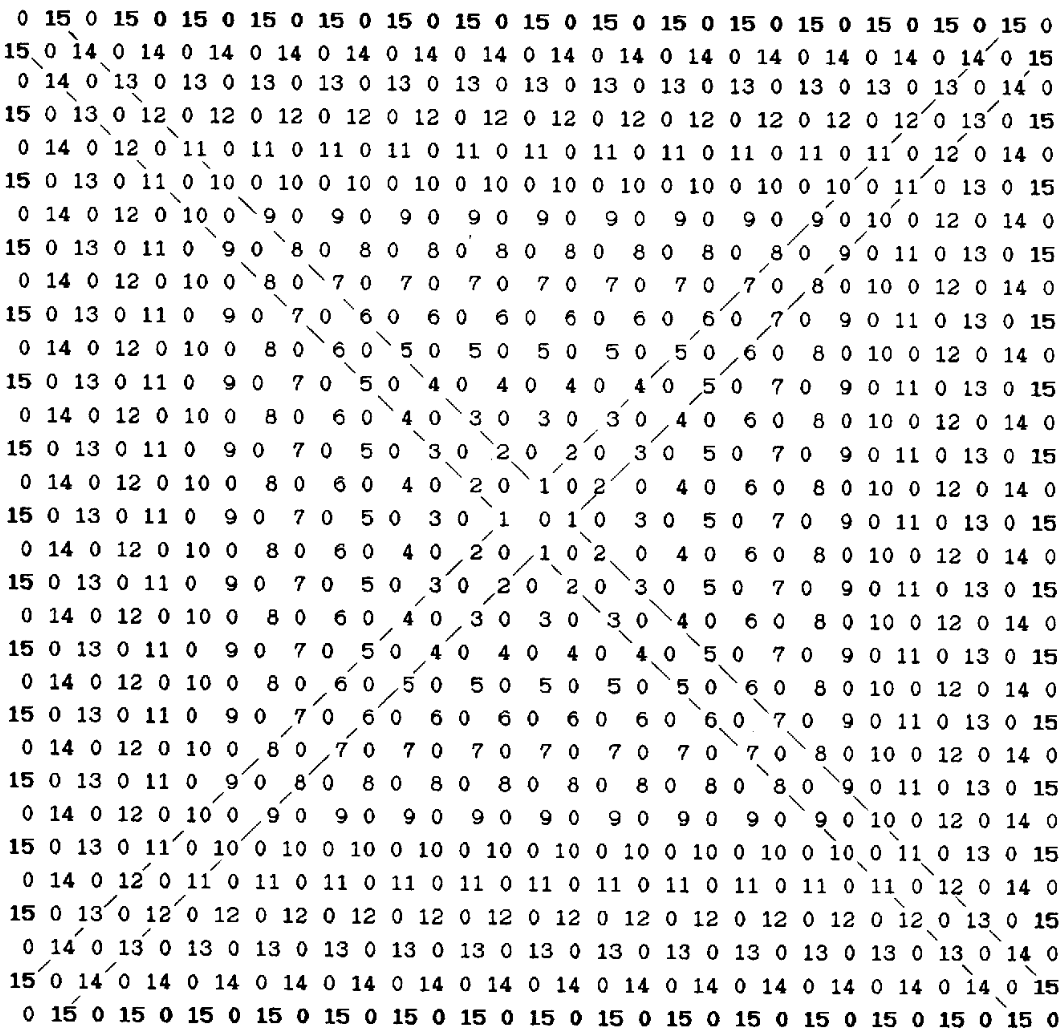


Fig. 3. Integers 1-**15** in a St. Andrews cross array sum to 4960.

Malkuth appears in the context of the representation of the dimension of the superstring symmetry groups $E_8 \times E_8$ and SO(32) with dimension **496**. This cross pattée representation of 4960 — the number of space-time components of the **496** gauge particles predicted by superstring theory — encodes the number 480 of non-zero roots of this group as the number of non-zero integers summing to 4960. Alternatively, a **31**×**31** square array of integers 1-**15** in the form of a St. Andrews cross represents the number 4960 (Fig. 3). This demonstrates how the Godname EL with number value **31** prescribes the number of components of the gauge fields of superstrings.

