

ARTICLE 43

The Tree of Life Nature of the {3,7} Tessellation of the 168 Automorphisms of the Klein Quartic on the 3-torus

by

Stephen M. Phillips

Flat 3, 32 Surrey Road South. Bournemouth. Dorset BH4 9BP. England.

Website: <http://www.smphillips.mysite.com>

Abstract

The Godnames assigned to the ten Sephiroth of the Tree of Life prescribe properties of the dodecagon. As the tenth regular polygon, the dodecagon is unique in embodying the superstring structural parameter 168, the dimension 248 of the superstring gauge symmetry group E_8 and the number of chromosomes in the human cell. Moreover, its geometrical composition conforms to the 3:7 pattern of yods in the tetractys — the Pythagorean counterpart of the Tree of Life. The 24 hexagonal yods on its sides and the 144 internal yods surrounding its centre denote, respectively, the 24 leptons and 144 subquarks belonging to the family of 168 particles hypothesized in Part 4 of Article 40. Six 7-fold systems exhibiting different kinds of duality and characterized by the number 168 are compared. The basic 24:144 division of any holistic system is displayed in the 168 yods other than corners associated with each set of the first six enfolded polygons of the inner Tree of Life, the pentagon having 24 such yods and the five other polygons having 144 yods. The same division appears in the two separate Type B dodecagons, which have 24 corners and 144 vertices and sides of their 72 tetractyses. It also manifests in the disdyakis triacontahedron, whose properties are analogous to the {3,7} tessellation of the 3-torus. Constructed from four triangular prisms and six square antiprisms, the 3-torus is composed of 260 geometrical elements. This indicates that it conforms to the archetypal pattern of the inner form of the Tree of Life, whose seven enfolded polygons contain 260 yods outside their shared edge. The Sri Yantra has the same geometrical composition. Just as 336 yods are on the edges of the separate prisms and antiprisms, so 336 yods line the sides of the 42 triangles of the Sri Yantra when they are turned into tetractyses. Their counterparts in the pair of joined dodecagons are the 336 yods other than their corners. In each case, they denote the number of circularly polarized oscillations of each closed curve of the heterotic superstring as it winds once around the axis of spin of the particle. The 24:144 division appears in the tessellated 3-torus as the 24 triangles in the prisms and the 144 triangles in the antiprisms when their triangular faces are each divided into three triangles. Additional evidence that the tessellated 3-torus conforms to the Tree of Life is the fact that 264 hexagonal yods lie on the edges of the 56 triangular faces of the separate prisms and antiprisms, for this is the number of yods in the seven enfolded polygons. Alternatively, the separate prisms and antiprisms of the 3-torus and its turned inside-out version have 264 sides when their triangular faces are divided into three tetractyses. The 3-torus with single tetractyses as faces comprises 248 yods that denote the 248 roots of E_8 , the yods at the centres of the eight triangular faces of the four prisms denoting its eight simple roots. The two halves of the Sri Yantra are the counterpart of the 3-torus and its turned inside-out version. Each half of the first two layers has nine triangles corresponding to the four prisms. Each half of the last two layers with 12 triangles corresponds to the six antiprisms. 192 yods line the sides of the 56 triangles of the {3,7} tessellation when they are turned into tetractyses. 24 of them are vertices of the prisms and the other 168 are hexagonal yods on the 48 edges of the antiprisms. Their counterpart in the Pythagorean musical scale are the 168 harmonics between the 10th overtone with tone ratio 24 and the 24th overtone with tone ratio 192. 1680 yods surround the vertices and centres of the 56 triangles in the tessellated 3-torus when their 168 sectors are each divided into three tetractyses. The {3,7} tessellation embodies not only the dimension 248 of the superstring symmetry group E_8 but also the superstring structural parameter 1680 — the number of circularly polarized oscillations in each closed curve of the $E_8 \times E_8$ heterotic superstring.

Table 1. Gematria number values of the ten Sephiroth.

	SEPHIRAH	GODNAME	ARCHANGEL	ORDER OF ANGELS	MUNDANE CHAKRA
1	Kether (Crown) 620	EHYEH (I am) 21	Metatron (Angel of the Presence) 314	Chaioth ha Qadesh (Holy Living Creatures) 833	Rashith ha Gilgalim First Swirlings. (Primum Mobile) 636
2	Chokmah (Wisdom) 73	YAHVEH, YAH (The Lord) 26, 15	Raziel (Herald of the Deity) 248	Auphanim (Wheels) 187	Masloth (The Sphere of the Zodiac) 140
3	Binah (Understanding) 67	ELOHIM (God in multiplicity) 50	Tzaphkiel (Contemplation of God) 311	Aralim (Thrones) 282	Shabathai Rest. (Saturn) 317
4	Daath (Knowledge) 474				
4	Chesed (Mercy) 72	EL (God) 31	Tzadkiel (Benevolence of God) 62	Chasmalim (Shining Ones) 428	Tzadekh Righteousness. (Jupiter) 194
5	Geburah (Severity) 216	ELOHA (The Almighty) 36	Samael (Severity of God) 131	Seraphim (Fiery Serpents) 630	Madim Vehement Strength. (Mars) 95
6	Tiphareth (Beauty) 1081	YAHVEH ELOHIM (God the Creator) 76	Michael (Like unto God) 101	Malachim (Kings) 140	Shemesh The Solar Light. (Sun) 640
7	Netzach (Victory) 148	YAHVEH SABAOTH (Lord of Hosts) 129	Haniel (Grace of God) 97	Tarshishim or Elohim 1260	Nogah Glittering Splendour. (Venus) 64
8	Hod (Glory) 15	ELOHIM SABAOTH (God of Hosts) 153	Raphael (Divine Physician) 311	Beni Elohim (Sons of God) 112	Kokab The Stellar Light. (Mercury) 48
9	Yesod (Foundation) 80	SHADDAI EL CHAI (Almighty Living God) 49, 363	Gabriel (Strong Man of God) 246	Cherubim (The Strong) 272	Levanah The Lunar Flame. (Moon) 87
10	Malkuth (Kingdom) 496	ADONAI MELEKH (The Lord and King) 65, 155	Sandalphon (Manifest Messiah) 280	Ashim (Souls of Fire) 351	Cholem Yesodoth The Breaker of the Foundations. The Elements. (Earth) 168

The Sephiroth exist in the four Worlds of Atziluth, Beriah, Yetzirah and Assiyah. Corresponding to them are the Godnames, Archangels, Order of Angels and Mundane Chakras (their physical manifestation). This table lists their number values obtained by the practice of gematria, wherein a number is assigned to each letter of the alphabet, thereby giving to a word a number value that is the sum of the numbers associated with its letters.

(All numbers from this table referred to in the article are written in **boldface**).

1. Godnames prescribe the dodecagon

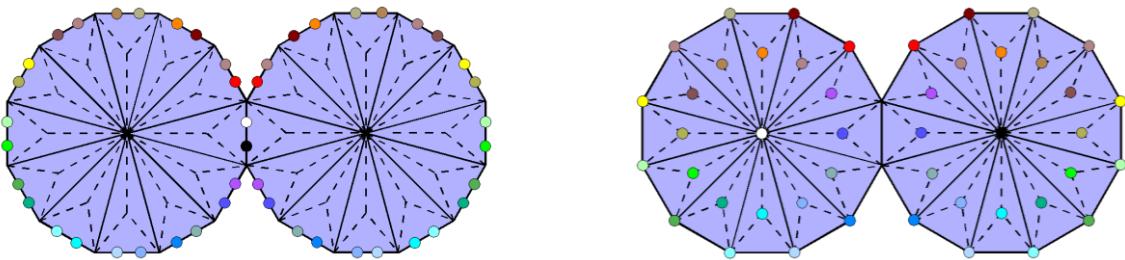
The Godnames (Table 1) prescribe the dodecagon and pair of dodecagons as follows:

EHYEH (21): number of yods in Type A dodecagon = **73** = **21**st prime number.

YAH (15): number of yods in the root edge and pair of Type A dodecagons = **73 + 4 + 73 = 150 = 10×15**.

YAHWEH (26): number of vertices of 24 sectors in pair of separate Type A dodecagons = **26**. Number of geometrical elements in pair of Type A dodecagons + root edge = **98 + 3 = 101 = 26**th prime number. **260 (=26×10)** yods in two joined dodecagons outside root edge other than vertices & centres of tetractyses.

ELOHIM (50): number of vertices & triangles in pair of separate Type A dodecagons = **26 + 24 = 50**. Number of corners of **72** tetractys sectors of pair of Type B dodecagons = **50**. There are **282** yods on the 118 external sides of the two separate Type B dodecagons. **282** is the number value of *Aralim* ("Thrones"), the Order of Angels assigned to Binah.



46 hexagonal yods lie on the boundary of the pair of Type B dodecagons. The pair of black & white yods on the root edge denotes the two sex chromosomes. The pair of 22 coloured yods denotes the two sets of 22 autosomes.

The **72** tetractyses in the pair of Type B dodecagons have 46 corners outside the root edge. The black and white central yods denote the pair of sex chromosomes. The pair of 22 coloured yods denotes the two sets of 22 autosomes.

Figure 1. The pair of dodecagons embodies the diploid number 46 of the human cell in two different ways.

EL (31): number of yods on sides of 12 tetractyses in Type A dodecagon = **61** = **31**st odd integer. **310 (=31×10)** hexagonal yods in pair of joined, Type B dodecagons.

ELOHA (36): number of yods on boundary of dodecagon = **36**. Number of yods surrounding centres of pair of separate, Type B dodecagons = **360 = 36×10**.

YAHWEH ELOHIM (76): number of yods on boundaries of two separate dodecagons and in root edge = **36 + 36 + 4 = 76**. **YAHWEH (26)** prescribes their **26** corners and endpoints; **ELOHIM (50)** prescribes the number of hexagonal yods.

