ARTICLE 50 (Part 1)

The Golden Ratio, Fibonacci & Lucas Numbers in Sacred Geometries

by

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This article explores the presence and role of the Golden Ratio, the Fibonacci numbers F_n and the Lucas numbers L_n in the sacred geometries of the Platonic solids, the outer & inner Trees of Life and their polyhedral counterparts - the 144 Polyhedron and the disdyakis triacontahedron. F_8 is the number of geometrical elements in the Lower Face of the Tree and F_9 is the number of elements in its Upper Face, F_{10} being the number of elements in the whole Tree. L_{10} is the number of geometrical elements in the three lowest Trees of Life mapping three-dimensional space. F_{10} measures the inner Tree of Life as the 55 corners of the 48 sectors of its seven separate polygons. L_9 is the number of corners of the seven enfolded polygons that are not also their centres. F_9 (=34) is the number of corners associated with the seven enfolded polygons enfolded in successive overlapping Trees of Life. It is also the number of geometrical elements added by them. The length of successive twists of the DNA molecule is about 34 angstroms. The 21:34 division of geometrical elements in the outer Tree of Life manifests in the 21 vertices & centres of the first three Platonic solids and the 34 vertices & centres in the last two. It also appears in the first four Platonic solids as the 210 triangles, polyhedral vertices & sides in their faces and as the remaining 340 elements in all five solids. The former embody the dimension 248 of the exceptional Lie group E_8 describing superstring forces as the 248 corners & sides of the 120 sectors of their 38 faces. This number is embodied in the lowest Tree of Life as the 248 yods below its apex. The (248+248) yods other than corners in the root edge and (7+7) separate polygons symbolize the (248+248) roots of $E_8 \times E_8$. The 1370 yods lining edges of the tetractyses needed to construct the five Platonic solids is the counterpart of the 1370 yods in the inner Tree of Life. L_{10} is the average number of geometrical elements in each half of these solids. F_{10} is the average number of elements in the faces of each half. The average number of internal geometrical elements (including centres) is 137, showing how the Platonic solids embody the number determining the fine-structure constant in physics. The relation $L_{10} = F_{10} + 2F_9$, which appears in the three Trees of Life mapping three-dimensional space, manifests in the five Platonic solids - the only regular polyhedra that can exist in such a space. These Trees encode the 206 bones in the human skeleton and the 361 classical acupuncture points. L_{10} is the number of Pythagorean intervals between the notes of the seven musical scales.

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Figure 1

In the ancient practice of gematria, consecutive letters of an alphabet are assigned the integers 1-10, 20-100 & 200-900. This means that words have numerical values that are the sum of their letter values. Words in religious texts with the same number values, however different they were, were considered to possess some kind of analogy, implying that the textual passages in which these words were located were connected in a subtle way and allowing levels of meaning beyond the literal to be uncovered in these sacred writings. In Kabbalah, the ten Divine Qualities, or Sephiroth, manifest in the four Worlds of Atziluth (Archetypal World), Beriah (World of Creation), Yetzirah (Formative World) and Assiyah (World of Action). The divine names that are assigned to the Sephiroth embodying the archetypal essence of God function in Atziluth. The Archangels assigned to each Sephirah operate in Beriah, the ten orders of angels exist in Yetzirah, whilst the Mundane Chakras are regarded as the physical manifestations of the Sephiroth, some being assigned one of the sacred planets (to be understood in their astrological, not astronomical, sense). The table in Figure 1 lists the gematria numbers of the Hebrew names of the Sephiroth, the Godnames, Archangels, Orders of Angels & Mundane Chakras. A few of the numbers, such as those of Elohim, Elohim Sabaoth and Cholem Yesodoth differ from those stated by standard works on Hebrew gematria. This is because these texts provide only numbers that are the sums of their letter values. They take no account of the possibility of contraction, wherein a sum of such values, or even an individual one, can be reduced to another number if it is a multiple of 10. For example, the letter value 40 of the Hebrew letter mem (מ) can be reduced to 4 because 4 + 0 = 4, as can the letter value 400 of tav (ת). Whether such contraction is required depends upon the context in which these words occur. Thirty years of correlating the gematria number values of the Sephiroth in the four Worlds with the properties of sacred geometries and with the mathematics underlying superstring theory has proved to the author that contraction occurs in a few of these number values. An example germane to superstring structure will be discussed later on in this article.

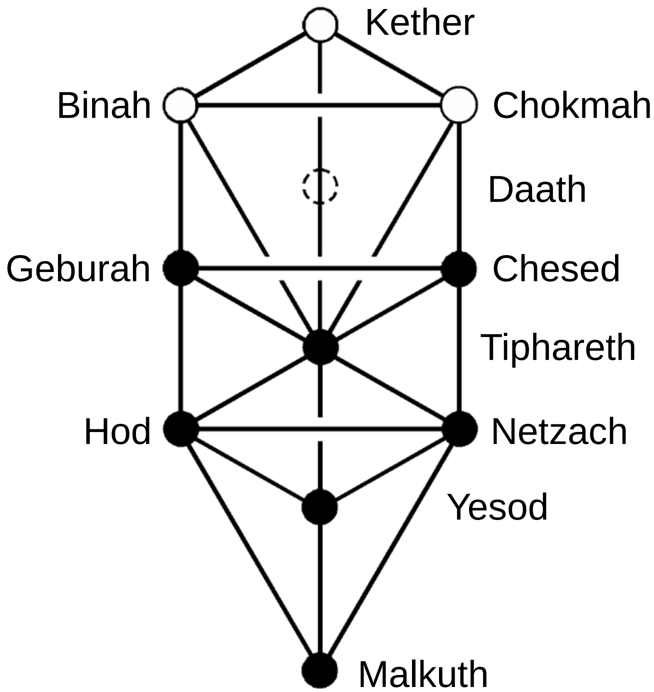
	SEPHIRAH	GODNAME	ARCHANGEL	ORDER OF ANGELS	MUNDANE CHAKRA
1	Kether (Crown) **620**	EHYEH (I am) **21**	Metatron (Angel of the Presence) **314**	Chaioth ha Qadesh (Holy Living Creatures) **833**	Rashith ha Gilgalim First Swirlings. (Primum Mobile) **636**
2	Chokmah (Wisdom) **73**	YAHVEH, YAH (The Lord) **26**, **15**	Raziel (Herald of the Deity) **248**	Auphanim (Wheels) **187**	Masloth (The Sphere of the Zodiac) **140**
3	Binah (Understanding) **67**	ELOHIM (God in multiplicity) **50**	Tzaphkiel (Contemplation of God) **311**	Aralim (Thrones) **282**	Shabathai Rest. (Saturn) **317**
	Daath (Knowledge) **474**				
4	Chesed (Mercy) **72**	EL (God) **31**	Tzadkiel (Benevolence of God) **62**	Chasmalim (Shining Ones) **428**	Tzadekh Righteousness. (Jupiter) **194**
5	Geburah (Severity) **216**	ELOHA (The Almighty) **36**	Samael (Severity of God) **131**	Seraphim (Fiery Serpents) **630**	Madim Vehement Strength. (Mars) **95**
6	Tiphareth (Beauty) **1081**	YAHVEH ELOHIM (God the Creator) **76**	Michael (Like unto God) **101**	Malachim (Kings) **140**	Shemesh The Solar Light. (Sun) **640**
7	Netzach (Victory) **148**	YAHVEH SABAOTH (Lord of Hosts) **129**	Haniel (Grace of God) **97**	Tarshishim or Elohim **1260**	Nogah Glittering Splendour. (Venus) **64**
8	Hod (Glory) **15**	ELOHIM SABAOTH (God of Hosts) **153**	Raphael (Divine Physician) **311**	Beni Elohim (Sons of God) **112**	Kokab The Stellar Light. (Mercury) **48**
9	Yesod (Foundation) **80**	SHADDAI EL CHAI (Almighty Living God) **49**, **363**	Gabriel (Strong Man of God) **246**	Cherubim (The Strong) **272**	Levanah The Lunar Flame. (Moon) **87**
10	Malkuth (Kingdom) **496**	ADONAI MELEKH (The Lord and King) **65**, **155**	Sandalphon (Manifest Messiah) **280**	Ashim (Souls of Fire) **351**	Cholem Yesodoth The Breaker of the Foundations. The Elements. (Earth) **168**

The Sephiroth exist in the four Worlds of Atziluth, Beriah, Yetzirah and Assiyah. Corresponding to them are the Godnames, Archangels, Order of Angels and Mundane Chakras (their physical manifestation). This table gives their number values obtained by the ancient practice of gematria, wherein a number is assigned to each letter of the alphabet, thereby giving a number value to a word that is the sum of the numbers associated with its letters. When some of these numbers are referred to in the article, they will be written in ***boldface****.*

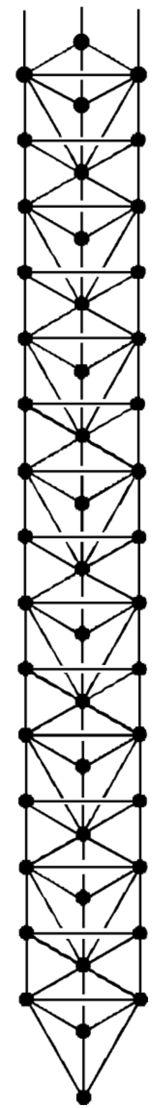
Figure 2

At the heart of the Jewish mystical tradition called ‘Kabbalah’ is the Tree of Life, or *Otz Chiim*. It represents Adam Kadmon, or “heavenly man.” The ten “Divine Qualities,” or Sephiroth (singular: Sephirah) are represented by ten circles. The first three - Kether (Crown), Chokmah (Wisdom) and Binah (intelligence) constitute the Supernal Triad. They head the Pillars of Equilibrium (central column), Mercy (right-hand column) and Judgement (left-hand column). The seven remaining Sephiroth are called the Sephiroth of Construction. They are Chesed (Mercy), Geburah (Severity), Tiphareth (Beauty), Netzach (Victory), Hod (Glory), Yesod (Foundation) & Malkuth (Kingdom). Between Binah and Chesed on the Pillar of Equilibrium is Daath (knowledge). It is not a Sephirah but a stage of transition from the subjective level of God to the seven Sephiroth of Construction expressing the objective nature of God. The Sephiroth are connected by 22 straight lines, or Paths.

As a three-dimensional object, the Tree of Life consists of 16 triangles with 22 edges and ten corners. Traditional Kabbalah considers only the generic single Tree of Life and the four overlapping Trees that represent the Archetypal World (the domain of the Divine Names), the Creative World (the archangelic level), the Formative World (the angelic realms) and the World of Action (physical universe). However, it has been shown in earlier articles that 91 overlapping Trees of Life (called the Cosmic Tree of Life, or CTOL) map all levels of physical and superphysical reality. Figure 2 shows the lowest seven Trees. They map the **26**-dimensional space-time that is predicted by string theory and which is prescribed by YAHWEH, the Godname of Chokmah whose number value is **26**.



Tree of Life



Overlapping Trees of Life

Figure 3

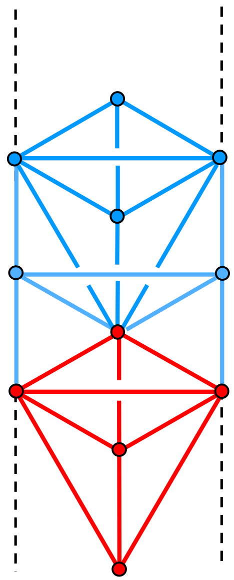
In Kabbalah, the kite-shaped group of Sephiroth of Construction:

Tiphareth-Netzach-Hod-Yesod-Malkuth

is called the ‘Lower Face’ of the Tree of Life and its ‘Upper Face’ is the kite-shaped group of four Sephiroth:

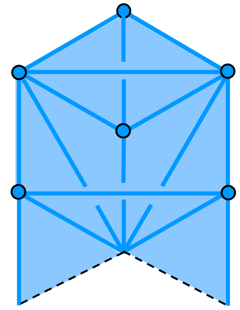
Kether-Chokmah-Binah-Tiphareth

For the sake of simplicity of nomenclature, these two names will be meant here to refer not only to corners of triangles (the traditional sense) but also to the space enclosed by them, as well as to their edges (Paths). The Upper Face will be regarded as the blue corners, edges & triangles of the Tree of Life outside its Lower Face which, geometrically speaking, comprises not only five red corners but also red triangles and their red edges. This means that the blue triangular space with Chesed, Tiphareth & Netzach at its corners and the blue triangular space with Geburah, Tiphareth & Hod at its corners belong to the Upper Face, whereas two of their edges - the dotted lines joining Tiphareth & Netzach and Tiphareth & Hod - are regarded as belonging to the Lower Face. It also means that - in the geometrical connotation used here - the term ‘Upper Face’ excludes Tiphareth as a corner. This change in definition allows the Tree of Life to be regarded as a simple combination of these two geometrical structures.



A member of a set of
overlapping Trees of Life

Upper Face



Lower Face

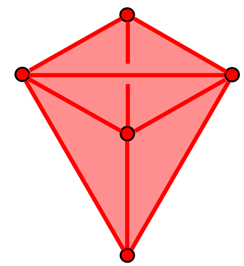


Figure 4

The nth Fibonacci number (1) is defined as:

$$F_n = F_{n-1} + F_{n-2}, \text{ if } n>1,$$

where $F_0 = 0$ & $F_1 = 1$. They belong to the infinite series:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ….

Each integer is the sum of the previous two numbers. The French mathematician, François Édouard Anatole Lucas (1842-1891), who gave this series of numbers the name of Fibonacci Numbers, found a similar series occurs often when investigating Fibonacci number patterns:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, ...

The Fibonacci rule of adding the previous two to get the next is kept, but here we start from 2 and 1 (in this order) instead of 0 and 1 for the (ordinary) Fibonacci numbers. Called the Lucas numbers after him, the nth number in this series is defined as:

$$L_n = L_{n-1} + L_{n-2}, \text{ for } n>1$$

where $L_0 = 2$ & $L_1 = 1$. To emphasis their presence, all Fibonacci and Lucas numbers will henceforth be written in colour.