The number of YAHWEH defines the dimension **248** of E_8 in the following way: using the identity

$$6201 = 1^2 + 2^2 + 3^2 + \ldots + \mathbf{26}^2,$$

then

$$24800 = 4\times 6200 = 4(2^2 + 3^2 + \ldots + \mathbf{26}^2).$$

Fig. 4 shows the cross pattée representation of this number. 1400 integers are present. The sum of the 296 integers forming the boundary of the cross is

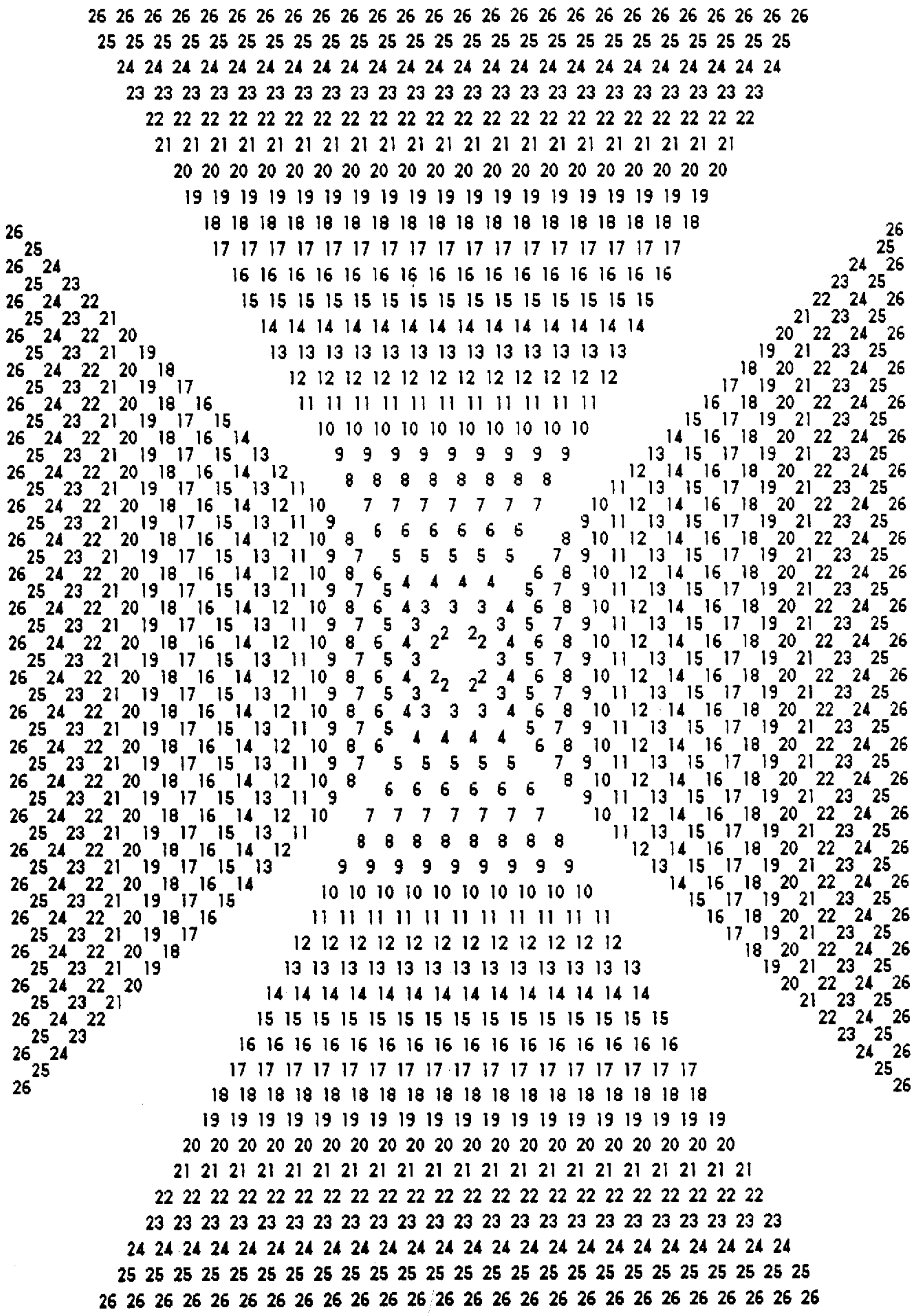


Fig. 4. Integers 2–**26** in a cross pattée array sum to 24800.