YAHWEH SABAOTH (129): 130 yods outside the root edge in dodecagon other than vertices & centres of tetractyses. **130 = 129**th integer after 1.

ELOHIM SABAOTH (153): **153** yods outside the root edge other than 24 yods on the internal edges of sectors.

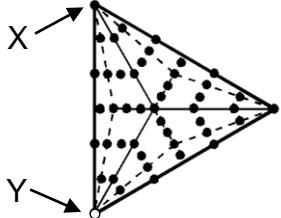


Figure 2. The 46 yods in the Type B triangle symbolize the 46 chromosomes in the human cell.

EL ChAI (49): number of sides in root edge and pair of separate Type A dodecagons = **24 + 1 + 24 = 49**.

Number of vertices, sides & triangles in pair of separate Type A dodecagons = **26 + 48 + 24 = 98 = 49**th even integer.

Number of vertices, sides & triangles in Type A dodecagon = **13 + 24 + 12 = 49**.

ADONAI MELEKH (155): **155** hexagonal yods associated with each joined, Type B dodecagon.

Of the 181 yods in a dodecagon, **36** are centres of tetractyses, leaving 145 yods on their 60 sides. 141 yods are on sides outside the root edge. The pair of joined dodecagons has $(141+4+141=286)$ yods on 61 sides. Of these, 46 yods are vertices of **72** tetractyses, leaving 240 hexagonal yods on sides. The pair of joined dodecagons has 358 yods. 46 of these are vertices of **72** tetractyses, leaving 312 hexagonal yods. The pair of joined dodecagons (Fig. 1) embodies the human diploid number 46. The X and Y sex chromosomes (non-autosomes) correspond to the centres of the two dodecagons. The 44 vertices outside the root edge correspond to the 44 autosomal chromosomes. Alternatively, there are 46 hexagonal yods on the boundaries of the two dodecagons, the pair of hexagonal yods on the root edge symbolizing the X and Y chromosomes. The 46 yods in a Type B triangle (Fig. 2) also symbolize the 46 human chromosomes, the sex chromosomes corresponding to the two ends of the root edge.

2. 240 roots of E_8

A Type B dodecagon has 25 vertices, 60 sides & **36** triangles. 84 vertices & sides surround its centre. The pair of separate dodecagons has $(84+84=168)$ vertices & sides surrounding their centres. Each dodecagon has 181 yods, i.e., **168** yods other than its centre and corners. The two separate dodecagons have **72** triangles. There are 240 geometrical elements surrounding the centres of the pair of dodecagons. The split:

$$240 = 168 + 72$$

mirrors the division between the **72** roots of E_6 , a subgroup of the superstring gauge symmetry group E_8 , and the **168** roots of E_8 that are not roots of E_6 . It corresponds to the **168** vertices & sides and the **72** triangles surrounding the centre of the dodecagon.

This **72:168** division appears, alternatively, as follows: a Type A dodecagon has 13 vertices and 24 sides of its 12 sectors, i.e. **49** geometrical elements. The 240 geometrical elements of the two Type B dodecagons comprise **72** elements found in the pair of Type A dodecagons and **168** extra elements. Their composition is:

$$(12+12) \times (1 \text{ vertex} + 3 \text{ sides} + 3 \text{ triangles}) = 24 \text{ vertices} + (3 \times 24 = 72) \text{ sides} + (3 \times 24 = 72) \text{ triangles}$$

The numbers of the three types of geometrical element are displayed below:

Number of vertices (V), sides (E) & triangles (T) surrounding centre of dodecagon

	V	E	F	Total
Type A:	12 (24)	24 (48)	12 (24)	48 (96)
Type B:	24 (48)	60 (120)	36 (72)	120 (240)

(Numbers in brackets refer to the pair of dodecagons). Surrounding the centres of the pair of Type A dodecagons are 24 vertices & triangles and 24 sides. This 24:24 division of the **48** elements is universally found in holistic systems.¹

The **72:168** division appears in the tetractys representation of 240:

$$\begin{array}{ccccccc}
 & & 24 & & & & \\
 & & 24 & 24 & & & \\
 & & 24 & 24 & 24 & & \\
 24 & 24 & 24 & 24 & = & \begin{array}{c} 24 \\ \diagdown \quad \diagup \\ 24 & & 24 \\ \diagup \quad \diagdown \\ 24 & & 24 \end{array} & + \begin{array}{c} 24-24 \\ | \\ 24-24 \\ | \\ 24-24 \end{array} \\
 & & 240 & & = & 72 & = 168
 \end{array}$$

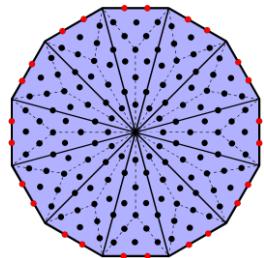
The triplet of 24s at the corners of the triangle corresponds to the 24 corners of the two Type A dodecagons, the 24 boundary sides and the 24 radial sides of their sectors. The central 24 corresponds to the 24 centres of the sectors of the dodecagons. One triplet of 24s (blue dashed triangle) correspond to the (3×24) sides inside the sectors and the

other triplet of 24s (red dashed triangle) corresponds to the (3x24) triangles.

A separate Type A and a Type B dodecagon have **168** geometrical elements surrounding their centres. They comprise 84 sides and 84 vertices & triangles. This 84:84 division is intrinsic to holistic systems. The joined pair of dodecagons has **248** yods surrounding their centres. They symbolize the **248** roots of E₈.

3. Comparison with 168 particles

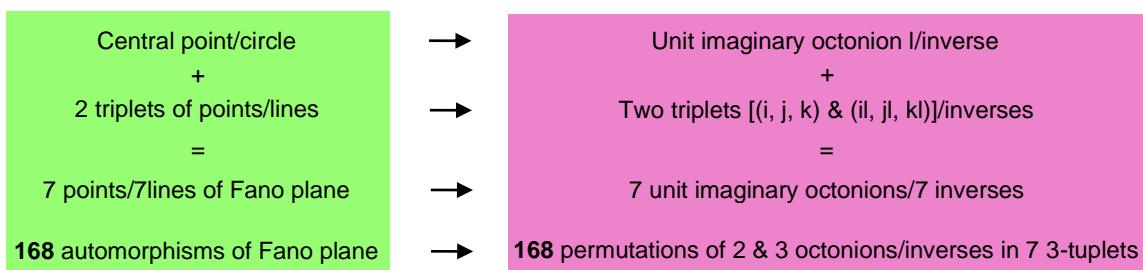
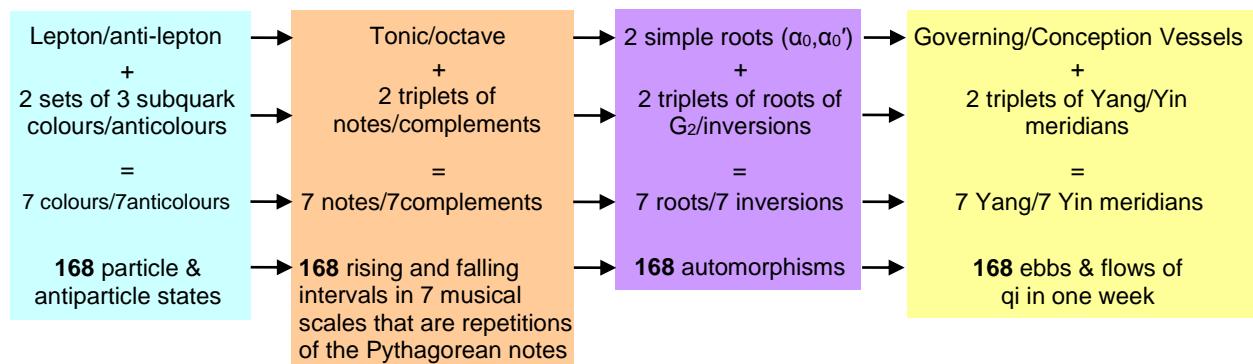
It was hypothesized in Article 40 (Part 4) that the family of supersymmetric particles consists of **168** members. They comprise 12 leptons & 12 anti-leptons, **72** subquarks & **72** anti-subquarks. They correspond in the Type B dodecagon (Fig. 3) to the 12 pairs of yods on its boundary, the **72** hexagonal yods on **36** sides of the **36** tetractyses forming Y shapes and the **72** yods that are either hexagonal yods on radial sides, centres of



$$168 = 24 (\bullet) + 144 (\circ)$$

Figure 3. The 24 red yods denote 12 leptons and 12 anti-leptons; the 144 black yods denote **72** subquarks and **72** anti-subquarks.

tetractyses or centres of sectors: **168** = 12x(2+12). The 14 yods per sector comprise the two hexagonal yods on the boundary of the dodecagon and the 12 internal yods. The counterpart of this in acupuncture is the distinction between the 12 acupoint meridians and the Governing and Conception Vessels. It also mirrors the distinction between the two simple roots (α_0, α_0') of G₂, the group of automorphisms of the Fano plane, and its 12 roots forming four triplets arranged three-dimensionally as two nested Stars of David. Finally, it corresponds to the 14 types of notes making up the seven musical scales: the tonic, octave, and the 12 notes between them. As **168** = 12x(2+12), the following correspondences exist between various manifestations of 14-fold holistic systems:



Every two hours, the vital energy that Chinese acupuncturists call ‘qi’ ebbs and flows in

successive members of the 12 meridians other than the Conception and Governing Vessels. In one week, the 12 meridians undergo ($7 \times 12 \times 2 = 168$) ebbs and flows. The 24 ebbs and flows in one day correspond to the 24 particles of a given colour. The weekly cycle of seven days corresponds to the seven colour states of particles, the seven notes in the Pythagorean scale above the tonic, the seven roots of G_2 , the seven Yang or Yin meridians, the seven points or the seven lines of the Fano plane and the seven unit imaginary octonions e_i ($i = 1-7$).