The Lower Face of any Tree of Life in a set of overlapping Trees contains **21** points, lines & triangles. This is F_8. It is also the number value of EHYEH, the Godname of Kether (see Fig. 1). Its Upper Face contains 34 geometrical elements. This is F_9. A whole Tree of Life has 55 elements. This is the tenth Fibonacci number F_{10}. Thirty-four more elements are needed to construct successive, overlapping Trees. Two overlapping Trees have 89 elements. This is F_{11}. Three overlapping Trees possess 123 elements. This is the tenth Lucas number L_{10}. The significance of this particular set of Trees will be revealed later.

The natural appearance of both Fibonacci and Lucas numbers in the geometrical composition of the Tree of Life and its two basic components that become replicated in successive Trees is the first indication of how these numbers, which manifest in the philotaxis of plants and flowers, are also intrinsic to the growth of the Tree of Life as a geometrical object. The general relationship between them:

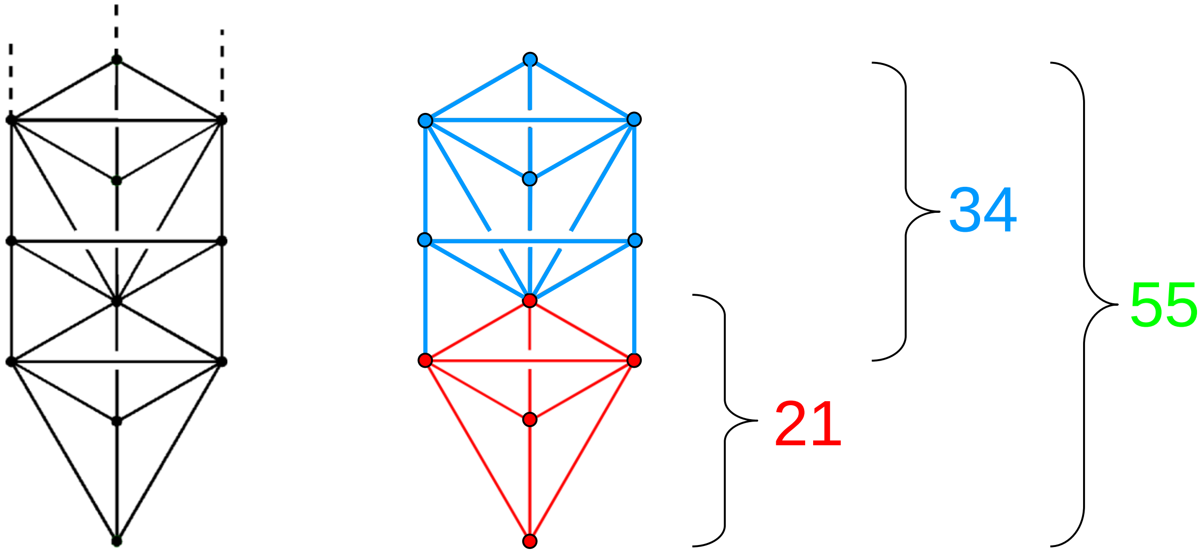
$$L_n = F_{n-1} + F_{n+1}$$

Is actualised geometrically in the three overlapping Trees of Life as

$$123 = 34 + 89,$$

because 34 is the number of geometrical elements in the Upper Face of the third Tree and 89 is the number of elements below it.

n	0	1	2	3	4	5	6	7	8	9	10
F_n	0	1	1	2	3	5	8	13	21	34	55
L_n	2	1	3	4	7	11	18	29	47	76	123



The lowest n Trees of Life

Number of vertices = 6n + 5

Number of sides = 16n + 9

Number of triangles = 12n + 7

Number of geometrical elements ≡ N(n) = **34**n + **21**

N(1) = **21** + **34** = **55** = F_{10}

N(2) = N(1) + **34** = **34** **+** **55** = **89** = F_{11}

N(3) = **34** + N(2) = **34** + **89** = 123 = L_{10}

Figure 5

The lowest three Trees serve to illustrate another relationship between Fibonacci and Lucas numbers. The nth numbers obey the equation:

$$L_n = F_n + 2F_{n-1}$$

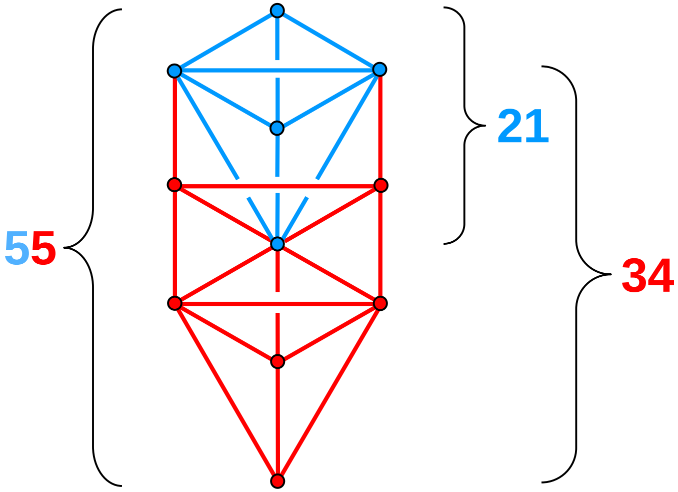
This follows from the equation just discussed:

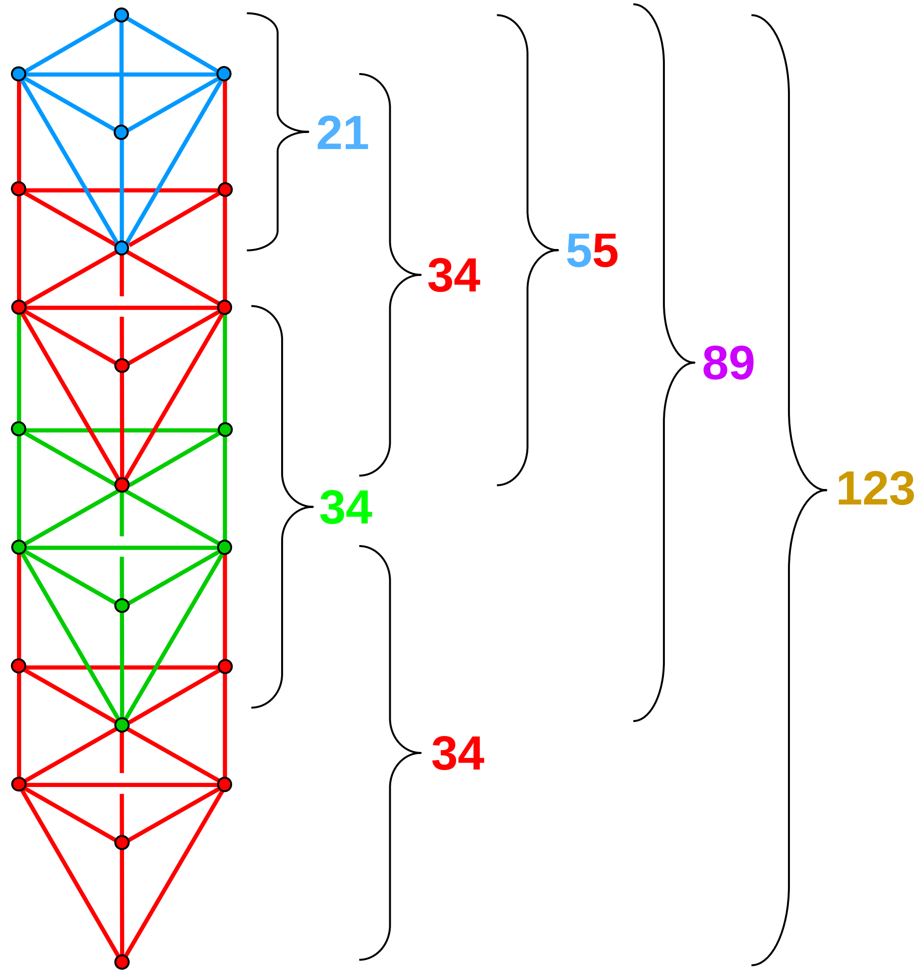
$$L_n = F_{n-1} + F_{n+1}$$

because

$$F_{n+1} = F_{n-1} + F_n.$$

For n = 10, $L_{10} = F_{10} + 2F_9 = 55 + 2\times34$. This has the natural meaning in the context of the three lowest overlapping Trees of Life as the 55 geometrical elements of either the lowest or highest one and as the 34 elements added by each of the next two Trees. This relation between Lucas and Fibonacci numbers, illustrated for n = 10, will be shown to apply to the total geometrical composition of several other holistic systems that possess sacred geometry. Earlier articles proved the equivalence of these systems, so that it is safe to infer that all such systems conform to this equation, at least for n = 10, if not for all n<10.





Number of geometrical elements in the 1-tree = 55 = F_{10}

$$L_n = F_n + 2F_{n-1}$$

$$L_{10} = F_{10} + 2F_9$$

$$\boxed{123 = 55 + 2\times 34}$$

Number of geometrical elements in the 3-tree = 123 = L_{10}

Figure 6

Two similar circles that overlap centre to circumference create the *Vesica Piscis*. As four overlapping circles generate the locations of the ten Sephiroth of the Tree of Life, the latter is created by the extremities of a vertical stack of four *Vesicae Piscis* (shown shaded). Their horizontal extremities can be regarded as the centres of two more circles. The eight ends (white dots) of the horizontal diameters of the four overlapping circles are not points of intersection of any two circles, unlike the eight black dots. If we stack more *Vesicae Piscis* so as to form the next higher Tree of Life, the black dots are translated into blue dots and the white dots are shifted to red dots. The black and white dots constitute two independent sets of eight points. They possess the amazing property that straight lines passing through any two dots intersect at the corners of two similar sets of seven regular polygons:

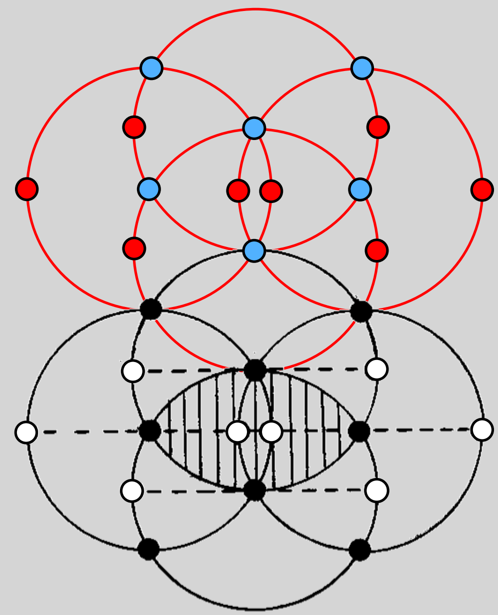
triangle, square, pentagon, hexagon, octagon, decagon, dodecagon.

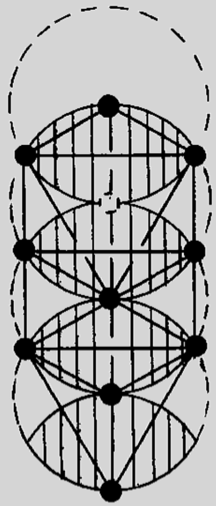
Those in one set are the mirror image of their counterparts in the other set. They share one edge - the vertical “root edge,” as it has been called in previous articles.

This geometrical object is implicate in the geometry of the circles whose intersection create overlapping Trees of Life. They represent a hidden geometrical potential that is possessed not only by a single Tree but by every overlapping Tree because, when straight lines are drawn through the blue and red dots of the circles generating the next higher Tree, they intersect at the corners of another set of 14 regular polygons.

8 ends of horizontal diameters

8 intersections of circles





The generation of the inner Tree of Life

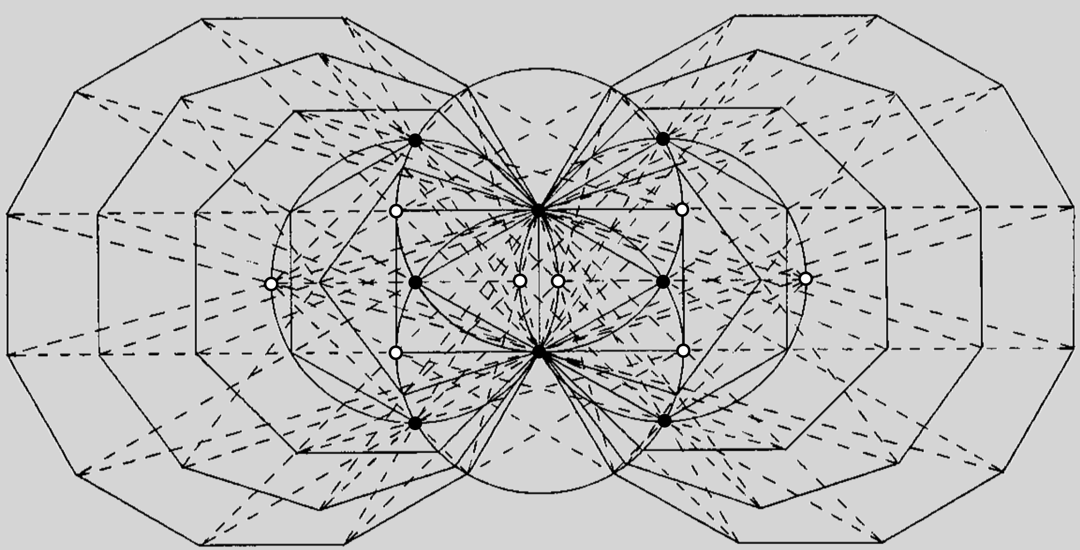
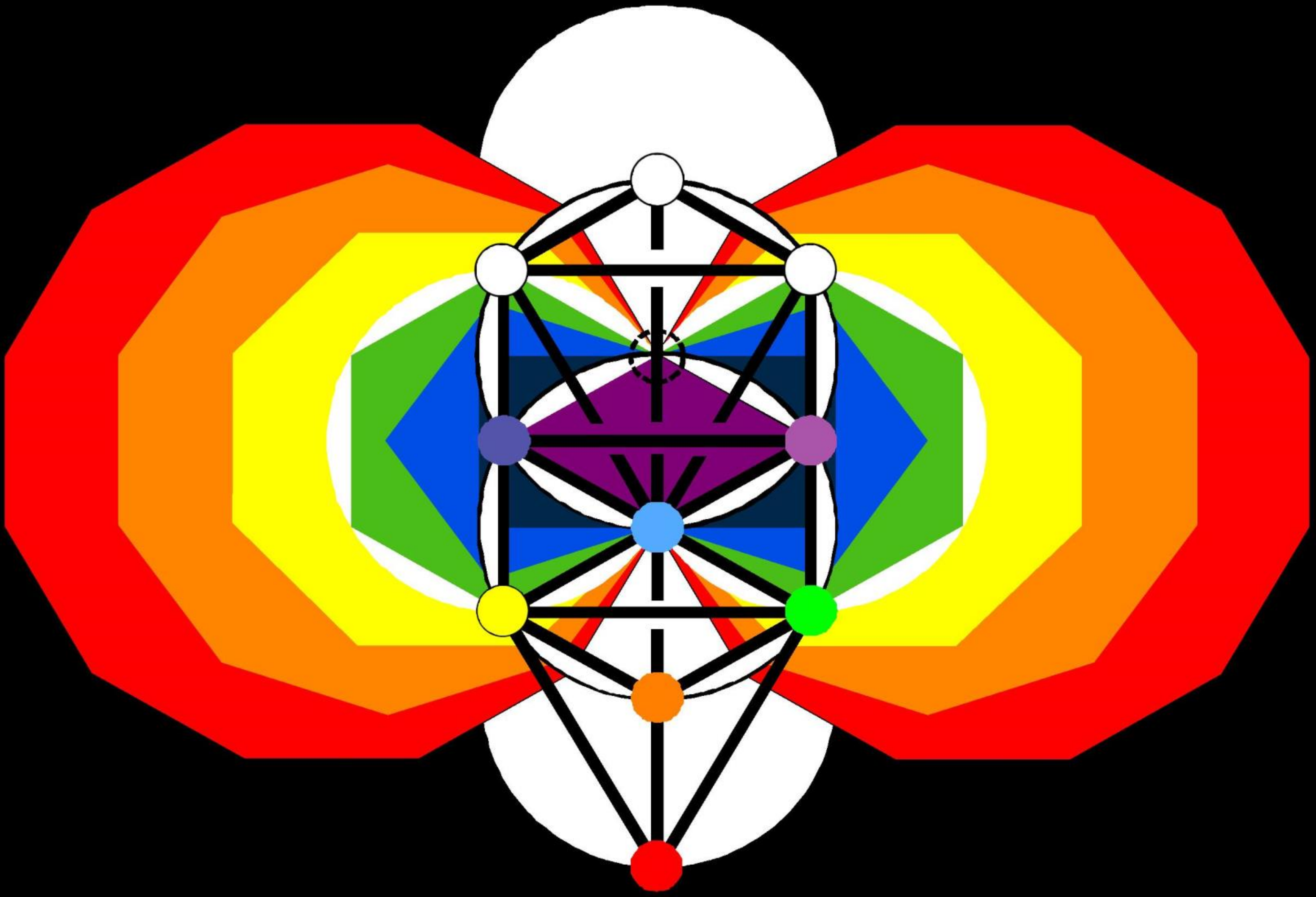