$$5246 = \begin{array}{cccc} 331 & 331 & 331 & 331 \\ 331 & 331 & 331 & 331 \\ 331 & 331 & 331 & 331 \\ 331 & 331 & 331 & 331, \end{array}$$

where 331 is the **67**th prime number, **67** being the number value of Binah. Observe how, through this 4×4 square array representation, the Pythagorean Tetrad reveals that the number of Binah — the Sephirah embodying the most abstract archetypes of form — defines the *shape* of an archetypal pattern of numbers whose sum characterizes the physics of superstrings! Since 24800 is the sum of 1400 integers, 49600 is the sum of 2800 integers, where

$$2800 = \begin{array}{c} \mathbf{280} \\ \mathbf{280}\ \ \mathbf{280} \\ \mathbf{280}\ \ \mathbf{280}\ \ \mathbf{280} \\ \mathbf{280}\ \ \mathbf{280}\ \ \mathbf{280}\ \ \mathbf{280}, \end{array}$$

thus relating the second perfect number 28 to the third perfect number **496** as well as the number **496** of Malkuth to the number value **280** of *Sandalphon*, its Archangel. All these representations of **248** and **496** in terms of the two Godname numbers of Chokmah have a *four*-fold, rotational symmetry: their appearance is unaltered by a rotation of 90º. They illustrate the fundamental importance of the Pythagorean Tetrad in expressing parameters of the Tree of Life.

3. Godnames of Hod & Netzach prescribe 248 & 496

It should not be supposed that only two of the ten Godname numbers prescribe the dimensions of the superstring gauge symmetry groups E_8, SO(32) and $E_8 \times E_8$. The Godnames of *all* the Sephiroth participate in this prescription, their specificity increasing, the lower the position of the Sephirah in the Tree of Life. As an example, we shall now consider how the numbers of the Godnames YAHWEH SABAOTH of Netzach and ELOHIM SABAOTH of Hod conspire to determine the numbers **248** and **496**.
The identity

$$24800 = \sum_{n=2}^{\mathbf{26}} (2n)^2 = 4^2 + 6^2 + 8^2 + \ldots + 52^2$$

can be written as follows in terms of the odd integers composing each square number in this summation:

$4^2 = 1 + 3 + 5 + 7.$
$6^2 = 1 + 3 + 5 + 7 + 9 + 11.$
$8^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + \mathbf{15}.$
•
•
•
$52^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + \mathbf{15} + \ldots + \mathbf{101} + \mathbf{103}.$

Adding,

$$24800 = 25(1+3+5+7) + 24(9+11) + 23(13+\mathbf{15}) + \ldots + 1(\mathbf{101}+\mathbf{103}).$$

The number 24800 is the sum of a stack of **26** pairs of odd integers, the base of which comprises 25 ‘1's and the apex of which is the number **103**. Fig. 5 shows two similar