These holistic systems manifest a fundamental duality:

7 colour states	\longleftrightarrow	7 anti-colour states
7 musical notes	\longleftrightarrow	7 complements of notes
7 roots of G_2	\longleftrightarrow	7 inversions of roots
7 Yang meridians	\longleftrightarrow	7 Yin meridians
7 points of Fano plane	\longleftrightarrow	7 lines
7 unit imaginary octonions (e_i)	\longleftrightarrow	7 inverses ($e_i^{-1} = -e_i$)

Each holistic system is invariant with respect to interchange of the members of each class of polar opposites. This duality is represented in the inner form of the Tree of Life as the two sets of seven regular polygons, one set being the mirror image of the other.

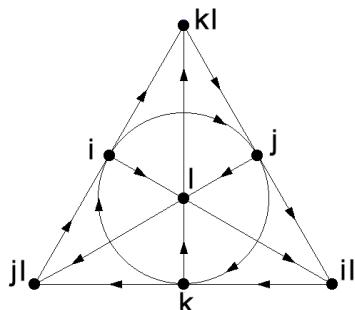


Figure 4. The seven unit imaginary octonions (i, j, k), (il, jl, kl), & I assigned to the seven points of the Fano plane.

Each septet of members of the holistic system comprises a singlet and two triplets:

7 colour states = colour singlet (1), colour triplet (3) + colour triplet (3').

7 musical notes = tonic (1) + (T, T^2, T^3) + (T^2L^2, T^3L^2, T^4L^2) ($T = 9/8$ & $L = 256/243$)

7 roots of $G_2 = \alpha_0 + (\alpha_1, \alpha_2, \alpha_3) + (\alpha_4, \alpha_5, \alpha_6)$.

7 meridians = Governing or Conception Vessel + 3 hand meridians + 3 foot meridians.

7 points of Fano plane = central point + triplet of points on circle + triplet of points at vertices of triangle (Fig. 4).

7 unit imaginary octonions = $I + (i, j, k) + (il, jl, kl)$ (Fig.4).

This 1:3:3 pattern reflects how the seven Sephiroth of Construction comprise Malkuth and the two triads: Chesed-Geburah-Tiphareth & Netzach-Hod-Yesod.

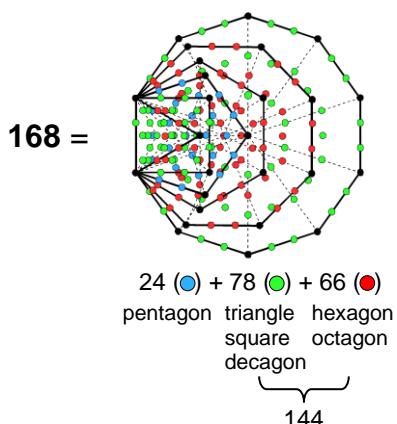
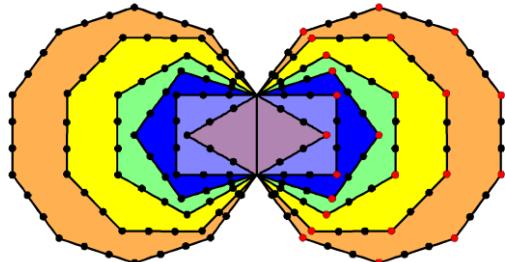


Figure 5. The first 6 enfolded polygons constitute a Tree of Life pattern. The 24:144 division of **168** is embodied in the 24 yods of the pentagon and the 144 yods in the five other enfolded polygons that are not corners of polygons.

The pattern of 24 leptons and 144 subquarks appears in the first six enfolded polygons (Fig. 5). Associated with them are **168** yods other than corners. Of these, 24 yods belong to the pentagon, the five other polygons having 144 yods. The number value 78 of *Cholem* is the number of yods in the triangle, square & decagon. The number value 90 of *Yesodoth* is the number of yods in the pentagon, hexagon & octagon.

The 24:144 division of the number **168** manifests not only in the first six enfolded polygons but also in both sets of these polygons. **168** yods lie outside their shared root



24 (●) 144 (●)

Figure 6. The **168** yods on the boundaries of the two sets of six enfolded polygons comprise 24 corners (red) of one set and 144 other yods.

edge on their 60 sides (Fig. 6). They include 24 corners of each set of polygons. There are 24 corners of one set of polygons (red yods) and 144 other (black) yods.

The same 24:144 division manifests in the pair of Type B dodecagons. 120 vertices, sides & triangles surround the centre of each polygon. The 240 geometrical elements surrounding the centres of the two dodecagons consist of **72** triangles and **168** vertices

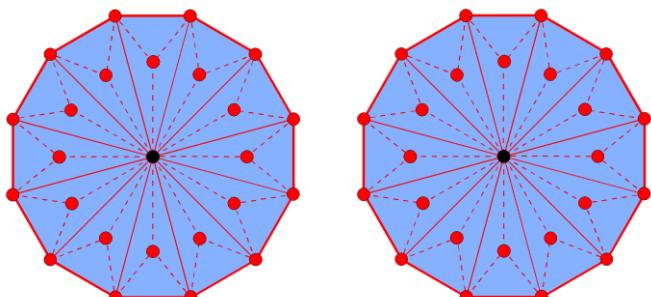


Figure 7. Surrounding the centres of the two Type B dodecagons are 240 geometrical elements. They comprise **72** triangles (blue) and **168** vertices & sides (red). The latter comprise 24 corners of dodecagons and 144 vertices & sides.

& sides (Fig. 7). The latter comprise 24 corners of the dodecagons and 144 vertices & sides. The number **168** measures the number of geometrical elements that shape the **72** tetractyses making up the two Type B dodecagons.

The 24:144 division is also exhibited in the disdyakis triacontahedron (Fig. 8). Surrounding a vertical axis joining two diametrically opposite A vertices are 60 vertices. They comprise 24 vertices above the equator with 12 vertices and 24 vertices below it. 84 edges and 60 triangles (144 geometrical elements) are above and below the

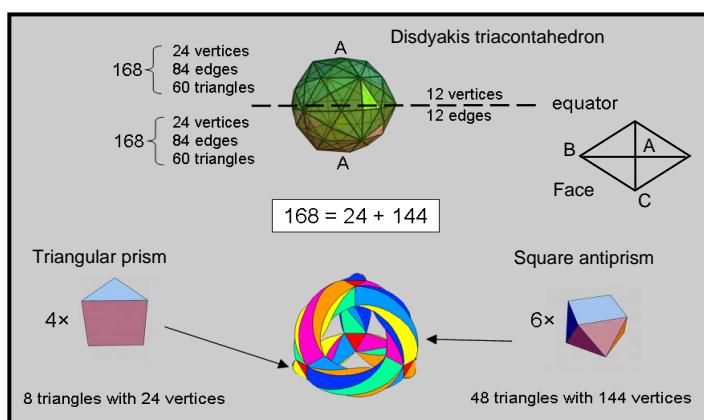


Figure 8. There are 24 vertices and 144 edges & triangles above or below the equator of the disdyakis triacontahedron. They correspond to the 24 vertices of the 8 triangles that form four triangular prisms and to the 144 vertices of the **48** separate triangles that form six square antiprisms. When stuck together, the 10 prisms & antiprisms create a 3-torus whose 56 triangles are the {3,7} tessellation on the 3-torus of the **168** automorphisms of the Klein quartic.

equator. The **168** geometrical elements composing each half of the disdyakis triacontahedron above and below its equator are made up of 24 0-dimensional elements and 144 1- & 2-dimensional elements.

Fig. 8 also shows the {3,7} tessellation onto the 3-torus of the 56 orbits of order 3 that correspond to the 56 bitangent points of the Klein quartic.² The 3-torus is topologically equivalent to six square antiprisms with their 12 square faces stuck to the 12 square faces of four triangular prisms in a tetrahedral formation, each antiprism being given a twist. This leaves 56 triangular faces, which, as separate triangles, have **168** vertices (eight from the triangular prisms with 24 vertices, **48** from the square antiprisms with 144 vertices). Each triangular prism has nine edges and each square antiprism has 16 edges, of which eight edges coincide with edges of the former when they are stuck together. This leaves $(4 \times 9 + 6 \times 8 = 84)$ edges. The 84 sides of the 56 triangles in the

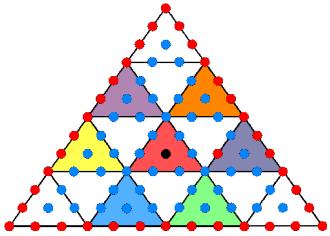


Figure 9. **36** (red) yods lie on the boundary of the 2nd-order tetractys and **48** internal (blue) yods surround its centre (black yod).

tetrahedron and the 84 sides of its version turned inside out correspond to the 84 edges above the equator of the disdyakis triacontahedron and their 84 mirror images.

The four triangular prisms at the apices of the tetrahedron have all their ($4 \times 9 = 36$) edges joined to edges of the six twisted, square antiprisms forming its edges. The latter have ($6 \times 8 = 48$) edges that are not joined. This **36:48** division of shape-forming edges has its counterpart in the 2nd-order tetractys, which is the abstract representation of holistic

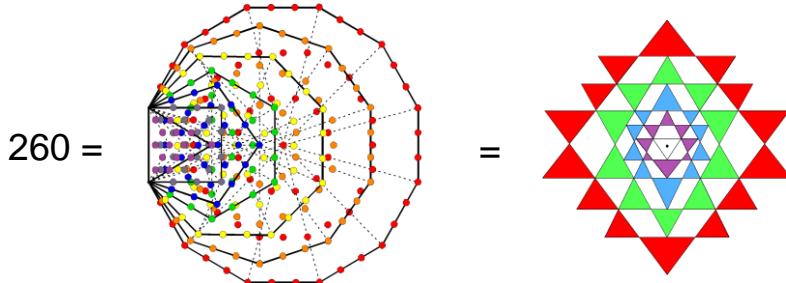


Figure 10. The 260 yods outside the root edge in the inner Tree of Life symbolize the 260 geometrical elements in the 10 prisms and antiprisms making up the 3-torus and in the 3-dimensional Sri Yantra.

systems (Fig. 9). **36** (red) yods lie on its boundary and **48** (blue) yods are inside it surrounding its centre (black yod). The 84 degrees of freedom symbolized by these yods characterize holistic systems. The 3-torus is revealed by its construction from four triangular prisms and six square antiprisms to be a holistic system because their edges possess the same **36:48** pattern displayed by the 2nd-order tetractys.