Figure 7

The form of the Tree of Life known to Kabbalists is but its outer form. The 14 regular polygons constitute its inner form. They have 70 corners. Their 94 sectors have **80** corners, where **80** is the number value of Yesod. Notice that the corners of the triangles coinciding with Chesed and Geburah are the centres of the two hexagons. Also, the outermost corners of the two pentagons coincide with the centres of the decagons. This means that the sectors of the 14 polygons have **76** corners that are not also centres of polygons. **76** is the number value of YAHWEH ELOHIM, the Godname of Tiphareth. It is also the ninth Lucas number.



The outer and inner Trees of Life

Figure 8

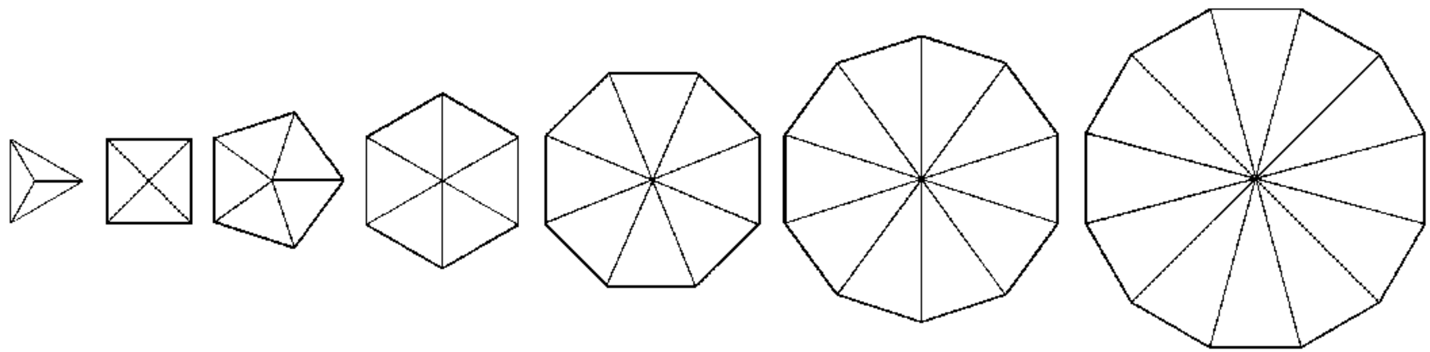
When separated, each set of seven polygons have **48** corners. According to the table of gematria number values of the Sephiroth shown in Fig. 1, this is the number value of *Kokab*, the Mundane Chakra of Hod. It is not coincidental that this number appears in this context because many previous articles have demonstrated that the inner Tree of Life is prescribed by the Godnames and manifests all the number values of the Sephiroth in the four Worlds.

Including the centres of the seven polygons, their **48** sectors have (**48**+7=55) corners. 55 is the tenth Fibonacci number F_{10}. This is the counterpart in the inner Tree of Life of the 55 geometrical elements in its outer form. In either case, it is a measure of the *form* or *shape* of a holistic system.

The numbers in the Fibonacci series less than 55 can be identified by distinguishing between the corners of each polygon and its centre (denoted "1" in Fig. 8). The sectors of the triangle, pentagon & decagon have **21** corners and the sectors of the square, hexagon, octagon & dodecagon have 34 corners. Each Fibonacci number divides into smaller Fibonacci numbers that measure corners of subsets of polygons.

The twelfth Fibonacci number 144 is the number of corners and sides of the **48** sectors of the seven separate polygons.

4+1=5 6+1=7 10+1=11



3+1=4 5+1=6 8+1=9 12+1=13

13

3+1=4 5+1=6 10+1=11

8

21

4+1=5 6+1=7 8+1=9 12+1=13

21

34

48 corners of 7 polygons

7 centres

55 corners of **48** triangles

144 corners & sides surround centres

Figure 9

The outer Tree of Life has 16 triangles with ten corners and 22 edges. These **48** geometrical elements are the counterpart of the **48** corners of the seven separate polygons. Just as seven elements are added when the Tree of Life becomes the lowest Tree of Life, so seven corners are added by dividing the seven polygons into their sectors.

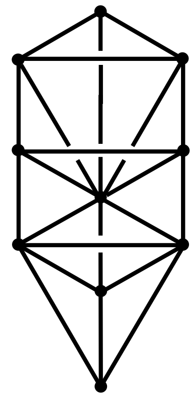
Outer Tree of Life

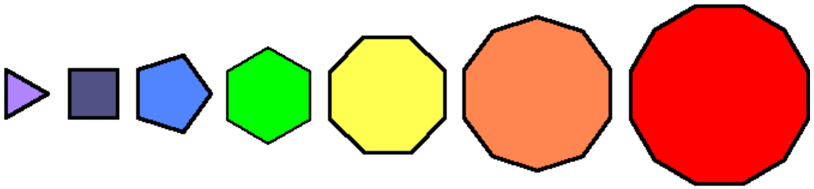
10 corners

22 sides

16 triangles

48 geometrical elements





48 corners

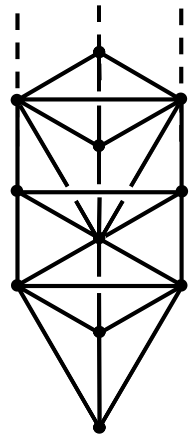
1-tree

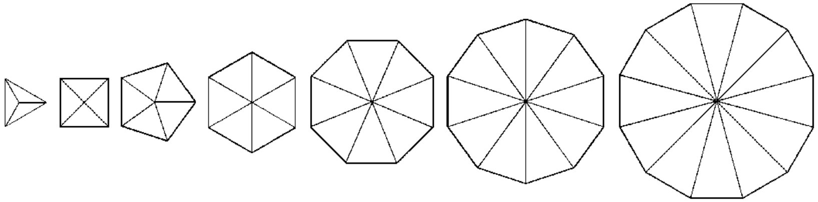
11 corners

25 sides

19 triangles

55 geometrical elements





55 corners

Figure 10

The topmost corners of the two hexagons enfolded in the inner Tree of Life coincide with the lowest corners of the two hexagons enfolded in the next higher Tree. There are 68 corners per set of 14 polygons, 34 per set of seven polygons. This is the counterpart of the addition of 34 geometrical elements in the Upper Face of every higher Tree of Life. In algebraic terms, the number of geometrical elements in the lowest n Trees of Life (what has been called the 'n-tree' in previous articles) is:

$$N(n) = 34n + \mathbf{21},$$

so that $N(n+1) - N(n) = 34$.

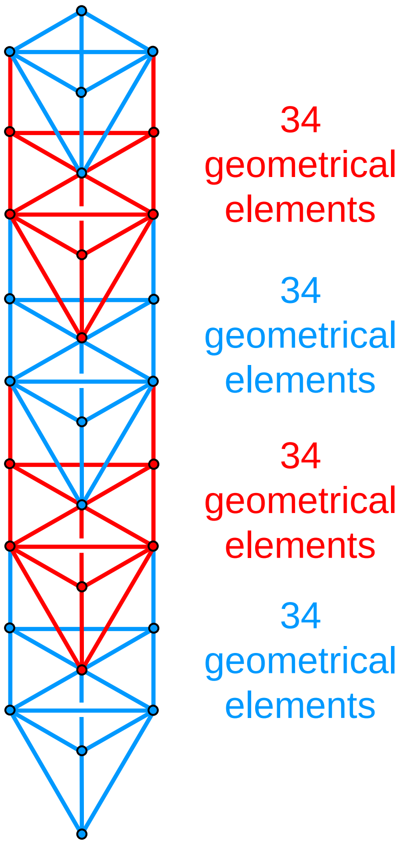
Compare this with the fact that the number of corners of the 14n polygons enfolded in the n-tree is:

$$C(n) = 68n + 2,$$

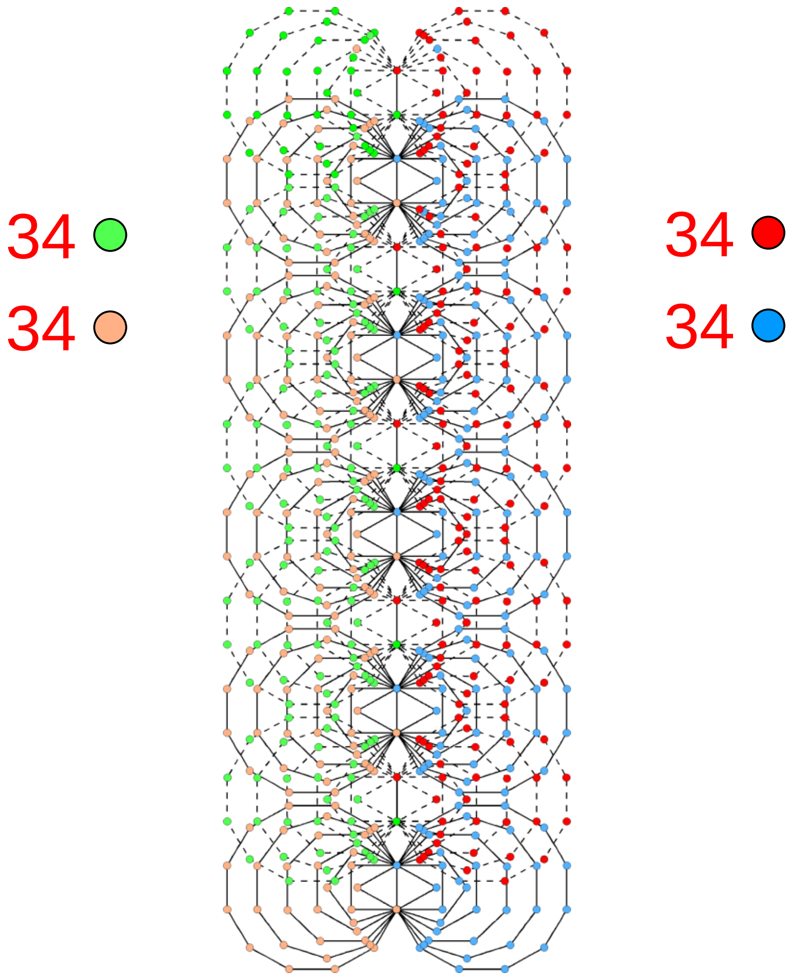
so that the number of corners associated with each set $\equiv C'(n) = C(n)/2 = 34n + 1$ and

$$C'(n+1) - C'(n) = 34.$$

The ninth Fibonacci 34 number measures the geometrical composition of successive Trees of Life and the corners associated with the seven polygons enfolded in them.



34 geometrical elements are added in each higher or lower Tree of Life



There are 34 corners per 7 polygons in each higher or lower Tree of Life

Figure 11

The width of the DNA molecule is 20 angstroms, to the nearest integer (1 angstrom = 10^{-8} cm). One 360 degree turn of DNA measures about 34 angstroms in the direction of the axis. These lengths, 34:20, are in the ratio of the Golden Mean Φ, within the limits of the accuracy of the measurements (compare 1.7 with 1.618…). It is remarkable that its two dimensions, rounded off to the nearest integer, are so close to two Fibonacci numbers. Each DNA strand contains periodically recurring phosphate and sugar subunits. There are ten such phosphate-sugar groups in each full 360 degree revolution of the DNA spiral, the average vertical distance between base pairs being 3.4 angstroms. The amount of rotation of each of these subunits around the DNA cylinder is therefore **36** degrees. This is exactly half the pentagon rotation, showing a close relation of the DNA sub-unit to the Golden Mean and refuting the suggestion that the closeness of the length and width of one helical turn of the DNA double helix is merely coincidental.