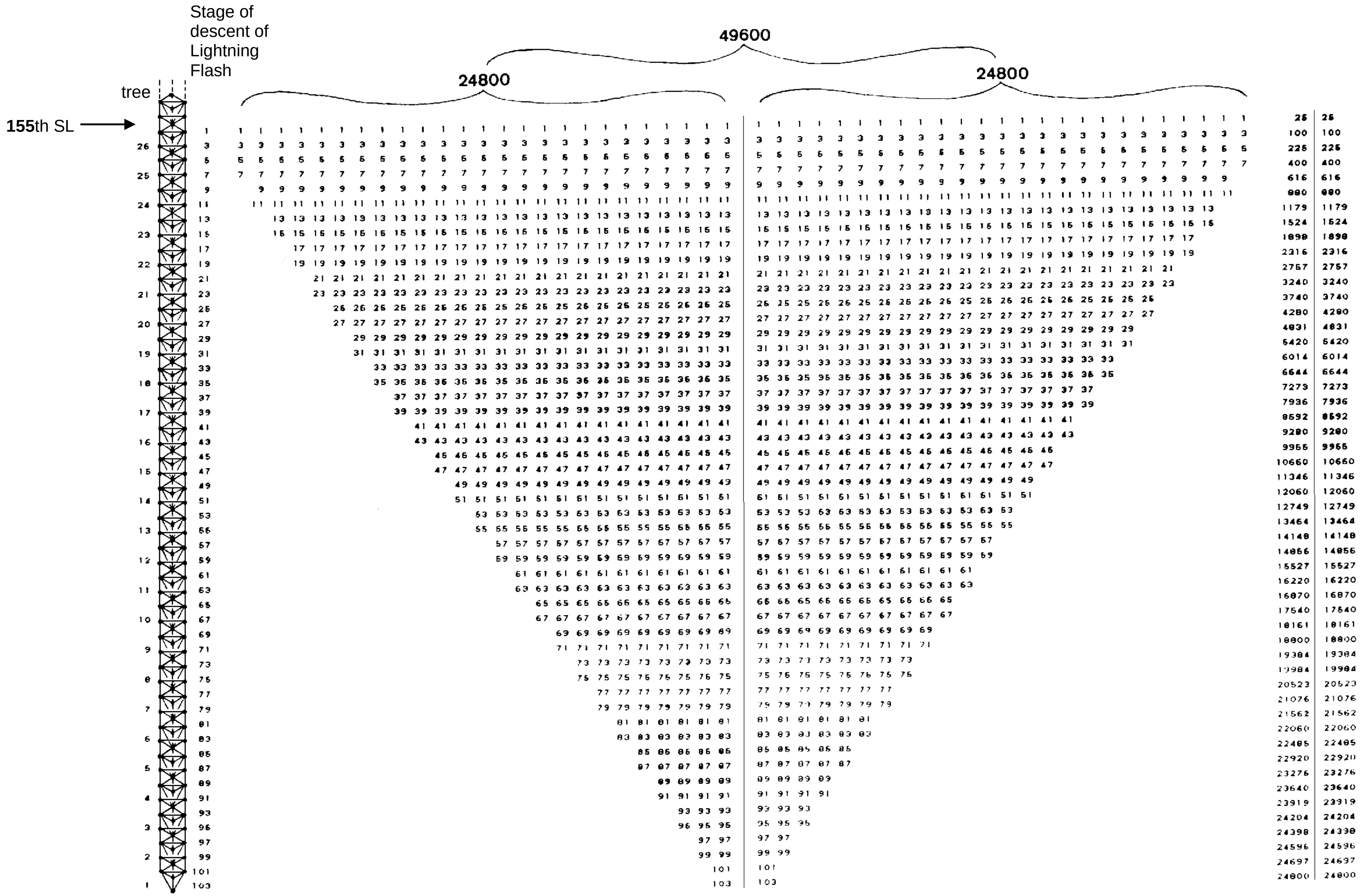

Stage of descent of Lightning Flash
tree
155th SL
49600
24800
24800

Figure 5

stacks placed side by side to represent the number 49600. The reason why they are depicted as inverted will be given shortly. The base of the pair of stacks consists of **50** ‘1's. The height and width of the pair of stacks is prescribed by the numbers of, respectively, YAHWEH and ELOHIM. The range of integers is specified by the number **103**, which is the number of SABAOTH ("Hosts"). Each stack is prescribed by YAHWEH SABAOTH, whilst both stacks are prescribed by ELOHIM SABAOTH. This is how these two Godnames define patterns of integers adding up to 24800 and 49600. Ignoring the factor of 100, which merely reflects the 10-dimensional nature of superstring space-time, we see that the number **496** naturally splits up into two **248**s, reproducing what mathematicians call the "direct product" of two similar E_8 groups with dimension **248**. These Godnames prescribe in an arithmetic way the $E_8 \times E_8$ heterotic superstring.

The number of yods in the n-tree (the lowest n Trees of Life) with all their triangles turned into tetractyses is given by

$$Y(n) = \mathbf{50}n + 30.$$

The **49**-tree represents what Theosophists call the ‘cosmic physical plane,’ each Tree mapping one of its **49** subplanes. It has Y(**49**) = 2480 yods. This is the number of space-time components of the **248** 10-dimensional gauge fields of E_8. It can be expressed as the sum

$$2480 = \sum_{n=0}^{4} t_n + \sum_{n=1}^{4} T_n,$$

where

$$t_n = \begin{matrix} & & & 4^n & & & \\ & & 3^n & & 3^n & & \\ & 2^n & & 2^n & & 2^n & \\ 1^n & & 1^n & & 1^n & & 1^n \end{matrix} \quad \text{and} \quad T_n = \begin{matrix} & & & 1^n & & & \\ & & 2^n & & 2^n & & \\ & 3^n & & 3^n & & 3^n & \\ 4^n & & 4^n & & 4^n & & 4^n. \end{matrix}$$