4. {3,7} tessellation of 3-torus with tetractyses

Consider the triangular faces of the triangular prisms and the square antiprisms making up the 3-torus, firstly, turned into tetractyses and, secondly, divided into their three sectors and the latter then turned into tetractyses. We shall calculate the numbers of yods in each case, noting that the 12 squares of the prisms become stuck to the 12 squares of the antiprisms, so that they play no part in the properties of the surface of the 3-torus. Their properties are listed below:

A triangular prism has 6 vertices, 9 edges, 2 triangles & 3 squares.

A square antiprism has 8 vertices, 16 edges (8 unshared), 8 triangles & 2 squares.

Table 2. Yod composition of the four triangular prisms & the six square antiprisms.

	Tetractys			Three tetractyses		
	Triangular Prism	Square Antiprism	Total	Triangular Prism	Square Antiprism	Total
Number of vertices	$4 \times 6 = 24$	$6 \times 8 = 48$	72	$24 + 4 \times 2 = 32$	$48 + 48 \times 1 = 96$	128
Number of sides	$4 \times 9 = 36$	$6 \times 16 = 96$	132	$36 + 8 \times 3 = 60$	$96 + 48 \times 3 = 240$	300
Number of triangles	$4 \times 2 = 8$	$6 \times 8 = 48$	56	$8 \times 3 = 24$	$48 \times 3 = 144$	168
Number of geometrical elements	68	192	260	116	480	596
Number of hexagonal yods on sides of tetractyses	2x36 = 72	$2 \times 96 = 192$	264	$2 \times 60 = 120$	$2 \times 240 = 480$	600
Number of hexagonal yods	$8 + 72 = 80$	$48 + 192 = 240$	320	$24 + 120 = 144$	$144 + 480 = 624$	768
Number of yods on sides of tetractyses	$24 + 72 = 96$	$48 + 192 = 240$	336	$32 + 120 = 152$	$96 + 480 = 576$	728
Number of yods	$8 + 96 = 104$	$48 + 240 = 288$	392	$24 + 152 = 176$	$144 + 576 = 720$	896

The four triangular prisms have 24 vertices and the six square antiprisms have **48** vertices. The ten separate prisms and antiprisms have **72** vertices. This is the **36**th even integer, showing how ELOHA, Godname of Geburah with number value **36**, prescribes the geometry of the 3-torus. They have 132 sides, where 132 is the **65**th even integer after 2. This shows how ADONAI, Godname of Malkuth with number value **65**, prescribes the form of the 10 prisms and antiprisms.

A triangular prism has six vertices, nine edges and two triangles. A square antiprism has eight vertices, 16 edges and eight triangles. The two structural units are composed of **49** geometrical elements prescribed by EL ChAI, Godname of Yesod with number value **49**. The four triangular prisms have 68 geometrical elements and the six square

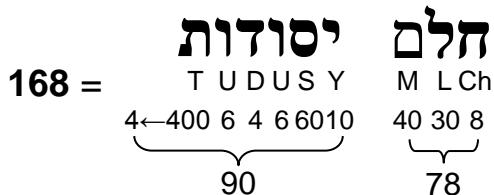
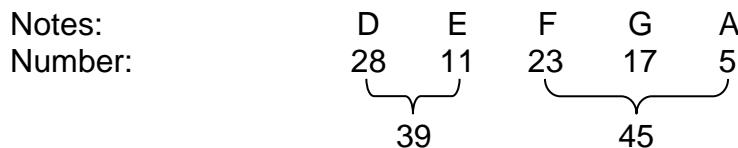


Figure 11. The gematria number value of *Cholem Yesodoth* denotes the number of rising and falling intervals between notes in the seven musical scales that are repetitions of the notes of the Pythagorean scale.

antiprisms have 192 geometrical elements. The ten separate prisms and antiprisms have 260 (=26×10) geometrical elements. This demonstrates the holistic nature of the tessellated 3-torus that they form, for it is not only prescribed by YAHWEH with number value **26** but these elements are symbolized by the 260 yods outside the root edge of the seven enfolded polygons making up the inner form of the Tree of Life (Fig. 10). The 3-dimensional Sri Yantra is also composed of 260 geometrical elements.³

The 84 repeated, rising, Pythagorean intervals⁴ between notes of the seven musical scales have the composition:



Similarly for the 84 repeated, falling intervals. There are therefore ($2 \times 39 = 78$) rising and falling major seconds and thirds and ($2 \times 45 = 90$) rising and falling intervals of perfect fourths, perfect fifths & major sixths. Remarkably, 78 is the gematria number value of *Cholem* and 90 is the number value of *Yesodoth*, the two Hebrew words making up the Mundane Chakra of Malkuth (see Table 1) with number value **168** (Fig. 11). There are also 39 repeated intervals of a major third (E), perfect fourth (F) & major sixth (A), and

45 intervals of a major second (D) or perfect fifth (F). However, the former choice seems the correct one, intuitively speaking, because the two subsets follow the *natural* order of consecutive tone ratios of the notes D, E, F, and G & A.

Table 2 indicates that 264 hexagonal yods lie on the 132 sides of the 56 triangles of the 10 prisms and antiprisms. This confirms the Tree of Life character of the 3-torus formed from these building blocks because the seven enfolded polygons that constitute the

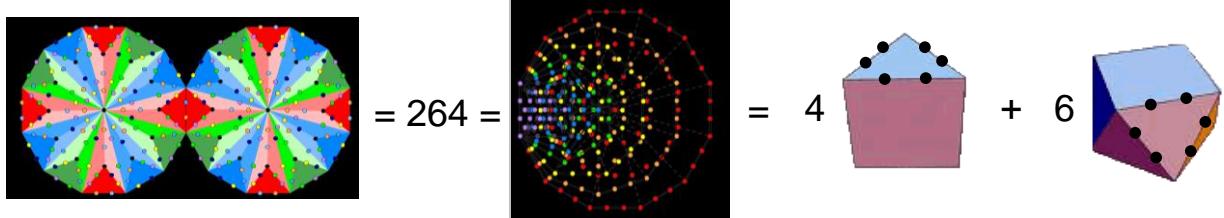


Figure 12. The 264 yods in the seven enfolded polygons symbolize the 264 hexagonal yods on the 132 sides of the 56 triangles in the 10 separate prisms and antiprisms forming a 3-torus. This number is embodied in the pair of joined Type B dodecagons as the 264 yods other than corners that lie on sides of their **72** tetractyses.

inner form of the Tree of Life have 264 yods when their 47 sectors are turned into tetractyses (Fig. 12). Alternatively, the pair of 3-tori has 264 sides. Furthermore, there are 264 yods other than corners needed to delineate the sides of the **72** tetractyses in the pair of joined dodecagons when their sectors are each divided into three tetractyses.

According to Table 2, 336 yods lie on the sides of the 56 tetractyses of the 10 prisms and antiprisms. It is further confirmation of the Tree of Life nature of the {3, 7} tessellation of the 3-torus because this number has been discussed in many previous articles as a structural parameter of the $E_8 \times E_8$ heterotic superstring. It is the order of $SL(2,7)$, the group describing the 336 automorphisms and antiautomorphisms of the

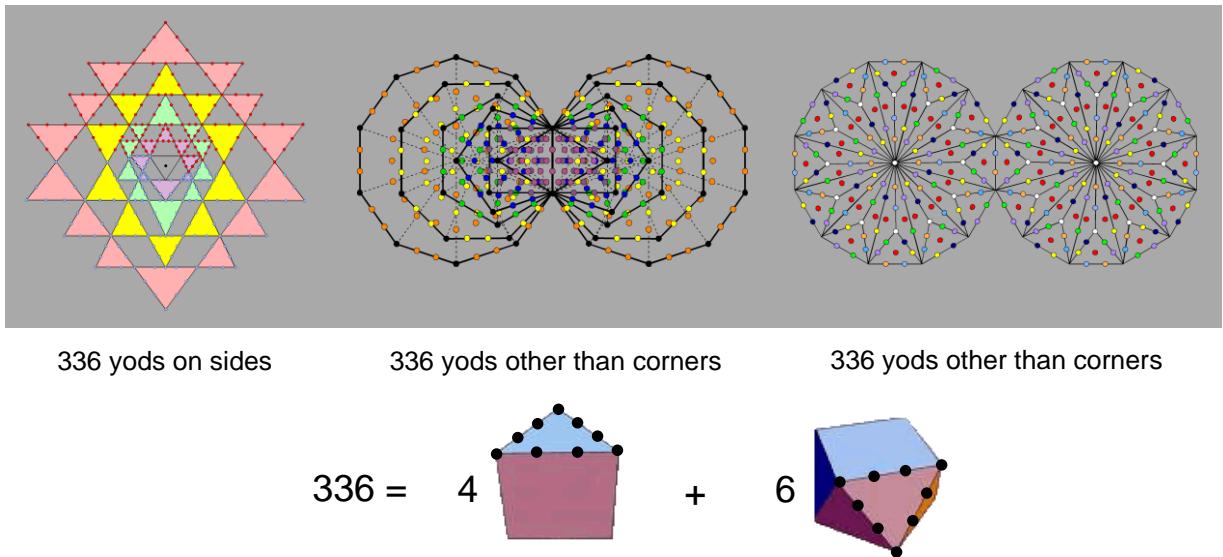


Figure 13. 336 yods line the edges of the 10 separate prisms and antiprisms. This is the number of yods on the sides of the 42 triangles in the Sri Yantra, the number of yods other than corners in the pair of six enfolded polygons and the number of yods other than corners in the pair of Type B dodecagons.