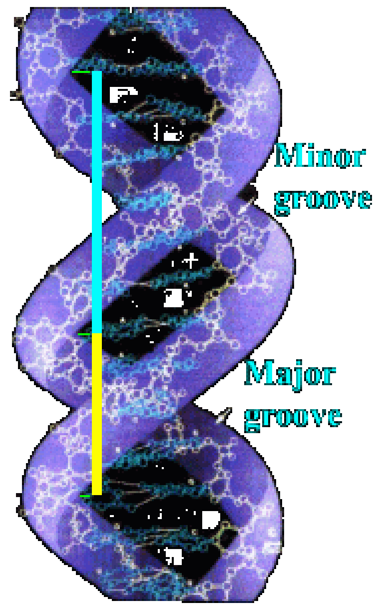
The coiling of the two helical strands of the DNA molecule creates a major groove 22 angstroms wide and a minor groove 12 angstroms wide. Their relative proportion is 22/12 $\approx$ 1.545… .This differs from the Golden Ratio Φ only by about 4.5%. Is that merely a coincidence? Or is it yet another indication that the “molecule of life” conforms to the same ideal proportions that are found in many macroscopic forms of life?

≈ 20 Å

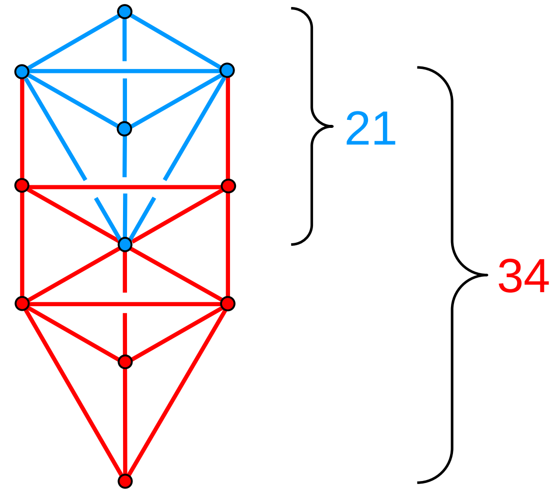
34/20 = 1.7

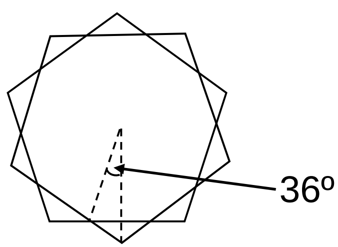
Φ = 1.618…

≈ 34 Å



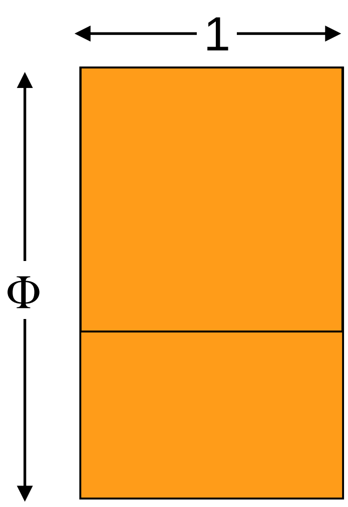
DNA molecule





10 nucleotide
pairs per
revolution

≈ 3.4 Å



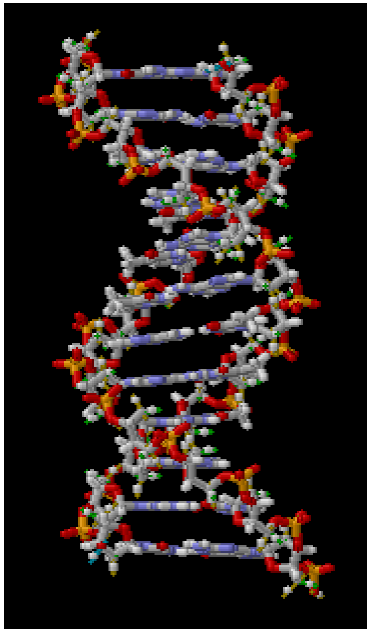


Figure 12

The two strands of the DNA molecule are held together by non-covalent hydrogen bonding between pairs of nitrogen bases. There are four types: adenine (A), thymine (T), guanine (G) & cytosine (C). They form the rungs of a ladder whose sides are alternating sugar and phosphate groups and which spiral around each other in opposite senses. Base A always pairs with base T and base C always pairs with base G. When a cell prepares to divide, the DNA helix splits down the middle and becomes two strands. These single strands serve as templates for building two new, double-stranded DNA molecules - each a replica of the original DNA molecule. In this process, an A base is added wherever there is a T base, a C where there is a G, and so on until all the bases once again have partners.

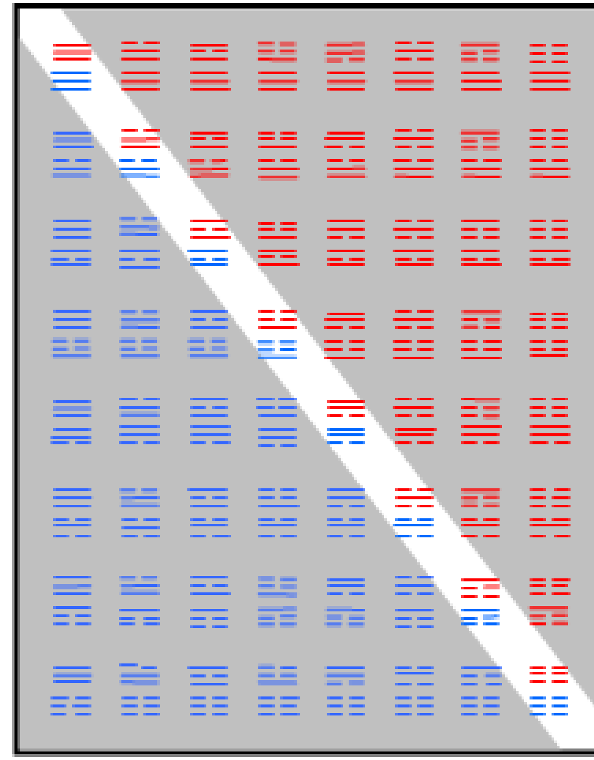
The genetic code consists of (4×4×4=**64**) triplets of nucleotides called codons (**64** is the number value *Nogah*, the Mundane Chakra of Netzach). With three exceptions, each codon encodes for one of the 20 standard amino acids used in the synthesis of proteins. RNA is a single-strand molecule with a much shorter chain of nucleotides than DNA. Instead of thymine, the complementary base to adenine is uracil (U), an unmethylated form of thymine. Figure 12 shows the **64** codons of messenger RNA (mRNA). An anticodon is a sequence of three adjacent nucleotides in transfer RNA (tRNA) that correspond to the three bases of the codon on the mRNA strand. An anticodon is complementary to the codon in mRNA that binds to it and designates a specific amino aid during protein synthesis. For example, the anticodon GUA is the complement of the codon CAU because G is the complement of C and U is the complement of A. The four bases appear (**64**×3=192) times in each set of **64** codons or anticodons. Each base appears (192/4=**48**) times, where **48** is the number value of *Kokab*, the Mundane Chakra of Hod, the next Sephirah after Netzach. There are 96 instances of A & C that bind to the 96 instances of their respective complements U & G.

Compare this pattern with the table of **64** hexagrams that form the basis of the ancient Chinese system of divination known as ‘I Ching.’ Each hexagram is a pair of trigrams (triplets of all combinations of lines & broken lines denoting the polarities of yang & yin). The **64** trigrams in one diagonal half of the 8×8 array comprise 192 lines & broken lines (96 yang lines & 96 yin lines). The **64** trigrams in the other half of the array similarly consist of 192 lines and broken lines. The 32 hexagrams in this half are the inversions of those in the other half, so that they comprise the *same* set of **64** trigrams. The **64** trigrams in in one diagonal half of the table correspond to the **64** mRNA codons and the **64** trigrams in its other half correspond to the **64** tRNA anticodons. The yang/yin duality of lines & broken lines manifests in RNA as pairs of complementary bases, which create **64** codons and **64** anticodons. The 96 lines in each half correspond to the 96 instances of the non-bonding A & C. The 96 broken lines in each half correspond to the 96 instances of their non-bonding complements U & G (for more details, see ref. 2).

64 codons of mRNA & **64** anticodons of tRNA

(**64**+**64**) trigrams of the I Ching table

Amino Acid	mRNA Base Codons	tRNA Base Anticodons
alanine	GCU, GCC, GCA, GCG	CGA, CGG, CGU, CGC
arginine	CGU, CGC, CGA, CGG, AGA, AGG	GCA, GCG, GCU, GCC, UCU, UCC
asparagine	AAU, AAC	UUA, UUG
aspartate	GAU, GAC	CUA, CUG
cysteine	UGA, UGC	ACA, ACG
glutamate	GAA, GAG	CUU, CUC
glutamine	CAA, CAG	GUU, GUC
glycine	GGU, GGC, GGA, GGG	CCA, CCG, CCU, CCC
histidine	CAU, CAC	GUA, GUG
isoleucine	AUU, AUC, AUA	UAA, UAG, UAU
leucine	UUA, UUG, CUU, CUC, CUA, CUG	AAU, AAC, GAA, GAG, GAU, GAC
lysine	AAA, AAG	UUU, UUC
methionine	AUG	UAC
phenylalanine	UUU, UUC	AAA, AAG
proline	CCU, CCC, CCA, CCG	GGA, GGG, GGU, GGC
serine	UCU, UCC, UCA, UCG, AGU, AGC	AGA, AGG, AGU, AGC, UCA, UCG
stop	UAA, UAG, UGA	AUU, AUC, ACU
threonine	ACU, ACC, ACA, ACG	UGA, UGG, UGU, UGC
tryptophan	UGG	ACC
tyrosine	UAU, UAC	AUA, AUG
valine	GUU, GUC, GUA, GUG	CAA, CAG, CAU, CAC



Each base occurs **48** times

192 instances of 4 bases in mRNA

192 instances of 4 bases in tRNA

96 { A (×**48**) binds to the complementary U (×**48**) } 96

C (×**48**) binds to the complementary G (×**48**)

192 —

192 – –

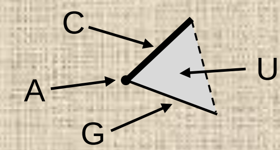
96 — 96 – –

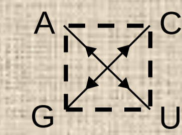
96 — 96 – –

Figure 13

The counterpart in the inner Tree of Life of the **48** instances of each of the four types of bases in the DNA molecule is the four types of geometrical elements making up the **48** sectors of the seven separate regular polygons. There are **48** vertices, **48** external sides, **48** internal sides & **48** triangles. The pair of geometrical elements (repeated **48** times) forming the boundaries of the polygons correspond to either the non-bonding pair (A,C) or (G,U) and the pair of internal elements (repeated **48** times) correspond to, respectively, either (G,U) or (A,C).

The four geometrical elements per sector of a polygon

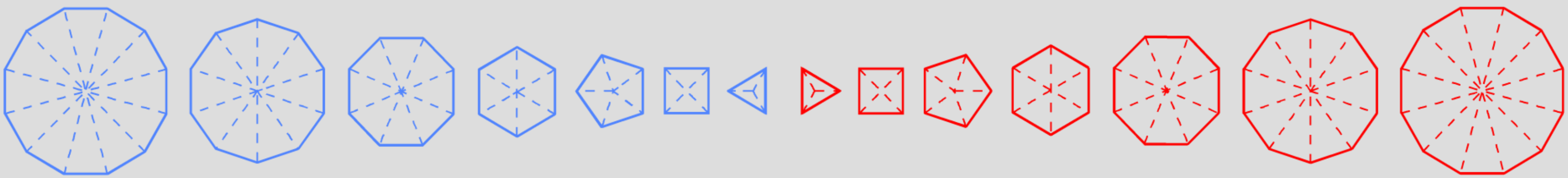




Double-headed arrows denote hydrogen bonding

The 192 geometrical elements in one half of the inner Tree of Life are the geometrical counterpart of the 192 instances of the four bases making up the **64** codons in mRNA and the 192 lines & broken lines in a diagonal half of the I Ching table. The 192 elements in the mirror image half of the inner Tree of Life are the counterpart of the 192 instances of the four bases making up the **64** anticodons in tRNA and the 192 lines & broken lines in the other half of the table. The mirror reflection of each element belonging to one set of polygons into its counterpart in the other set corresponds to replacing A, C, U & G in the **64** codons by their respective complements U, G, A & C in the **64** anticodons (for more details about the geometrical counterpart of the **64** hexagrams and the **64** codons & anticodons, see ref. 2).