It is remarkable that the number of integers in each stack summing to 24800 is

$$700 = \sum_{n=1}^{3} (t_n + T_n),$$

which is the number of yods in ten separate Trees of Life whose triangles are tetractyses. As well as illustrating the basic designing role of the Pythagorean Decad, this shows how yods can denote things such as numbers when they collectively define a Tree of Life parameter like the number **248**.

The column of numbers on the right-hand side of Fig. 5 is the running total of the integers in successive rows of either stack, addition commencing from the top row. Three partial sums are multiples of 100. The first two rows sum to 100, the first four rows sum to 400 and the first **36** rows add to 18800, after which a further **72** integers in 16 rows sum to 6000, making a total of 24800. The Godname number **36** of Geburah specifies a point in the summation of rows of integers yielding a partial sum that is a multiple of 100. Being specified by the number of a Godname, this stage has significance vis-à-vis the Tree of Life, as we now explain. There are **103** stages of descent of the Lightning Flash from Kether of the 25th tree, which, as the **155**th SL, is prescribed by the Godname ADONAI MELEKH of Malkuth. The first stage of descent reaches Hod of the **26**th tree, the **153**rd SL specified by ELOHIM SABAOTH with number value **153**. By comparing stages of descent of the Lightning Flash in the **26**-tree shown on the left in Fig. 5 with the integers 1, 3, 5... **103**, it will be seen that, for example, the 9th and 11th stages of descent occur at the Lower Face of the 24th tree, the 13th and **15**th stages occur at the Lower Face of the 23rd tree, etc. In other words, the integers in the paired rows of the stacks are, simply, the numbers of stages of descent of the Lightning Flash in successive trees. The 71st stage of descent, at which the running sum of the first **36** rows of integers is 18800, reaches Chesed of the eighth tree, which is the **49**th SL in the Cosmic Tree of Life. The Godname number of Yesod specifies the row where the partial sum is a multiple of 100 and therefore of possible physical significance. This is why we have considered an inverted stack of integers.

The summation of odd integers given above leading to the number 24800 has a simple interpretation vis-à-vis the **26**-tree that is the counterpart of **26**-dimensional space-time. Noting that: 1 + 3 = 2×2, 5 + 7 = 2×6, 9 + 11 = 2×10, etc., 24800 can be written:

$$24800 = \mathbf{50}\times2 + \mathbf{50}\times6 + \mathbf{48}\times10 + 46\times14 + \ldots + 4\times98 + 2\times102$$

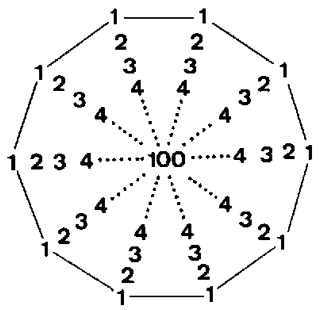
$$= \mathbf{50}\times2 + \sum_{n=1}^{25} 2n(106 - 4n).$$

Yesod of the nth tree is the 2nth SL on the central pillar. The Lightning Flash descends in (4n–3) stages from this SL. So the number of stages of descent from Kether of the 25th tree to this SL = **103** – (4n–3) = 106 – 4n. The sum

$$24700 = \sum_{n=1}^{25} 2n(106 - 4n)$$

is the sum of the numbers of SLs on the central pillar up to Yesod of each successive tree, each number being weighted with the number of stages of descent of the Lightning Flash from the **155**th SL specified by ADONAI MELEKH to that Yesod. The additional term **50**×2 is the product of the number (**50**) of SLs on the central pillar up to Yesod of the 25th tree and the number (2) of stages of descent of the Lightning Flash from the **155**th SL to Yesod of the **26**th (not the 25th) tree. Its difference from the other terms correlates with the fact that the **26**th tree is the counterpart of the dimension of time, whereas lower trees correspond to dimensions of space.