Klein quartic whose Riemann surface is topologically equivalent to a 3-torus. There are 336 yods on the sides of the 42 triangles of the Sri Yantra, 336 yods other than corners in the two sets of the first six enfolded polygons and 336 yods other than corners in the

pair of joined dodecagons when each sector is divided into three tetractyses (Fig. 13).

According to Table 2, the **168** tetractyses in the 10 separate prisms and antiprisms comprise **428** vertices & sides. **428** is the number value of *Chasmalim* ("Shining Ones"), the order of angels assigned to Chesed. They also contain 896 yods. Of these, 56 yods are centres of the 56 triangles. Hence, there are $(896 - 56 = 840)$ yods other than these centres. In the case of all triangular faces being tetractyses, there are 336 yods other than these same centres delineating their sides. This is remarkable, for, as many

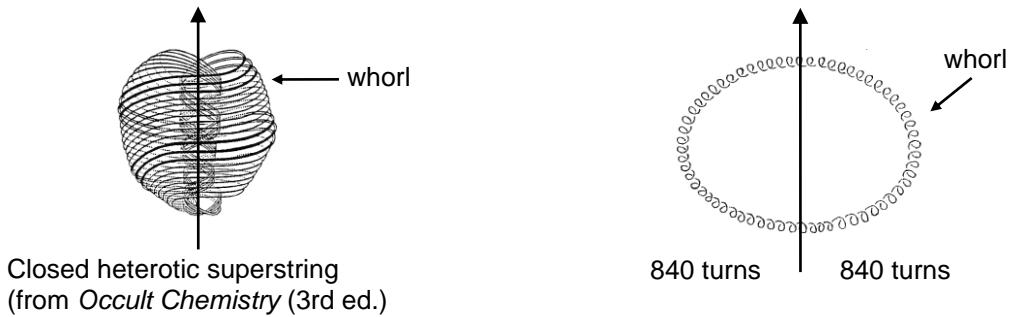


Figure 14. The unit of matter described over a century ago in the book *Occult Chemistry* by Annie Besant & C.W. Leadbeater is a subquark state of the $E_8 \times E_8$ heterotic superstring. Its 10 closed curves ("whorls") wind five times around its axis of spin. Each whorl is a helix with 1680 circular turns. It makes 840 turns in each inner and outer circuit of $2\frac{1}{2}$ revolutions and 336 turns in one revolution.

previous articles have discussed, both the numbers 840 and 336 are structural parameters of the heterotic superstring. Each of its ten closed curves, or "whorls", winds 1680 times around a torus as it twists five times around its spin axis (Fig. 14). Each revolution makes 336 circular turns and one-half of a revolution makes 840 turns. If the **168** tetractyses making up the prismatic version of the 3-torus were separated from one another, they would have 1680 yods. This is the number of turns in each whorl. It is remarkable that the ten prisms and antiprisms embody three structural parameters of the unit of physical matter described over a century ago by Besant and Leadbeater.⁵ This cannot be coincidence. Instead, it indicates a basic role for the 3-torus in the underlying physics of superstrings as the microscopic manifestation of holistic systems, for each yod in its **168** tetractyses surrounding the centres of the 56 triangles denotes a turn of a whorl.

Let us now consider the prisms and antiprisms joined together at their square faces to form the 3-torus. Table 3 displays their geometrical and yod composition.

Table 3. Yod composition of the 3-torus constructed from tetractyses.

	Tetractys			Three tetractyses		
	Triangular Prism	Square Antiprism	Total	Triangular Prism	Square Antiprism	Total
Number of vertices	$4 \times 6 = 24$	—	24	$24 + 4 \times 2 = 32$	$48 \times 1 = 48$	80
Number of sides	$4 \times 9 = 36$	$6 \times 8 = 48$	84	$36 + 8 \times 3 = 60$	$48 + 48 \times 3 = 192$	252
Number of triangles	$4 \times 2 = 8$	$6 \times 8 = 48$	56	$8 \times 3 = 24$	$48 \times 3 = 144$	168
Number of geometrical elements	68	96	164	116	384	500
Number of hexagonal yods on sides of tetractyses	$2 \times 36 = 72$	$2 \times 48 = 96$	168	$2 \times 60 = 120$	$2 \times 192 = 384$	504
Number of hexagonal yods	$8 + 72 = 80$	$48 + 96 = 144$	224	$24 + 120 = 144$	$144 + 384 = 528$	672
Number of yods on sides of tetractyses	$24 + 72 = 96$	96	192	$32 + 120 = 152$	$48 + 384 = 432$	584
Number of yods	$8 + 96 = 104$	$48 + 96 = 144$	248	$24 + 152 = 176$	$144 + 432 = 576$	752

With single tetractyses as their faces, the four triangular prisms and the six square antiprisms are composed of 56 tetractyses with **248** yods. This remarkable, for **248** is the number of gauge bosons that transmit the unified force between superstrings of

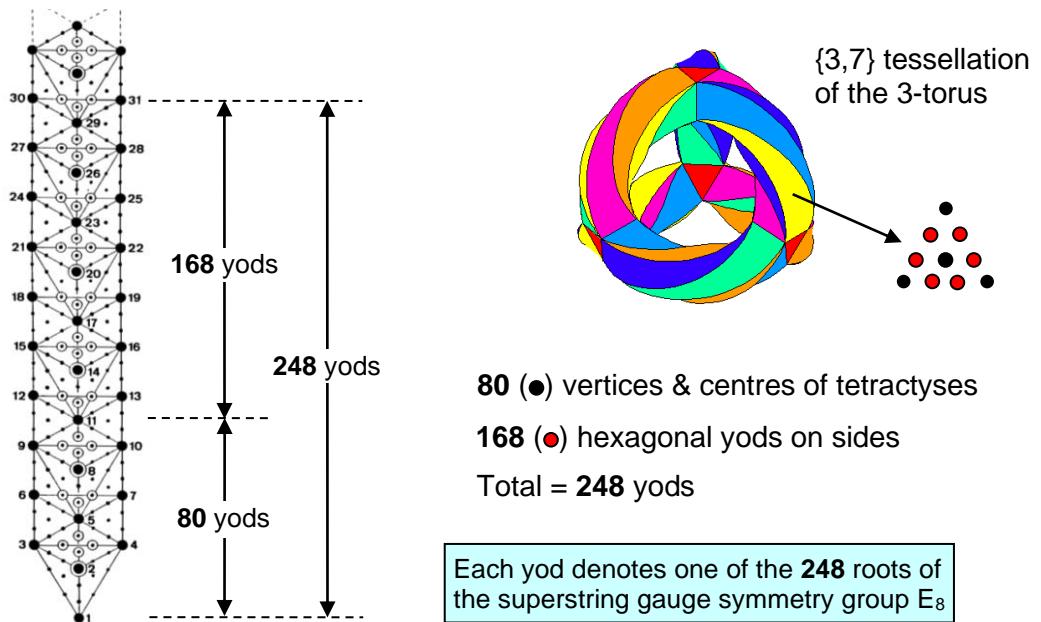


Figure 15. The **80** vertices & centres of the 56 tetractyses in the 3-torus correspond to the **80** yods in the lowest Tree of Life. The **168** hexagonal yods on the 84 sides of tetractyses correspond to the **168** yods up to Chesed of the fifth Tree (31st Sephirothic level). Alternatively, there are **80** hexagonal yods in the 4 prisms and **168** yods in the 6 antiprisms.

ordinary matter. Furthermore, the **248** yods comprise 8 yods at centres of the faces of the four triangular prisms, leaving 240 yods made up of 24 yods at vertices, **48** yods at centres of tetractyses, that is, **72** yods that either vertices or centres of tetractyses, and **168** hexagonal yods on their 84 sides. The division:

$$248 = 8 + 240$$

corresponds to the eight simple roots of E_8 and its 240 roots. The division:

$$240 = 72 + 168$$

corresponds to the **72** roots of E_6 , an exceptional subgroup of E_8 , and the **168**

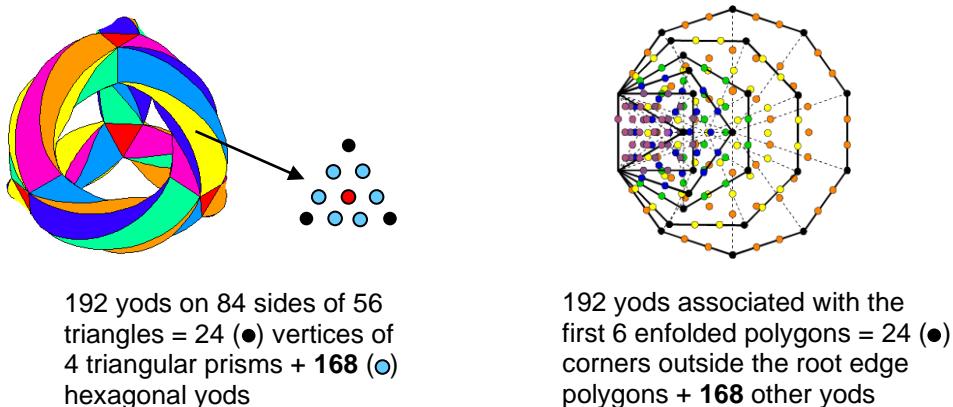


Figure 16. 192 yods lie on the 84 sides of the 56 tetractyses tiling the 3-torus. This is the number of yods associated with the first 6 enfolded polygons.

remaining roots of E_8 . The root structure of the superstring gauge symmetry group E_8 is embodied in the yod composition of the tessellation of the 3-torus in terms of tetractyses. As so often found in discussions of holistic systems, the number **168** defines the shape of the holistic system as — in the case of faces as single tetractyses — the number of hexagonal yods on the boundary of the tetractyses that compose it and — in the case of faces as three tetractyses — the number of triangles.