The (192+192) geometrical elements in the (7+7) separate polygons correspond to the (192+192) lines & broken lines in the I Ching table



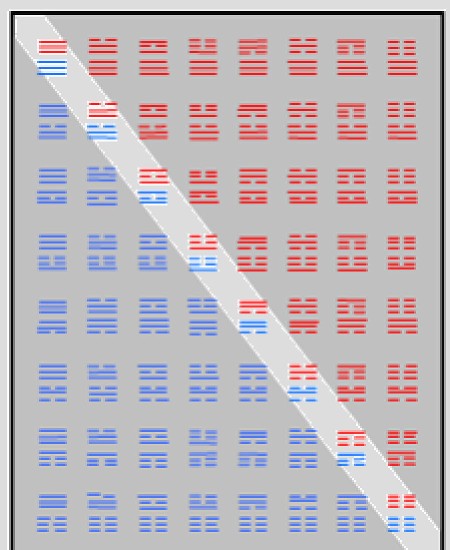
Number of corners = **48**
Number of triangles = **48**
Number of edges = 96
Total = 192

Number of corners = **48**
Number of triangles = **48**
Number of edges = 96
Total = 192

96
96
Total = 192

12 lines & 12 broken lines of lower 8 trigrams in diagonal

12 edges & 12 corners or triangles of hexagon



96
96
Total = 192

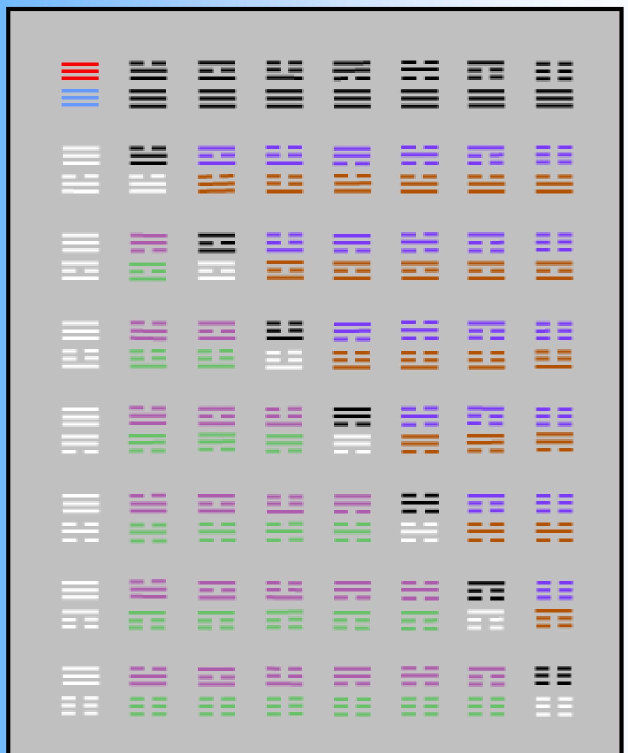
12 lines & 12 broken lines of upper 8 trigrams in diagonal

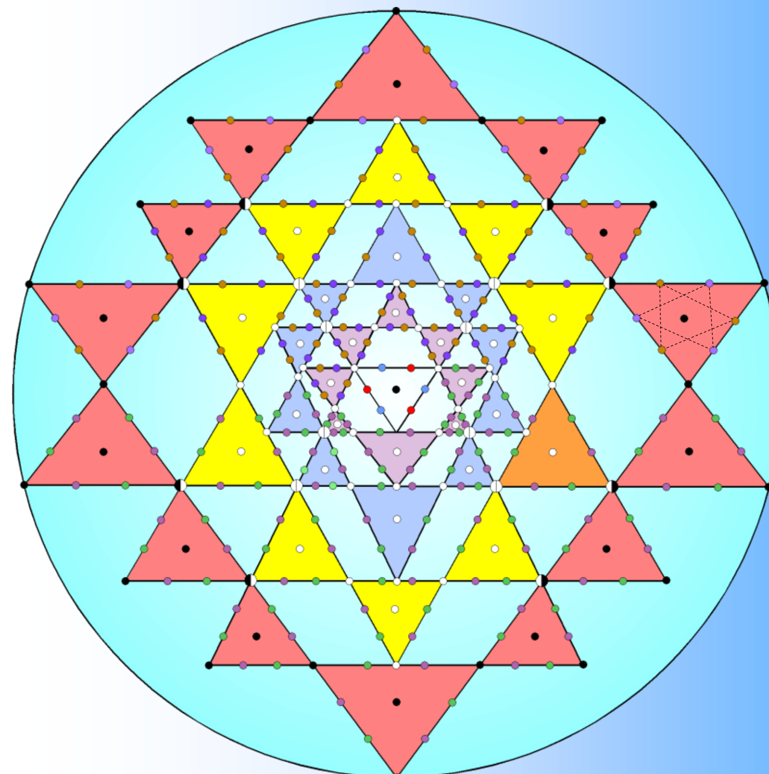
12 edges & 12 corners or triangles of hexagon

Figure 14

Yantras are the yogic equivalent of mandalas, used by Hindus and Buddhists as objects of meditation. The Sri Yantra is the most revered of these plans or charts that map Man’s inward journey from physical existence to spiritual enlightenment. It is generated from nine primary triangles. Five downward-pointing triangles symbolizing the feminine, creative energy of the Goddess Shakti intersect four upward-pointing triangles symbolizing the masculine, creative energy, popularly conceived in India as the God Shiva. Their overlapping generates 43 triangles. Forty-two triangles arranged in four layers of eight, ten, ten & 14 triangles surround a downward pointing triangle whose corners denote the triple Godhead, or Hindu Trimûrti of Shiva, Brahma & Vishnu. At its centre is a point, or bindu, representing the Absolute, or transcendental Unity.

When the 43 triangles in the 3-dimensional Sri Yantra are tetractyses, 378 yods surround the central tetractys (note that the outward-pointing corners of the triangles in the highest three layers lie *above* the joined corners in the layer next below the triangle in question. These corners are represented by circles that are split into two differently coloured halves; one half denotes a corner of a triangle in one set and the other half denotes a corner in the adjacent set). The two triplets of red and blue hexagonal yods in the central tetractys correspond to the two trigrams of the Heaven hexagram in the top left-hand corner of the I Ching table. Each pair of triplets of hexagonal yods in a triangle (one displays dashes connecting a pair) corresponds to a pair of trigrams in a hexagram. The 384 yods composing the Sri Yantra other than the corners and centre of the central triangle correspond to the 384 lines & broken lines of the **64** hexagrams (3).





Equivalence of the I Ching table and the Sri Yantra

Figure 15

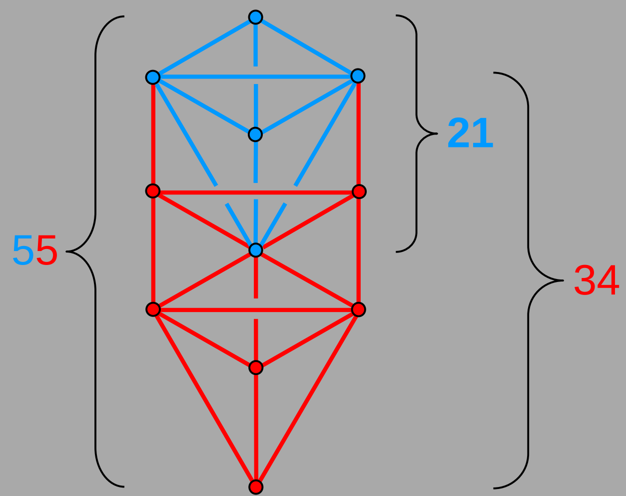
There are only five regular polyhedra. Known as the Platonic solids because of Plato's reference to them in *Timaeus*, his cosmological treatise expounding Pythagorean doctrine, they are:

tetrahedron   octahedron   cube   icosahedron   dodecahedron

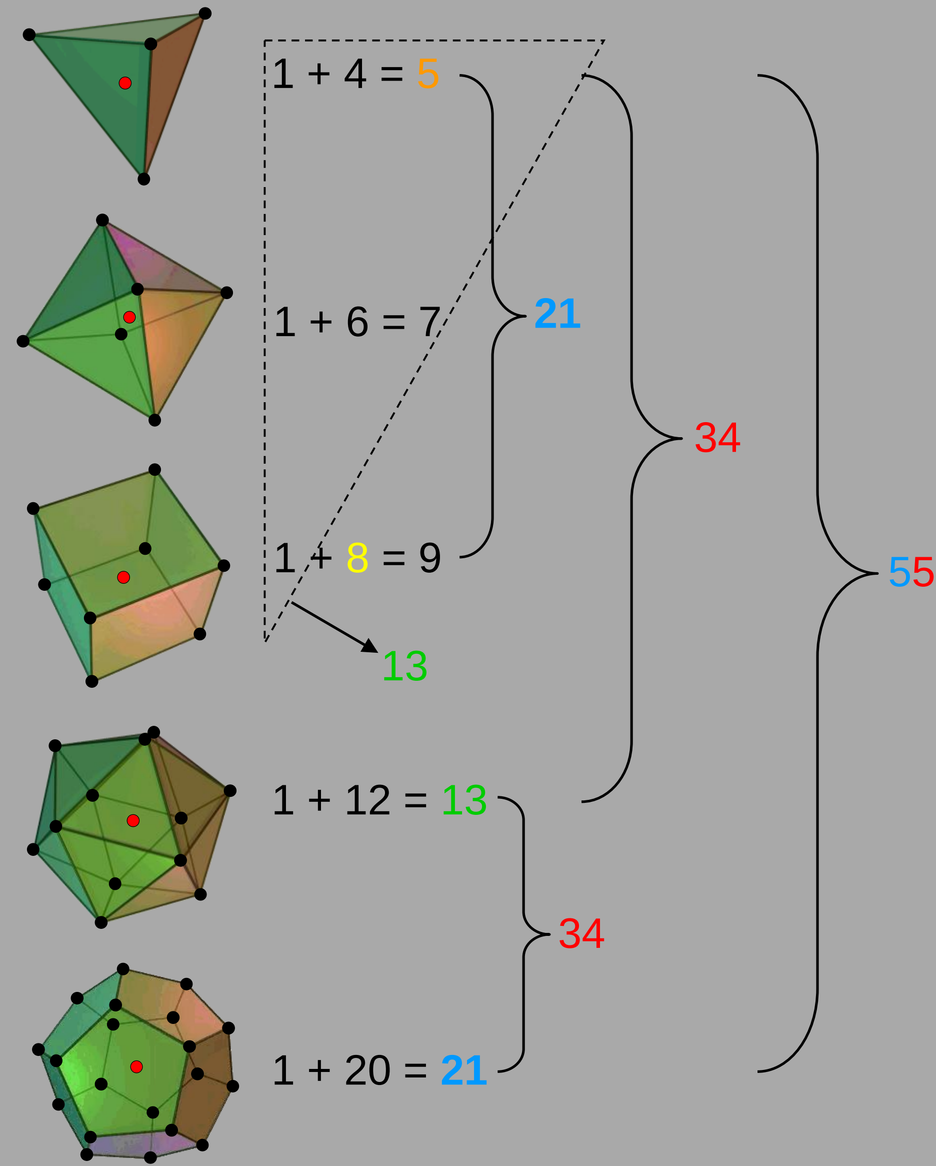
They have **50** vertices (black dots) and five centres (red dots), so that the 180 sectors of their faces and the 90 interior triangles formed by their edges have 55 corners. This is F_{10}. The tetrahedron has 5 centres & vertices, where 5 is F_5. The first three solids have **21** centres & corners and the icosahedron & dodecahedron have 34 centres & vertices. **21** is F_8 and 34 is F_9. The icosahedron has 13 centres & vertices. This is F_7. The dodecahedron has **21** centres & vertices. This is F_8. We see that the five Platonic solids display five of the first ten Fibonacci numbers. The other five are present as well, although less explicit because they mix points in several solids. As **21** = 8 + 13, F_6 appears as the 8 points that are either centres of the octahedron & cube or vertices of the former, whilst F_7 appears as the 5 centres & vertices of the tetrahedron and as the 8 vertices of the cube. The 5 points of the tetrahedron are further reducible to its centre (F_1), an apex (F_2) (this pair makes F_3 (=2)) and the 3 vertices of its base (F_4).

Comparing this pattern with the geometrical composition of the lowest Tree of Life, the 55 points in the set of five Platonic solids corresponds to the 55 points, lines & triangles making up the Tree, the **21** points in the first three solids correspond to the **21** geometrical elements in the kite-shape that is part of the Upper Face and the 34 points in the icosahedron and dodecahedron correspond to the 34 elements in the remainder of the Tree.

The 5 Platonic solids as a growth sequence in 3-d space governed by Fibonacci numbers



Number of centres (●) & vertices (●)



Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, …

Figure 16

The sum of the 180 angles subtended by the 90 edges of the five Platonic solids is 14400. This is the square of the sum of the squares of the first four even integers. The average of these angles is **80**^{o}. The sum is equivalent to 40 circles, where 40 = 4 + 8 + 12 + 16, and to **80** half-circles, where **80** is the number value of Yesod. These properties illustrate the Tetrad Principle (4), which states that the fourth member of a class of mathematical object, or the sum of the first four members, quantifies the properties of holistic systems that display sacred geometry, such as the Platonic solids.

When the **50** faces of the five Platonic solids are divided into their sectors, there are 550 points, lines & triangles. This is *ten* times the sum of the first *ten* integers, showing how the Decad determines the geometrical composition of the faces. Also, 550 = $10F_{10}$, where F_{10} (=55) is the *tenth* Fibonacci number.

The first four Platonic solids have 210 triangles and polyhedral vertices & sides making up their faces, leaving 340 elements distributed amongst all five solids. Hence,

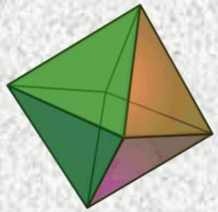
$$550 = 210 + 340 = 10\times(\mathbf{21}+34).$$

21 is the eighth Fibonacci number and 34 is the ninth such number. As the Lower Face of the lowest Tree of Life has 21 geometrical elements (Fig. 4) and the Upper face has 34 elements, this is their regular polyhedral counterpart.

tetrahedron

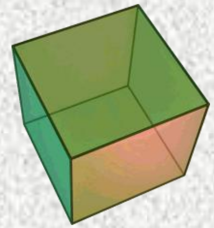
Sum of angles
= 720
≡ 2 circles

octahedron



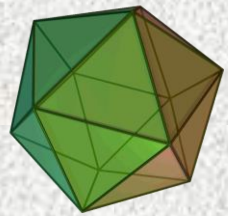
Sum of angles
= 1440
≡ 4 circles

cube



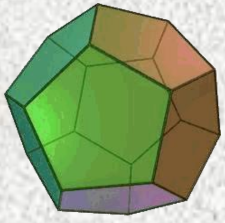
Sum of angles
= 2160
≡ 6 circles

icosahedron



Sum of angles
= 3600
≡ 10 circles

dodecahedron



Sum of angles
= 6480
≡ 18 circles

Sum of angles = 14400 = 120^2 = $(2^2+4^2+6^2+8^2)^2$ ≡ 40 circles, where 40=4+8+12+16.

Number of vertices = 100.
Number of sides = 270.
} 370 vertices & sides

Number of triangles = 180.

Number of vertices, sides & triangles = 550 = 10×(1+2+3+4+5+6+7+8+9+10).

Number of polyhedral vertices & sides & triangles in first 4 solids = 210.