The sum of the (700+700) integers in the two stacks is

49600 = 

$10 = 1 + 2 + 3 + 4$

$100 = 10^2 = 1^3 + 2^3 + 3^3 + 4^3$

This 10-fold array of integers 1–100 is a beautiful illustration of how the Pythagorean Decad and integers 1, 2, 3 & 4 define this superstring number. The sum of the rows of the twenty-five ‘1's and twenty-five ‘3's = 100 = $1^3 + 2^3 + 3^3 + 4^3$. The number of integers 5-**103** in the **50** remaining rows of a stack is

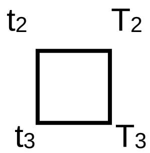
650 =

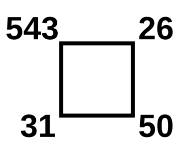
65

65 65

65 65 65

65 65 65 65

= 

= 

where **65** is the number value of ADONAI, the Godname of Malkuth, and **543**, **26**, **50** & **31** are the number values of the Godnames of the first *four* Sephiroth, illustrating once more the defining role of the Tetrad. Their sum is

$$
\begin{array}{cc}
 & 2470 \\
 & 2470\quad 2470 \\
24700 = & 2470\quad 2470\quad 2470 \\
 & 2470\quad 2470\quad 2470\quad 2470,
\end{array}
$$

where

$$2470 = \sum_{n=1}^{4} (t_n + T_n) = 1^2 + 2^2 + 3^2 + \ldots.. + 19^2.$$

The number of integers 5–**103** in the remaining (**50**+**50**)) rows of both stacks = 1300 = **26**×**50** = $1^5 + 2^5 + 3^5 + 4^5 = T_4$

$$
\begin{array}{cc}
 & 1^4 \\
 & 2^4\ 2^4 \\
= & 3^4\ 3^4\ 3^4 \\
 & 4^4\ 4^4\ 4^4\ 4^4.
\end{array}
$$

This illustrates the Tetrad Principle, through which archetypal patterns of numbers such as that in Fig. 5 are prescribed by the number 4. Their sum is

$$49400 = (1^3 + 2^3 + 3^3 + 4^3) \sum_{n=1}^{4} \left(1^n + 2^n + 3^n + 4^n\right).$$

The number of integers on the edge of each stack is **76**, demonstrating how the Godname number of Tiphareth also defines the number 24800. It is remarkable that the sum of the 152 integers on the boundary of both stacks is 5456 because this is the sum of the first **31** triangular numbers. This shows how the Godname EL of Chesed with number value **31** prescribes the pattern of integers generating the number 24800.

Another way of seeing how ELOHIM, the Godname of Binah, prescribes the number 24800 and thus the dimension **248** of the superstring symmetry group E_8 is as follows: any square number n^2 is the sum of the (n–1)th and (n+1)th triangular numbers because

$$n^2 = ½(n–1)n + n(n+1).$$

Applying this property to the squares of the 25 integers 4, 6, 8... 52:

$4^2 = 6 + 10.$
$6^2 =$ **15** + **21**.
$8^2 = 28 +$ **36**.
•
•
•
$52^2 = 1326 + 1378.$

The sum:

$$24800 = 4^2 + 6^2 + 8^2 + \ldots + 52^2$$

is the sum of the first **50** triangular numbers after 3. Therefore, the number of ELOHIM prescribes the number **248** as the 'kernel' of 24800. Since 24800 = **496**×**50**, the crucial dimension **496** of any gauge symmetry group that is free of quantum anomalies is the arithmetic mean of the first **50** triangular numbers after 3. This is the remarkable way in which ELOHIM prescribes the number of gauge fields that mediate the unified superstring interaction.

Finally, the number **155** of the Godname of Malkuth is related by the Pythagorean Tetrad to the number **248** by

$$\mathbf{155} = \frac{1+2+3+4}{4^2} \times \mathbf{248}.$$

In other words, a 4×4 square array of the number value of ADONAI MELEKH (4^2×**155**) generates the 2480 space-time components ((1+2+3+4)×248) of the 10-dimensional fields of the **248** gauge bosons of E_8. This fact demonstrates the profound Pythagorean principle whereby the Tetrad and its geometrical symbol — the square — define the basic, physical properties of nature, such as the four dimensions of Einsteinian space-time and the 25 spatial dimensions of bosonic strings predicted by quantum mechanics and symbolised by the 25 yods in a square with tetractyses as its sectors.