The 56 tetractyses in the 3-torus have **80** vertices and centres. Their 84 sides have **168** hexagonal yods. This **80:168** division appears in the lowest five overlapping Trees of Life constructed from tetractyses as the **80** yods of the lowest Tree and the **168** yods beyond it to Chessed of the fifth Tree — the **31st** Sephirothic level prescribed by EL, the Godname of Chessed with number value **31** (Fig. 15).

According to Table 3, 192 yods lie on the 84 sides of the 56 tetractyses. 24 yods are vertices of the four triangular prisms. This **24:168** division corresponds in the inner form of the Tree of Life to the 24 external corners and to the **168** yods other than corners associated with each set of the first six enfolded polygons (Fig. 16). One set of polygons corresponds to a 3-torus. Its mirror image corresponds to the 3-torus turned inside out.

With faces of the prisms and antiprisms constructed from three tetractyses, the 3-torus is composed of **168** tetractyses with **80** vertices. **80** is the number value of Yesod. The four prisms with single tetractyses as their triangular faces have **80** hexagonal yods. The four triangular prisms have 32 vertices and the six square antiprisms have 48

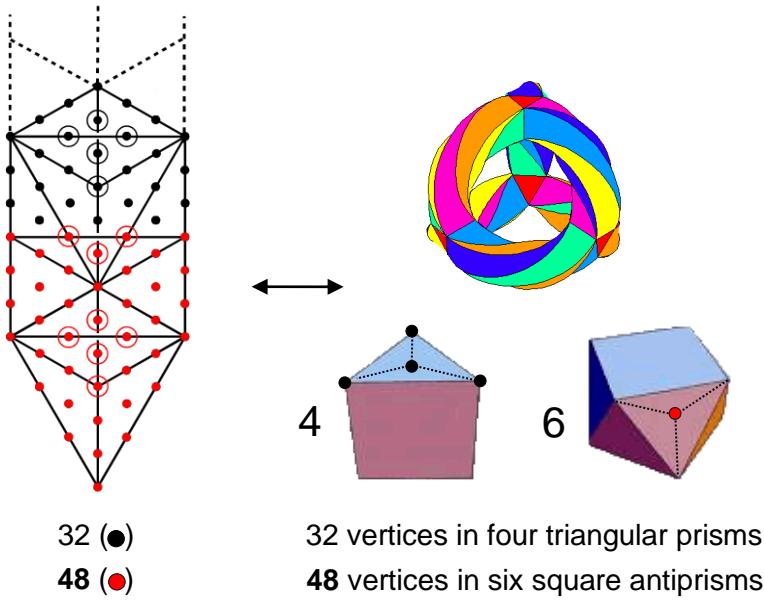


Figure 17. The Tree of Life character of the $\{3,7\}$ tessellation of the Klein Configuration on the 3-torus. The 32 vertices of the 24 triangles in the four triangular prisms at the apices of the tetrahedron correspond to the 32 black yods in the lowest Tree of Life above Chessed. The **48** vertices of the **168** triangles in the six square antiprisms correspond to the **48** red yods up to Chessed.

vertices. This **48:32** division manifests in the **80** yods of the lowest Tree of Life as the **48** yods up to Chessed and the 32 yods above it (Fig. 17). The correspondence between the yods and geometrical elements demonstrates the Tree of Life character of the 3-torus.

The **168** tetractyses comprise 24 tetractyses in the four triangular prisms and 144 tetractyses in the six square antiprisms. The same **24:144** pattern is displayed by the first six enfolded polygons and by the dodecagon (see Figs 6 & 7). According to Table 2, the 3-torus has **248** vertices & triangles. With single tetractyses as faces, it has **80** vertices & triangles. Hence, **168** vertices & triangles are added by turning each face into three tetractyses. **168** sides are added as well, i.e., 336 geometrical elements are added. This **168:168** division of added elements reflects the **168** automorphisms and **168** anti-automorphisms of $SL(2,7)$. The 3-torus with single tetractyses as faces has **168**

hexagonal yods on the 84 sides of 56 tetractyses. With its triangles turned into three tetractyses, it has 504 hexagonal yods on the 252 sides of **168** tetractyses. 336 hexagonal yods are added. The counterpart in the Sri Yantra of this **168**:336 division of

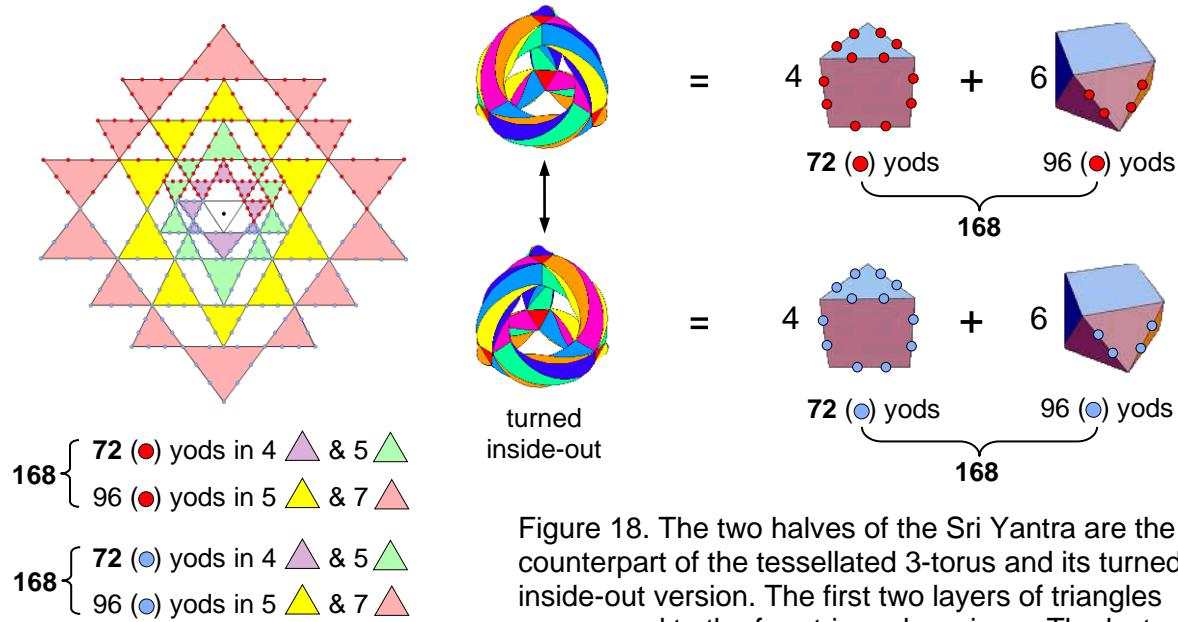


Figure 18. The two halves of the Sri Yantra are the counterpart of the tessellated 3-torus and its turned inside-out version. The first two layers of triangles correspond to the four triangular prisms. The last two layers correspond to the six square antiprisms.

the 504 hexagonal yods is the **168** geometrical elements in the fourth layer of 14 triangles and the 336 elements in the first three layers (see Table 3 of Article 35).

The 3-torus is constructed from 500 ($=50 \times 10$) geometrical elements (see Table 3). It is a remarkable demonstration of how ELOHIM, the Godname of Binah with number value

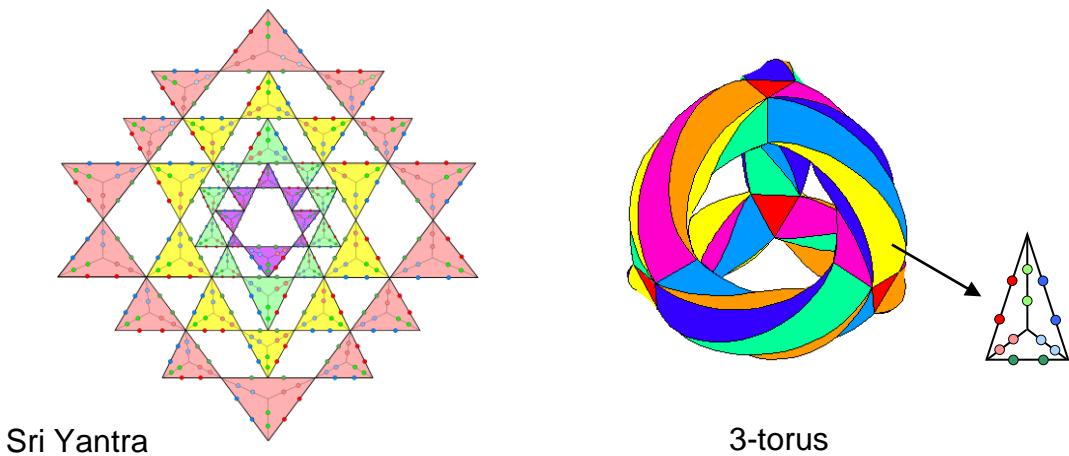


Figure 19. The 42 triangles of the Sri Yantra have 504 hexagonal yods on the 252 sides of their 126 tetractys sectors. This is the number of hexagonal yods on the 252 sides of the **168** tetractyses making up a 3-torus constructed from four triangular prisms and six square antiprisms.

50, prescribes the geometry of the 3-torus constructed from triangles. We found earlier that the *separate* prisms and antiprisms have 260 ($=26 \times 10$) geometrical elements when their triangular faces are regarded as single triangles. This is unmistakable evidence that YAHWEH and ELOHIM prescribe the geometrical composition of the {3,7} tessellation of the 3-torus. The Klein Configuration embodies the divine archetypes.