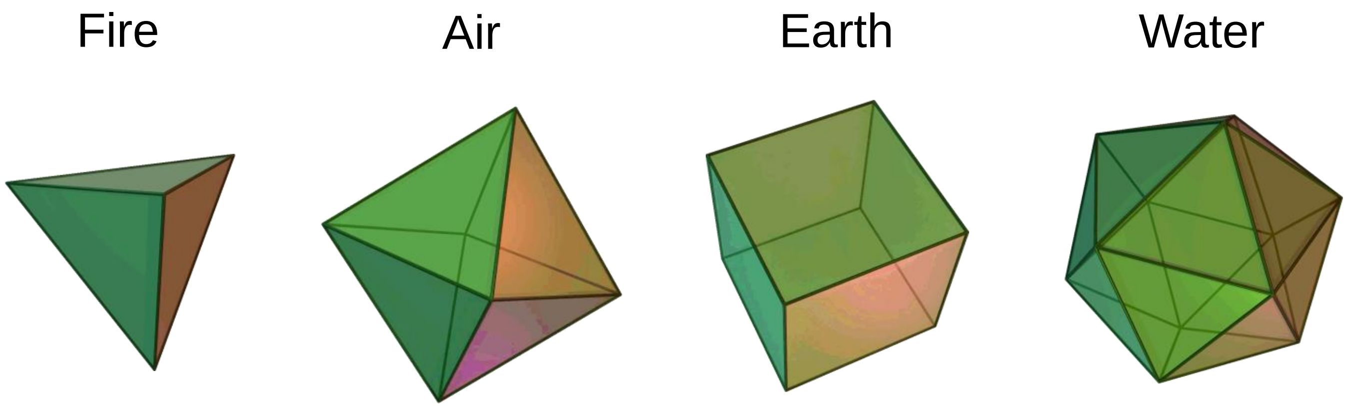
550 = 210 + 340

$$10F_{10} = 10\,F_8 + 10\,F_9$$

Figure 17

The first four Platonic solids have **248** vertices & sides in the 120 sectors of their 38 faces. This is the dimension of the rank-8, exceptional Lie group E_8 that plays a fundamental role in superstring theory. It demonstrates that the four regular polyhedra believed by the ancient Greeks to be the shapes of the particles of the elements Earth, Water, Air & Fire do, indeed, embody the physics governing all basic subatomic particles. **248** is the number value of *Raziel*, the Archangel of Binah (see table in Fig. 1).

The square is the symbol of the Pythagorean Tetrad, or number 4. When its sectors are so-called '2nd-order tetractyses' (tetractyses generated by replacing the ten points of a tetractys by a tetractys), there are **248** points (shown coloured) other than corners of tetractyses. The seven coloured points in each tetractys formally symbolize the seven Sephiroth of Construction and its corners denote the Supernal Triad of Kether, Chokmah & Binah.



Number of vertices & sides of triangular sectors of faces = **248**

Embodiment of the **248** roots of the superstring gauge symmetry group E_8 in the first four Platonic solids and in the square

248 =

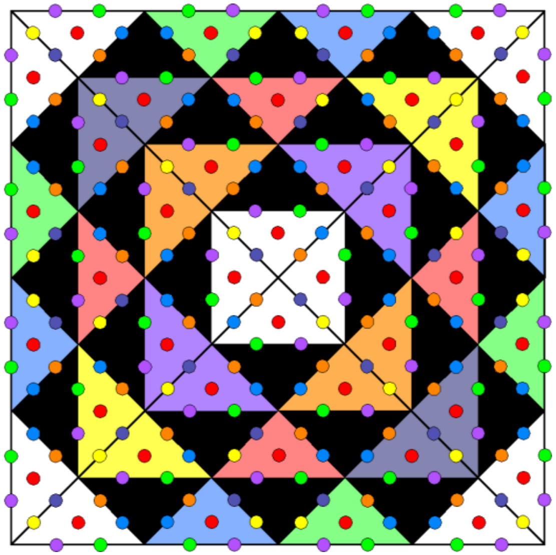
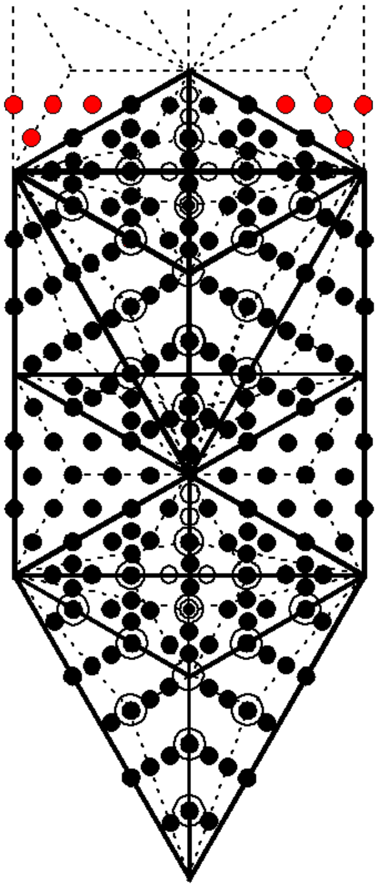


Figure 18

We saw earlier that the lowest Tree of Life has 55 geometrical elements. When its 19 triangles are divided into their 57 sectors and each sector then turned into a tetractys, there are 240 (black) yods generated by this transformation, i.e., yods other than the original 11 corners of the 19 triangles. A similar transformation of triangles outside this Tree generates eight (red) yods below its top. The 240 yods symbolise the 240 (non-zero) roots of E_8 and the eight yods denote its eight simple (zero) roots.

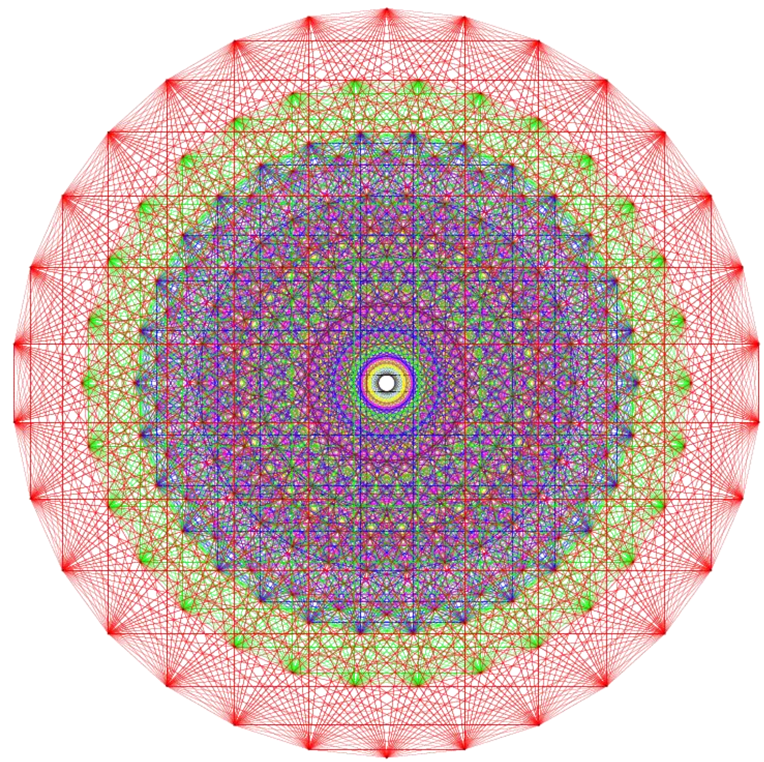
The E_8 root system consists of 240 vectors in an eight-dimensional space. Those vectors are the vertices (corners) of an eight-dimensional object called the Gosset polytope 4_{21}. In the 1960s, Peter McMullen drew (by hand) a two-dimensional representation of the Gosset polytope 4_{21}. The image shown here was computer-generated by John Stembridge, based on McMullen's drawing. (Credit: Image courtesy of American Institute of Mathematics).



= 248

240 (●) → 240 non-zero roots of E_8
8 (●) → 8 zero roots of E_8

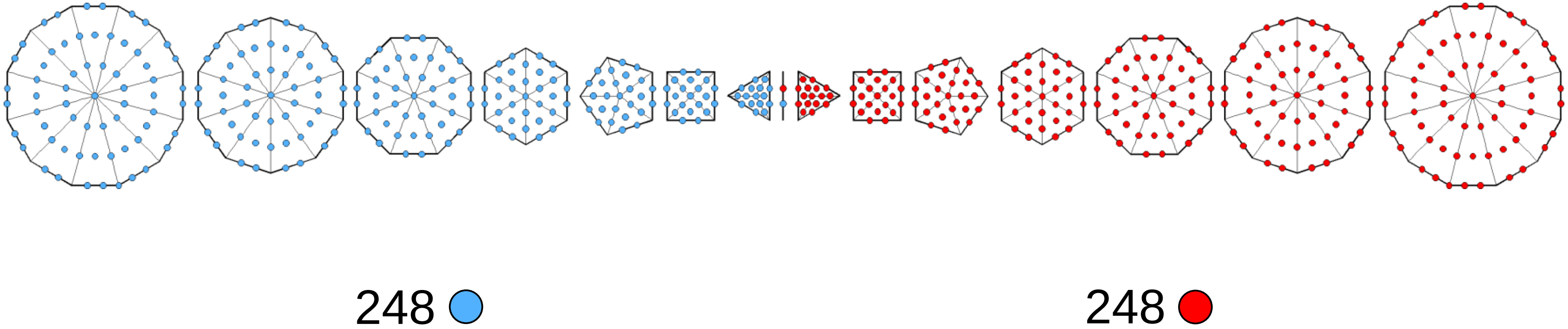
Below the apex of the 1-tree are **248** yods other than SLs that denote the **248** roots of the superstring gauge symmetry group E_8 and therefore the **248** gauge bosons transmitting the unified superstring force.



2-d representation of the 8-d Gosset polytope 4_{21} whose 240 vertices denote the 240 non-zero roots of E_8.

Figure 19

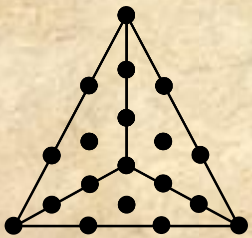
When the **48** sectors of the seven separate regular polygons in one half of the inner Tree of Life are converted into tetractyses, there are 247 yods other than the given **48** corners of the polygons, i.e., 247 new yods appear. Four yods lie along each edge, so that two extra yods appear when the root edge, now regarded as a separate straight line, is turned into such an edge. One of them is associated with one set of polygons and the second is associated with the other set. Hence, **248** yods are associated with each set and the root edge separating the two sets of polygons. They symbolise the (**248**+**248**=**496**) roots of E_8**×**E_8. The seven centres of each set and its associated yod on the root edge denote the eight simple roots of E_8 and the 240 other yods (called 'hexagonal yods' in previous articles because they are located at the corners and centre of hexagons) symbolise its 240 roots. The 240 yods belong to a geometrical object with 55 corners. They are the counterpart to the 240 yods that belong to the lowest Tree of Life that possesses 55 geometrical elements. This is how the tenth Fibonacci number F_{10} determines the superstring gauge symmetry group E_8.



The (248+248) yods other than corners that lie on the root edge and the (96+96) edges of the (48+48) sectors of the (7+7) separate polygons symbolize the (248+248) gauge bosons of the $E_8 \times E_8$ heterotic superstring

Figure 20

Suppose that the **50** faces of the five Platonic solids are divided into their 180 sectors and that their 90 internal triangles, created by joining vertices to their centres, are divided into their 270 sectors. Then suppose that these 450 sectors with 190 vertices surrounding the centres of Platonic solids are each turned into a tetractys. Four yods lie along every one of their 590 sides, two yods being between the ends of every side. The number of yods surrounding the centres of the five solids and lining the sides of their 450 tetractyses = 190 + 2×590 = 1370. This is the number of yods in 137 tetractyses. The number 137 shapes the archetypal set of five regular polyhedra. It is one of the most important numbers in modern physics, being the number whose reciprocal is approximately equal to the dimensionless fine-structure constant $\alpha = e^2/\hbar c \approx 1/137$, where e is the electric charge of the electron, ħ (=h/2π) is the reduced Planck's constant & c is the speed of light *in vacuo*. Its magnitude remains a mystery. The number 137 is a defining parameter of holistic systems, being embodied in all sacred geometries .



Number of yods lining sides of tetractyses = number of vertices + 2×number of sides

Number of vertices surrounding centres

	Tetrahedron	Octahedron	Cube	Icosahedron	Dodecahedron
Faces:	4 + 4 = 8	6 + 8 = 14	8 + 6 = 14	12 + 20 = 32	20 + 12 = 32
Interior:	6	12	12	30	30
Subtotal =	14	**26**	**26**	**62**	**62**

Total = 190

Number of sides

Faces:	6 + 4×3 = 18	12 + 8×3 = **36**	12 + 6×4 = **36**	30 + 20×3 = 90	30 + 12×5 = 90
Interior:	4 + 6×3 = 22	6 + 12×3 = 42	8 + 12×3 = 44	12 + 30×3 = 102	20 + 30×3 = 110
Subtotal =	40	78	**80**	192	200

Total = 590

Number of yods on edges of tetractyses = 190 + 2×590 = 1370

Figure 21

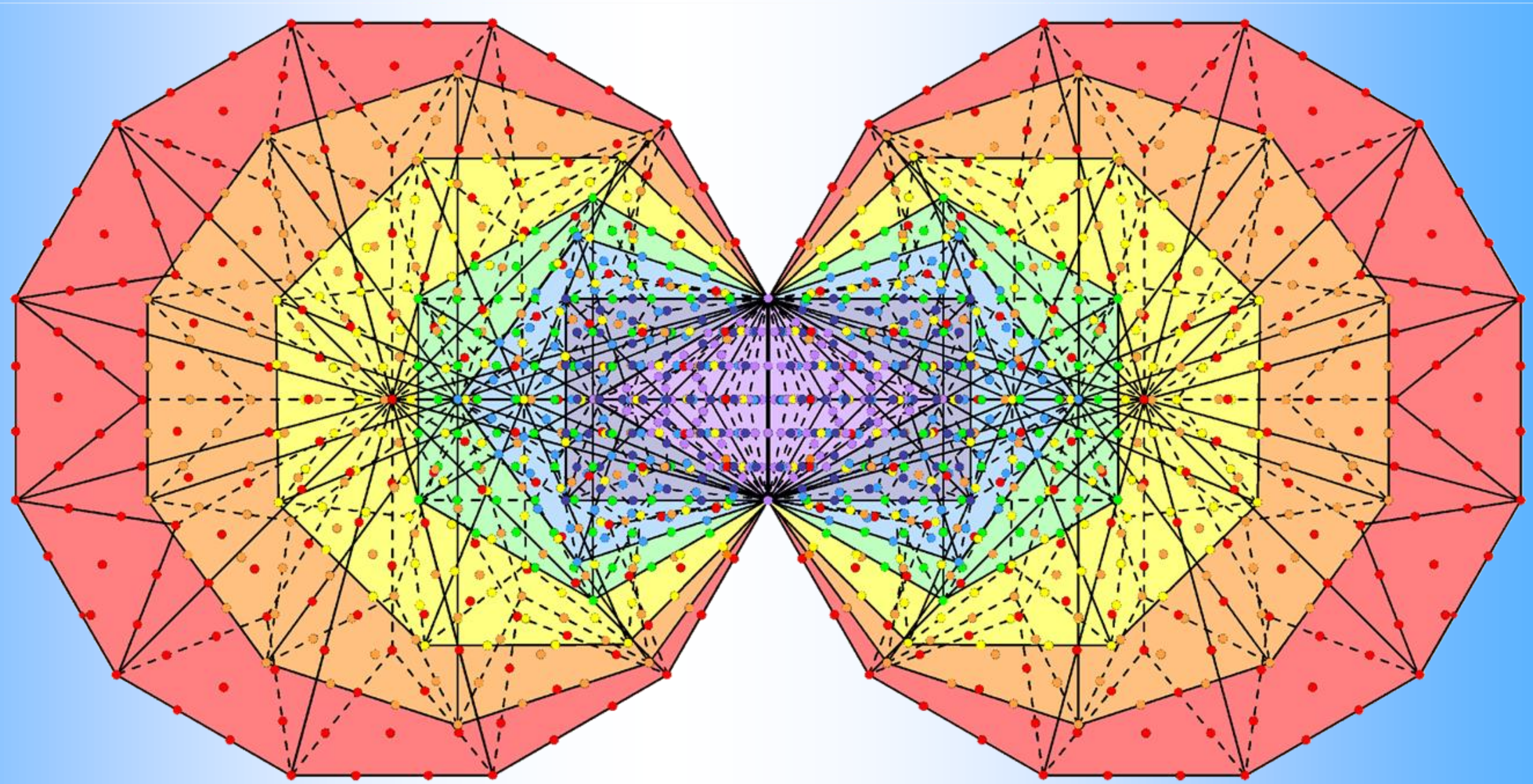
When the 94 sectors of the 14 enfolded, regular polygons making up the inner Tree of Life are themselves divided into their sectors and the latter then turned into tetractyses, the resulting **282** tetractyses have 1370 yods. This is the number of yods in 137 tetractyses. It is the same as the number of yods surrounding the centres of the five Platonic solids that line all the tetractyses needed to construct their faces and interiors. It is a remarkable illustration of how different holistic systems embody analogous properties.

The 14 enfolded polygons have 70 corners. This leaves 1300 yods that are added by the construction of the inner Tree of Life from tetractyses. The integers 1, 2, 3 & 4 symbolized by the four rows of dots in the tetractys express this number as

$$1300 = 1^5 + 2^5 + 3^5 + 4^5.$$

It is an example of the beautiful, mathematical properties of the inner Tree of Life.

1370 =



The Inner Tree of Life embodies the number 137 determining the fine structure constant $e^2/\hbar c \approx 1/137$

Figure 22

There are 90 triangles inside the five Platonic solids formed by their 90 polyhedral edges and by the **50** straight lines joining their **50** vertices and their five centres. They can be further divided into their 270 sectors. This generates (3×90=270) new sides, 90 new corners and 270 internal triangles. The number of points, lines & triangles inside the five solids that surround their centres = 90 + **50** + 270 + 270 = 680. According to Fig. 16, the number of points, lines & triangles in the **50** faces of the solids is 550. Therefore, the number of geometrical elements surrounding their centres = 550 + 680 = 1230 = $10L_{10}$, where L_{10} (123) is the tenth Lucas number. There are 340 internal elements in each half of the five solids. 340 = $10F_9$, where F_9 (34) is the ninth Fibonacci number. The relation

$$L_n = F_n + 2F_{n-1}$$

is geometrically realised for n = 10:

$$1230 = 550 + 680$$

$$10L_{10} = 10F_{10} + 10\times2F_9.$$

The factor '2' expresses the two halves of each solid. The factor '10' expresses the ten halves of the five solids. L_{10} (=123) is the average number of geometrical elements in each half that surround the centres of the five solids and F_{10} (= 55) is the average number of geometrical elements in each half of their faces. The Golden Ratio Φ determines the average geometrical composition of a Platonic solid because $L_{10} = \Phi^{10} + \Phi^{-10}$. This beautiful property shows how the Decad measures their geometrical composition.

The 55:68 distinction generated by the faces and the interior of the five Platonic solids corresponds in Fig. 5 to the 55 geometrical elements in the lowest Tree of Life and the 68 elements added in the second and third Trees.

Number of internal vertices of (3×90=270) triangles = 90.

Number of internal edges = **50** + (3×90=270) = 320.

Number of internal geometrical elements = 90 + 320 + 270 = 680.

Number of geometrical elements in faces = 550 = 10×F_{10}.

Total number of geometrical elements surrounding centres = 550 + 680 = 1230 = 10×L_{10}.

n	0	1	2	3	4	5	6	7	8	9	10
F_n	0	1	1	2	3	5	8	13	21	34	55
L_n	2	1	3	4	7	11	18	29	47	76	123

$$L_n = F_n + 2F_{n-1}$$

$$123 = 55 + 2\times 34 = 55 + 68$$

$$1230 = 550 + 680$$

external internal

$$F_n = \frac{\Phi^n - (-\Phi)^{-n}}{\sqrt{5}} \qquad L_n = \Phi^n + (-\Phi)^{-n}$$

$$F_{10} = \frac{\Phi^{10} - \Phi^{-10}}{\sqrt{5}} = \frac{\Phi^{10} - \Phi^{-10}}{\Phi + \Phi^{-1}} \qquad L_{10} = \Phi^{10} + \Phi^{-10}$$

$$1230 = 10\times(\Phi^{10}+\Phi^{-10})$$

Average number of elements in each half of 5 Platonic solids = 1230/10 = $L_{10} = \Phi^{10} + \Phi^{-10}$

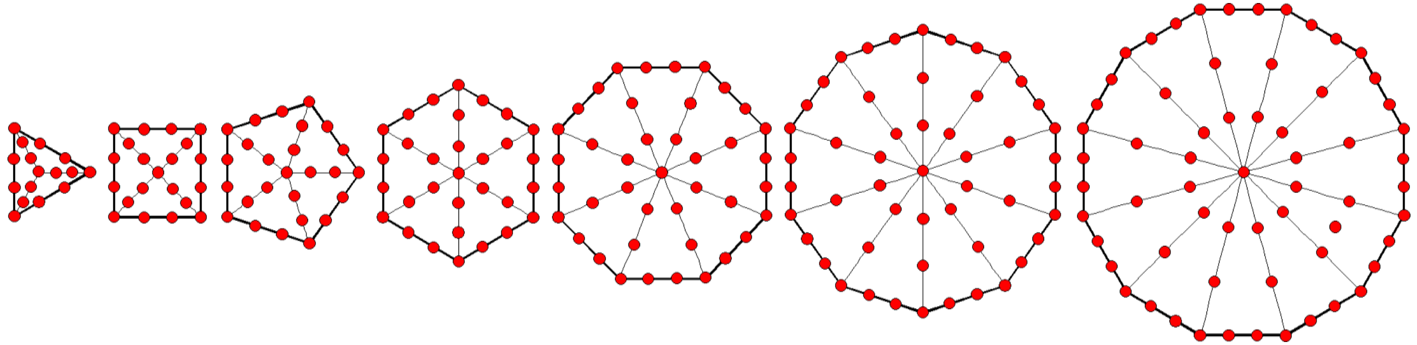
Figure 23

The five Platonic solids are made up of 1235 points, lines & triangles, including their centres. The average number of geometrical elements is 247. This is the number of yods lining the **48** tetractyses that make up the seven regular polygons of the inner Tree of Life. It is also the number of yods in the first four enfolded polygons when their sectors are divided into three tetractyses.

The average number of geometrical elements that surround the centres of the five Platonic solids is 1230/5 = **246**. This is the number value of *Gabriel*, the Archangel of Yesod.

Including 5 centres, number of geometrical elements = 1235.

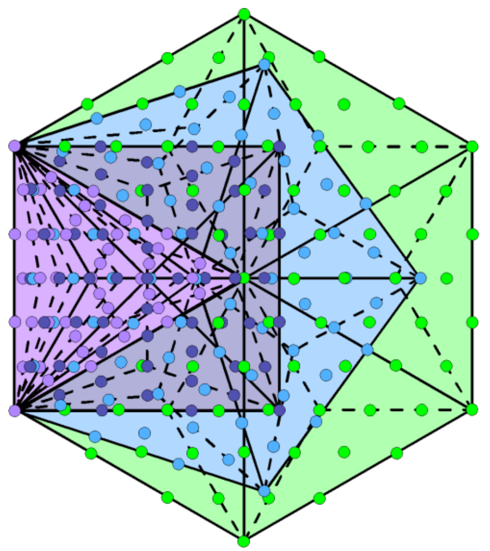
Average number = 1235/5 = 247.



247

247 yods line the **48** tetractys sectors of the 7 polygons

The first 4 enfolded polygons have 247 yods



Average number of geometrical elements in faces of each half of a Platonic solid = 550/10 = 55 = F_{10}.

Average number of geometrical elements in each half of a Platonic solid = 1230/10 = 123 = L_{10}.

Average number of internal elements in each half = $L_{10} – F_{10}$ = 68 = $2F_9$

Number of internal elements (including centres) = 680 + 5 = 685.

Average number of elements (including centres) inside 5 Platonic solids = 685/5 = 137.

Number of internal elements surrounding centres = 680 = $10(L_{10}-F_{10})$

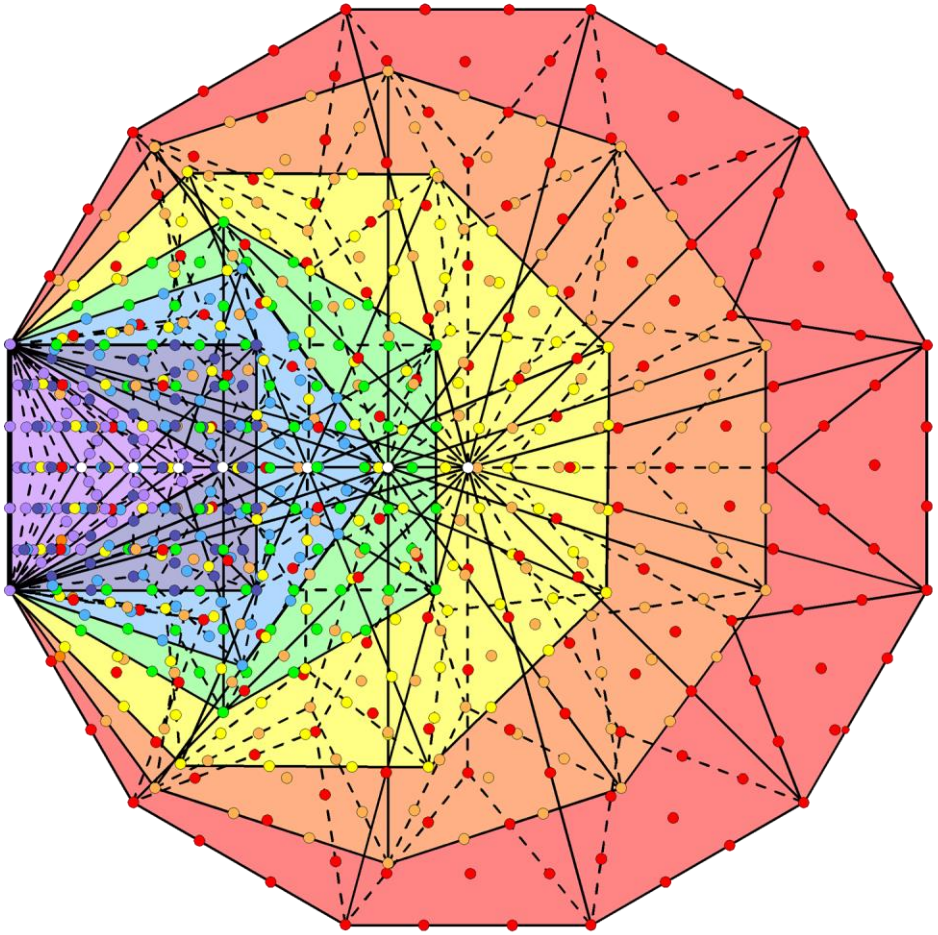
= $1^2 + 3^2 + 5^2 + 7^2 + 9^2 + 11^2 + 13^2 + \mathbf{15}^2$.

15 is the number value of YAH, the older Godname assigned to Chokmah.

Figure 25

We saw in Fig. 21 that the 14 enfolded polygons of the inner Tree of Life have 1370 yods. Each set of seven enfolded polygons has 687 yods. Therefore, 680 yods surround their centres. This is the counterpart of the 680 geometrical elements inside the five Platonic solids that surround their centres. Their interior constitutes a Tree of Life pattern characterised by the same parameter 680, which is determined by the ninth Fibonacci number 34.

680 =



The number of yods surrounding the centres of the seven enfolded polygons is the number of geometrical elements inside the five Platonic solids that surround their centres

Figure 26

We saw in Fig. 10 that the set of 14 polygons enfolded in each overlapping Tree of Life has 68 corners (two of the 70 corners coincide with corners of the two hexagons enfolded in the next higher Tree). The **140** polygons enfolded in ten overlapping Trees of Life have 680 corners that belong alone to those polygons. Ten such Trees are a representation of a single Tree, with each Sephirah replaced by a Tree of Life. This indicates that the five Platonic solids and ten Trees of Life are analogous sacred geometries. **140** is the number value of *Masloth*, the Mundane Chakra of Chokmah.