According to Table 3, the four triangular prisms have **72** hexagonal yods on the **36** sides of their eight tetractyses and the six square antiprisms have 96 hexagonal yods on the **48** sides of their **48** tetractyses that are unshared with the former. This **72:96** division of the **168** hexagonal yods has its counterpart in the Sri Yantra (Fig. 18). When its 42 triangles are turned into tetractyses, **168** yods line the 63 sides of the **21** triangles in each half. The first two layers of 18 triangles have 144 yods on their 54 sides, **72** yods in each half. The last two layers of 24 triangles have 192 yods on their **72** sides, 96 yods in each half. The **168** boundary yods in each half comprise **72** yods in the first two layers and 96 yods in the last two layers. The former corresponds to the four triangular prisms and the latter correspond to the six square antiprisms. The 3-torus and its turned inside-out version are the counterpart of each half of the Sri Yantra.

Table 3 indicates that 504 hexagonal yods lie on the 252 sides of the **168** tetractyses. For single tetractyses as faces, there are **168** such yods. Hence, 336 ($=2 \times 168$) hexagonal yods are added by the construction of each triangular face from three tetractyses. Remarkably, when the 42 triangles of the Sri Yantra surrounding the central one are each divided into their three sectors, the 126 sectors also have 252 sides with 504 hexagonal yods on them (Fig. 19). With their triangular faces divided into three tetractyses, **48** hexagonal yods are added to the triangular prisms and 288 hexagonal yods are added to the square antiprisms. Conversion of each triangle of the Sri Yantra into three tetractyses adds six hexagonal yods on sides. Compare the numbers of this type of yod for the two types of prism (red numbers denote number of hexagonal yods on sides *inside* the 56 primary triangles):

4 triangular prisms	6 square antiprism
$120 = 72 + \textcolor{red}{48}$	$384 = 96 + \textcolor{red}{288}$

with the number of hexagonal yods on the sides of the tetractyses making up the triangles of the Sri Yantra:

8 triangles	10 triangles	10 triangles	14 triangles
96	120	120	168

The (**72+96=168**) hexagonal yods in the 3-torus with tetractyses as the triangles correspond to the **168** hexagonal yods on the 84 sides of the 42 tetractyses in the last set of 14 triangles. The (**48+288=336**) hexagonal yods added by the conversion correspond to the ($96+120+120=336$) hexagonal yods on the **168** sides of the 84 tetractyses in the first three sets of triangles. We see that the fourth set of triangles corresponds to the original 3-torus and that the first three sets of triangles correspond to the change in the 3-torus when its 56 triangles are each divided into three tetractyses. Moreover, the 42 sectors of the last set of 14 triangles have 84 sides, whilst the 84 sectors of the first three sets of triangles have **168** sides. This compares with the 84 sides of the 56 triangles of the 3-torus and the **168** sides added by dividing each of these triangles into its three sectors. The correspondence exists for the sides as well. The 3-torus embodies the properties of the Sri Yantra because it is a holistic system.

The number of yods making up the **168** tetractyses in the 3-torus is 752. This is 16×47 , where 16 is the **15th** integer after 1 and 47 is the **15th** prime number. It shows how the Godname YAH with number value **15** prescribes the yod population of the 3-torus constructed from tetractyses. The Tetrad expresses the 176 yods in the four triangular prisms because $176 = 4 \times 44$. It expresses the 576 yods in the six square antiprisms because $576 = (1 \times 2 \times 3 \times 4)^2$. It defines the 104 yods in the four triangular prisms with single tetractyses as its triangular faces because there are **26** combinations of objects arranged in the four rows of the tetractys and $104 = 4 \times 26$. It expresses the 144 yods in

the six square antiprisms with single tetractyses because

$$144 = \begin{matrix} 1^0 & 2^0 & 3^0 & 4^0 \\ 1^1 & 2^1 & 3^1 & 4^1 \\ 1^2 & 2^2 & 3^2 & 4^2 \\ 1^3 & 2^3 & 3^3 & 4^3 \end{matrix}$$

We found earlier that the **168** tetractyses making up the 10 separate prisms and antiprisms making up the 3-torus have 840 yods surrounding their centres. The counterpart of this property in the disdyakis triacontahedron (Fig. 20) are the 840 yods

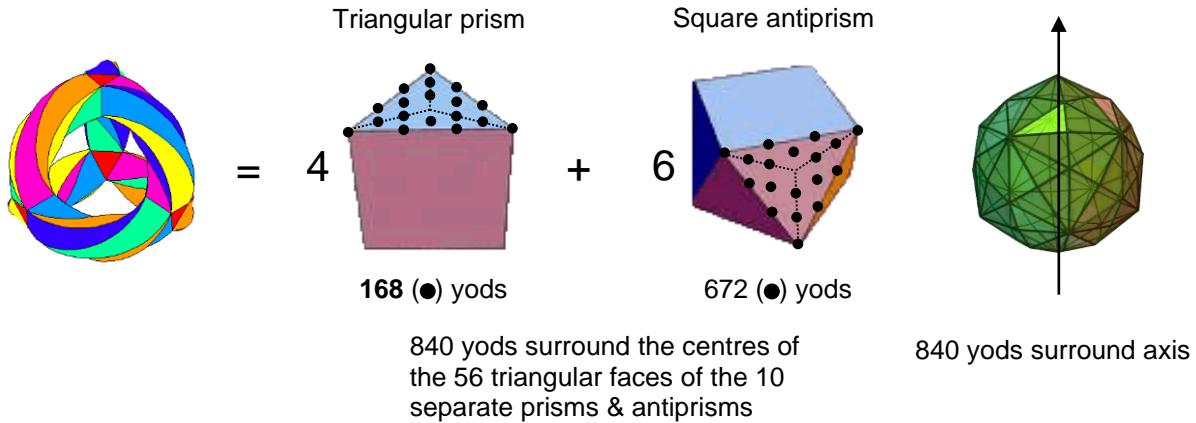


Figure 20. The disdyakis triacontahedron embodies the same superstring structural parameter 840 as the 10 separate prisms & antiprisms forming the 3-torus.

surrounding an axis that passes through two opposite vertices when its 120 faces are turned into tetractyses and when its 180 interior triangles with polyhedral edges as their sides are likewise transformed.⁶ Notice the ambient character of this number in both cases. The axis of the polyhedron is the counterpart of the centres of the 56 triangular faces of the 10 prisms and antiprisms. This is not coincidental, for the same character is

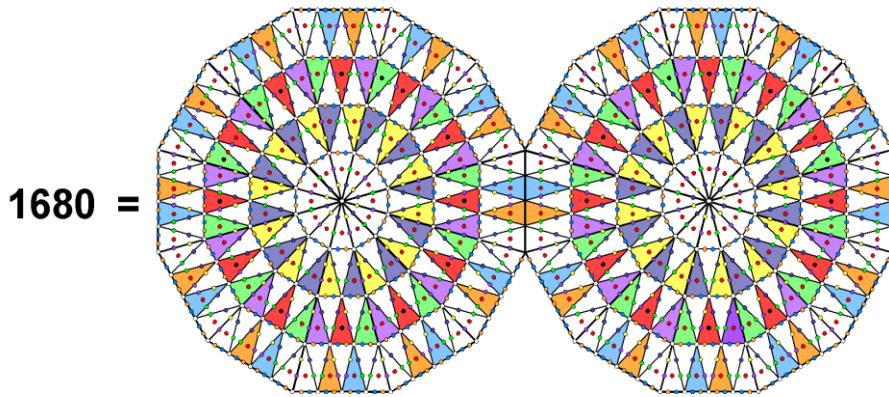


Figure 21. 840 yods surround the (●) centres of the 12 sectors of each dodecagon in the inner form of the Tree of Life.

displayed in the last regular polygon of the inner form of the Tree of Life (Fig. 21). When the 12 sectors of the dodecagon are transformed into 2nd-order tetractyses, one finds that there are 840 yods outside the root edge *surrounding* their 12 centres. The 1680 yods surrounding the 24 centres in both dodecagons symbolize the 1680 turns of each whorl of the heterotic superstring (see Fig. 14). Each dodecagon corresponds to an

inner or outer half of each curve with 840 helical turns and to the 3-torus or to its turned-inside out version. These 24 centres remind us of the 24 vertices of the four triangular prisms forming the apices of the tetrahedron into which the 3-torus can be deformed. The lines joining these points are their **36** edges. Defining the shape of these prisms, the 24 points express the meaning of Malkuth, as symbolized by the 24 centres of the 2nd-order tetractyses in the dodecagon. The pair of mirror-image dodecagons is the inner Tree of Life counterpart of the 3-torus built from prisms and antiprisms and its turned inside-out version, similarly constructed.

5. Particle and musical counterparts of the {3,7} tessellation

A regular tessellation, or tiling, is a covering of the plane by regular polygons so that the same number of polygons meets at each vertex. The regular tessellations of the Euclidean plane are well-known. They are: {3,6} in which equilateral triangles meet six at each vertex; {4,4} in which squares meet four at each vertex; and {6,3} in which hexagons meet three at each vertex. A notation like {3,6} is called a *Schlafli symbol*. In general, {n,k} is the tessellation by polygons with n vertices, k polygons meeting at each vertex. If $1/n + 1/k < 0$, {n,k} is a tessellation of the hyperbolic plane with negative curvature. If $1/n + 1/k = 0$, {n,k} is a tessellation of the Euclidean plane with zero curvature. If $1/n + 1/k > 0$, {n,k} is a tessellation of the elliptic plane with positive curvature. Poincaré described ways in which points in the hyperbolic plane can be