The (70+70) polygons enfolded in ten Trees of Life have 680 corners

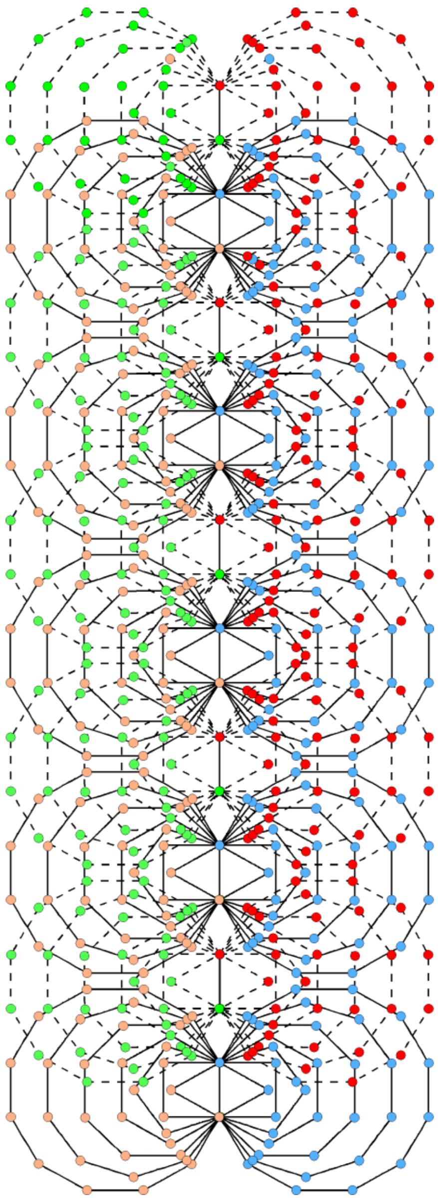


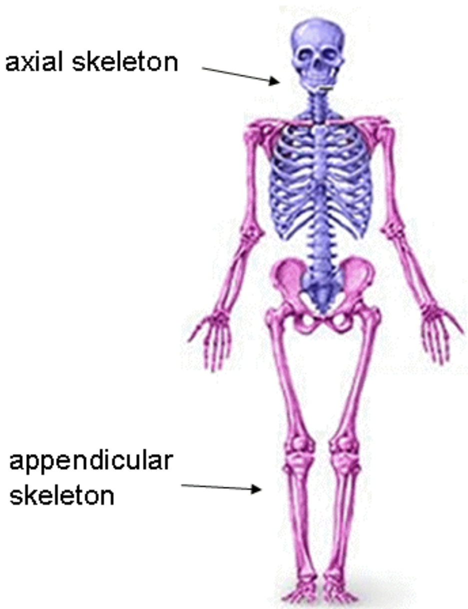
Figure 27

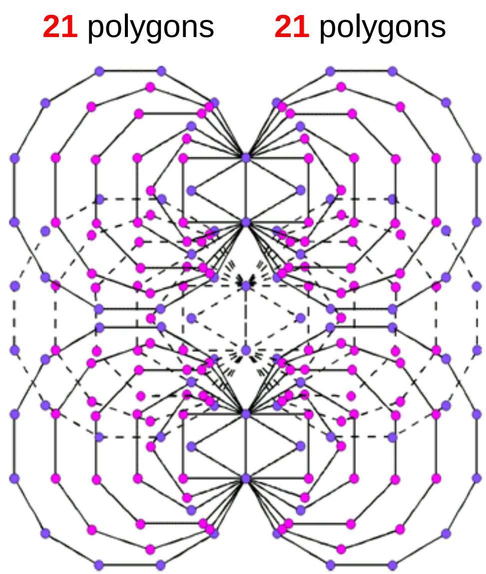
The two sets of **21** polygons enfolded in the lowest three Trees of Life have 206 corners. These Trees map the 3-dimensional aspect of Adam Kadmon, which for a human being is his or her physical body, its skeleton having 206 bones. This is why L_{10} measures the geometrical composition of these Trees (see Fig. 5). L_{10} also measures the geometry of their inner form because each set of **21** polygons has 123 sides outside their root edges.

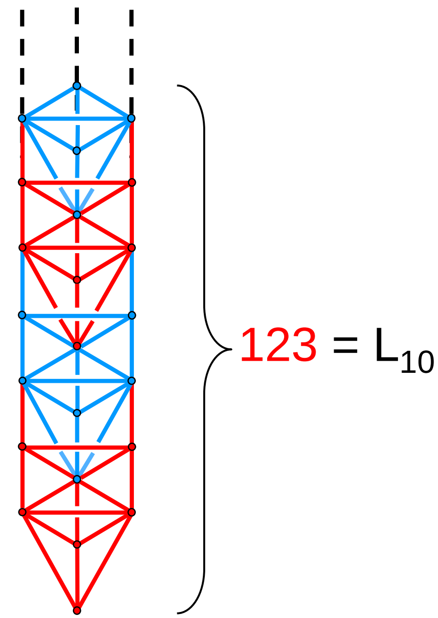
The **80** bones of the axial skeleton comprise 34 single bones and 23 pairs of bones, one on the left of the body and one on the right. 34 is F_9. **80** is the number value of Yesod, whose meaning, “foundation,” aptly describes this core set of bones.

Number of corners of the 14n polygons enfolded in n lowest Trees of Life ≡ C(n) = 68n + 2.

C(3) = 206.







80 bones in the axial skeleton

126 bones in the appendicular skeleton

80 (●) corners

126 (●) corners

The Lucas number L_{10} measures the geometrical composition of the outer form of the 3 lowest Trees of Life whose inner form has (21+21) polygons with 206 corners symbolizing the 206 bones of the human skeleton. L_{10} also measures the geometry of its inner form because each set of 21 polygons has 123 sides outside their shared root edges.

Figure 28

Number of corners of triangles in the lowest n Trees ≡ $S(n) = 6n + 5$.

Number of sides of triangles ≡ $E(n) = 16n + 9$.

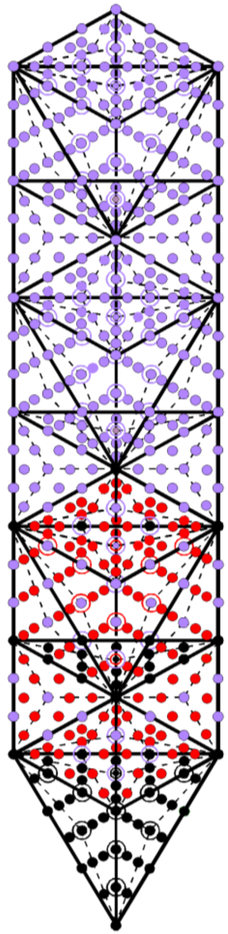
Number of triangles ≡ $T(n) = 12n + 7$.

Suppose that the three sectors of each triangle are tetractyses. Each triangle then has ten yods inside it.

Number of yods in the lowest n Trees = $S(n) + 10T(n) + 2E(n) = 158n + 93$.

The three lowest Trees have 567 yods, of which 206 yods symbolize the 206 bones of the human skeleton and 361 yods denote the 361 classical acupuncture points (5,6).

Circles denote yods behind other yods



As the representation of the 3-d form of Adam Kadmon, the lowest three Trees of Life encode the 206 bones and 361 classical acupuncture points in the human body

361 (●) → 361 acupoints

$\left.\begin{array}{r} \text{126 (●)} \\ \text{80 (●)} \end{array}\right\}$ 206 → 206 bones

Total = 567

Figure 29

For centuries the music of the Roman Catholic Church has been based upon eight ‘modes.’ Four modes:

Dorian, Phrygian, Lydian & Mixolydian

are called ‘authentic’ and four:

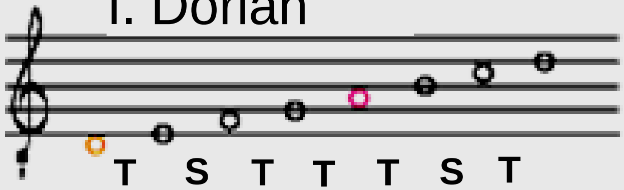
Hypodorian, Hypophrygian, Hypolydian & Hypomixolydian

are called ‘plagal.’ They are seven different musical scales with distinct orderings of intervals between their notes, the first (Dorian) and last (Hypomixolydian) having the same pattern of intervals but a different *dominant* (reciting note) and *finalis* (ending note) (7).

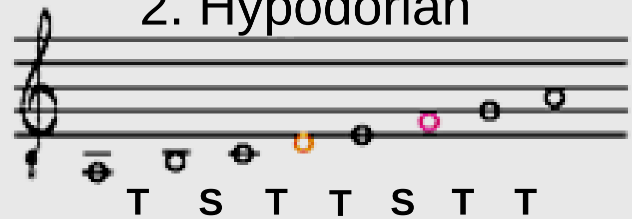
Church Musical Modes

Authentic | **Plagal**

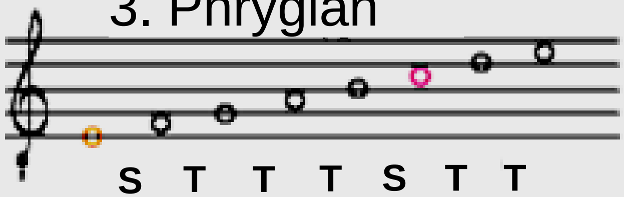
I. Dorian



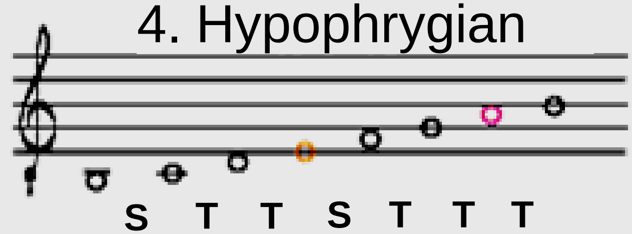
2. Hypodorian



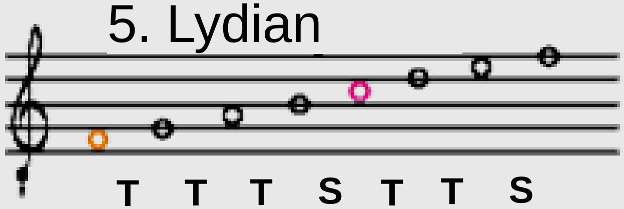
3. Phrygian



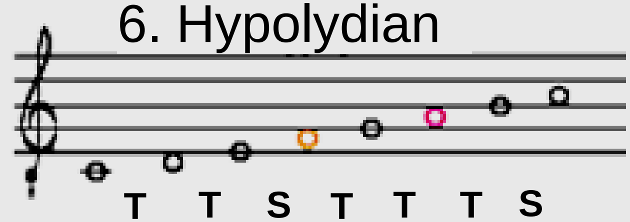
4. Hypophrygian



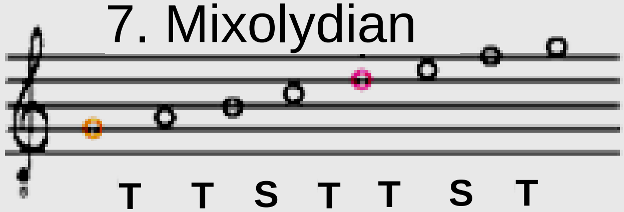
5. Lydian



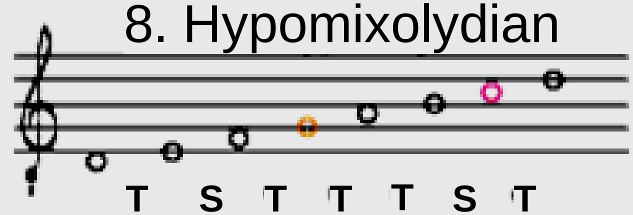
6. Hypolydian



7. Mixolydian



8. Hypomixolydian



S = semitone
T = whole tone

Finalis
(ending note)

Dominant
(Reciting note)

Figure 30

The pitches of the notes in the church modes are those of the modern equal-tempered scale. However, this is an invention by musicians to make their playing of music more convenient with more notes available than the ancient seven note Pythagorean scale:

T T L T T T L

where T is the tone interval of 9/8 and L (leimma) is the interval of 256/243 (the Pythagorean counterpart of the modern half-tone). Figure 30 shows the tone ratios of the seven types of musical scale (the C scale is the Pythagorean scale). Coloured cells denote notes belonging to the C scale and white cells denote non-Pythagorean notes. There are 12 types of notes between the tonic with tone ratio 1 and the octave with tone ratio 2. The last seven notes with tone ratio n are the inversions of their partners in the first seven notes with tone ratio m, where mn = 2. The thick vertical line separates notes and their inversions. Bold tone ratios refer to notes of the Pythagorean musical scale.

Tone ratios of notes in the seven octave species

F scale	1	9/8	81/64	729/512	3/2	27/16	243/128	2
E scale	1	256/243	32/27	4/3	3/2	128/81	16//9	2
D scale	1	9/8	32/27	4/3	3/2	27/16	16/9	2
C scale	1	9/8	81/64	4/3	3/2	27/16	243/128	2
B scale	1	256/243	32/27	4/3	1024/729	128/81	16/9	2
A scale	1	9/8	32/27	4/3	3/2	128/81	16/9	2
G scale	1	9/8	81/64	4/3	3/2	27/16	16/9	2

(white cells denote non-Pythagorean tone ratios)

1 256/243 **9/8** 32/27 **81/64** **4/3** 1024/729 | 729/512 **3/2** 128/81 **27/16** **16/9** **243/128** 2

Figure 31

The table in Fig. 31 lists the intervals below the octave between the notes in each of the seven musical scales and the numbers of each type (8). The 189 intervals comprise 125 intervals with tone ratios equal to the first six types of notes above the tonic and **64** intervals with tone ratios those of their inversions. Of these, 123 intervals have the tone ratios of Pythagorean notes. 123 is the tenth Lucas number L_{10}. They include 47 intervals that are either 9/8 or 27/16 and **76** other intervals. 47 is the eighth Lucas number and **76** is the ninth Lucas number. These are the only possible subsets of intervals that number either 47 or **76**.

There are 123 rising Pythagorean intervals (tone ratio n) below the octave and 123 falling intervals (tone ratio 1/n). The number value **246** of *Gabriel*, the Archangel of Yesod, is the number of rising & falling Pythagorean intervals below the octave between the notes of the seven musical scales. As found in the comment on Fig. 23, this is also the average number of geometrical elements in the five Platonic solids. The counterpart in the Platonic solids of the 123 rising Pythagorean intervals and the 123 falling Pythagorean intervals is the 123 geometrical elements on average in each half of the solids.

Number of intervals below the octave between the notes of the seven musical scales

256/243	14	**243/128**	4
9/8	35	16/9	10
32/27	24	**27/16**	12
81/64	18	128/81	9
4/3	30	**3/2**	24
1024/729	4	729/512	5
Total =	125		**64**

47

256/243 **9/8** 32/27 **81/64** **4/3** 1024/729 | 729/512 **3/2** 128/81 **27/16** 16/9 **243/128**

Number of Pythagorean intervals below the octave =

$35 + 18 + 30 + 4 + 12 + 24 = 123 = L_{10} = 47 + \mathbf{76} = L_8 + L_9$

References

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