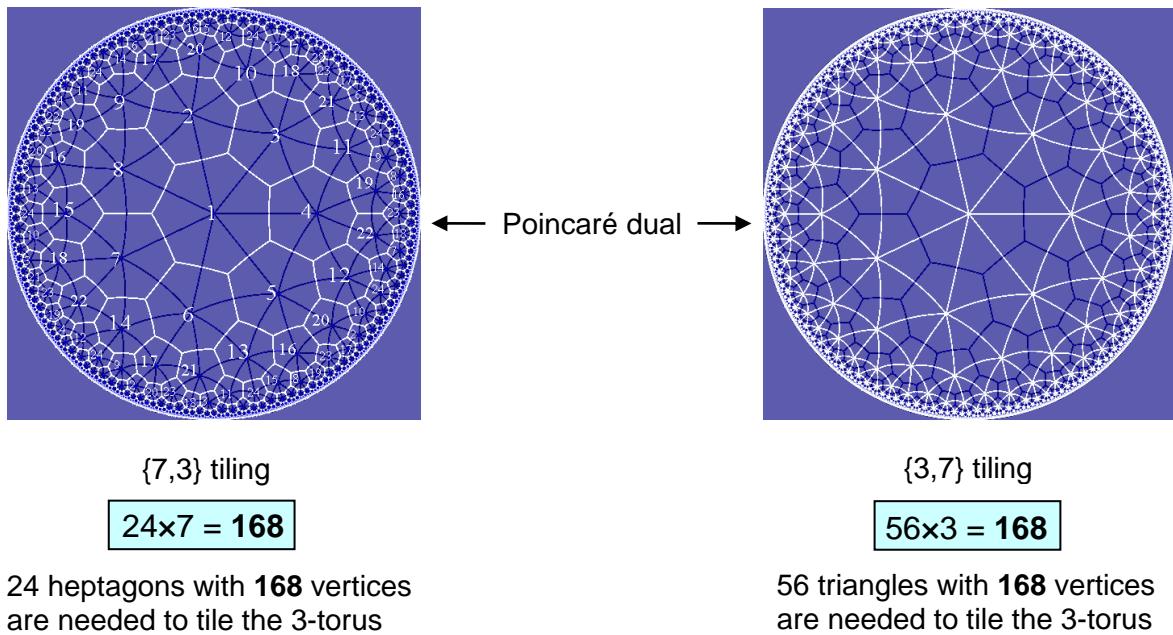


Figure 22. The {7,3} & {3,7} tiling on the Poincaré disk of the **168** symmetries of the Klein quartic.

represented conformally in the Euclidean plane. One of these is to represent the hyperbolic plane by points within the so-called ‘Poincaré disk’. {n,k} is the Poincaré dual of {k,n} in which vertices and polygons are interchanged.

$SL(2,7)$ is the double cover of the Klein quartic symmetry group $PSL(2,7)$. Its order 336 is the number of automorphisms and anti-automorphisms of the Klein quartic

$$x^3y + y^3z + z^3x = 0.$$

The **168** elements of $PSL(2,7)$ are represented either by the {7,3} tiling of the Poincaré disk with 24 heptagons having **168** vertices or by its {3,7} tiling with 56 triangles having **168** vertices (Fig. 22).

The Riemann surface of the Klein quartic is a **168**-sheeted covering of the sphere, branched over three points of the sphere.⁷

- Above one of these points, the **168** sheets join in groups of seven to give 24 points of the surface. These are the points of inflection of the curve described by the Klein quartic. They are also the Weierstrass points. The 24 orbits of subgroups of order 7 correspond to these points.
- Above another branch point, there are 84 points of the surface, where the sheets join in twos. These are the sextactic points through which pass a conic section that has six-fold contact with the curve. The 84 orbits of subgroups of order 2 correspond to them.
- Above the third branch point, the sheets join in threes to give 56 points of the surface. These 56 points are the points of contact of the Klein quartic with the 28 bitangents, or lines that are tangent to the curve at two points. The 56 orbits of subgroups of order 3 correspond to them.

$\text{PSL}(2,7)$ has two families of seven groups of order 24, isomorphic to the octahedral group. This means that the 56 ($=7 \times 8$) triangles of the 3-torus are associated with the eight vertices of seven cubes, each of whose 24 symmetries belong to the octahedral group; similarly for the turned-inside out version of the 3-torus, its 56 triangles define seven anticubes with 56 vertices.

In Article 40 (Part 4)⁸, a model of subatomic particles was presented that is analogous to the two groups of ($7 \times 24 = \mathbf{168}$) symmetries. Three generations of basic particles, each in seven colour states, were hypothesized to exist. Each state further belongs to a group of eight consisting of four sets of two (matter/antimatter, weak isospin doublet & two supersymmetric partners). There are left-handed and right-handed versions of each particle, so that there are **168** left-handed particles and **168** right-handed particles. The two chiral families form seven groups of 24, each of the same colour. The following correspondence is established:

168 automorphisms/antiautomorphisms of Klein quartic \rightarrow **168** left-/right-handed states;
 24 orbits of subgroups of order 7 \rightarrow 24 left-/right-handed states, each in 7 colours;
 56 orbits of subgroups of order 3 \rightarrow 56 left-/right-handed states, each in 3 generations;
 84 orbits of subgroups of order 2 \rightarrow 84 left-/right-handed states, each with its antimatter version.

The seven vertices of a heptagon in the {7,3} tiling of the Riemann surface of the Klein quartic are analogous to the seven colour states of basic particles, whilst the 24 heptagons correspond to the 24 particles of each colour. The three vertices of a triangle

Table 4. The tone ratios of the notes in eight octaves of the Pythagorean musical scale.

	C	D	E	F	G	A	B	Number of overtones
1	1	9/8	81/64	4/3	3/2	27/16	243/128	0
2	2	9/4	81/32	8/3	3	27/8	243/64	2
3	4	9/2	81/16	16/3	6	27/4	243/32	4
4	8	9	81/8	32/3	12	27/2	243/16	7
5	16	18	81/4	64/3	24	27	243/8	11
6	32	36	81/2	128/3	48	54	243/4	15
7	64	72	81	256/3	96	108	243/2	20
8	128	144	162	512/3	192	216	243	26

in the {3,7} tiling denote the three generations of each particle, whilst the 56 triangles correspond to the 56 states of each generation.

If this analogy is not merely coincidental, it means that the gematria number value **168** (Fig. 11) of *Cholem Yesodoth*, the Mundane Chakra of Malkuth, the tenth Sephirah, denoting the universe of physical matter, measures not only the number of helical turns made by each whorl of the heterotic superstring as it revolves 180° around its spin axis but also the number of fermionic states in which the superstring exists. These states have **168** spin-0 supersymmetric partners (so-called ‘sfermions.’)

Table 3 indicates that the 192 yods on the sides of the tetractyses making up the 3-torus are divided into their 24 vertices and the **168** hexagonal yods on their sides. Amazingly, this **24:168** division appears in the sequence of tone ratios of the notes of the Pythagorean scale shown in Table 4! The perfect fifth of the fifth octave has the tone ratio 24. It is the tenth overtone and the 32nd note above the tonic. There are 24 overtones up to the note G of the eighth octave with tone ratio 192, which is the 53rd note above the tonic. There are 14 overtones in **21** notes with a tone ratio difference of **168** between the 10th overtone with tone ratio 24 and the 24th overtone with tone ratio 192. Each yod on the boundaries of the prisms and antiprisms symbolizes a unit rise of pitch of a note of the Pythagorean scale. The 24 vertices of the four triangular prisms correspond to the first 24 harmonics up to the tenth overtone, their **72** hexagonal yods

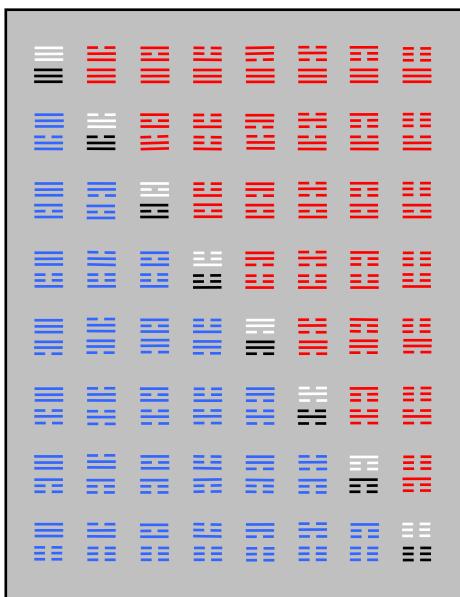


Figure 23. The **64** hexagrams of the I Ching table have 384 Yang (unbroken) and Yin (broken) lines. The 192 Yang/Yin lines in each diagonal half of the table comprise 24 lines & broken lines in the eight white or black trigrams along the diagonal and **168** lines & broken lines in the 56 red or blue trigrams outside the diagonal in each half.

168 — & — **168** - & -

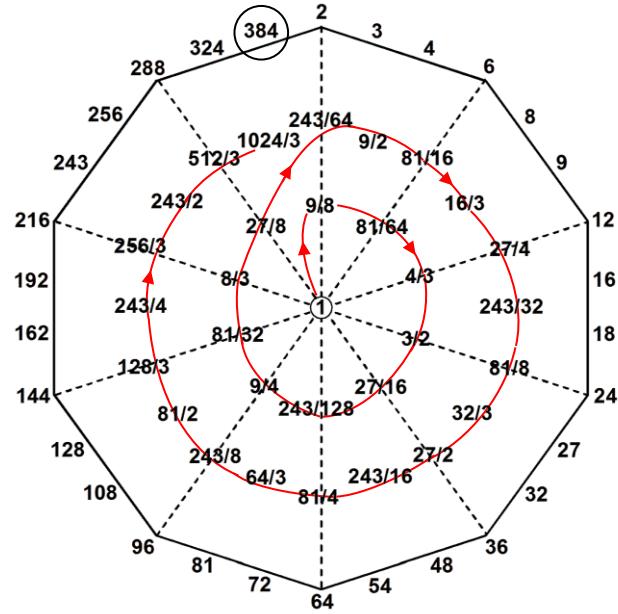
24 — & --

24 — & --

correspond to the next **72** harmonics up to 96, the 46th note above the tonic, and the 96 hexagonal yods on the edges of the six square antiprisms correspond to the final 96 harmonics up to 192, the 24th overtone. There are also **168** harmonics up to this note, 24 of which are overtones of the Pythagorean scale, leaving 144 non-musical harmonics. This **24:144** division in the number **168** was encountered in the two sets of the first six enfolded polygons (Fig. 6), the pair of dodecagons (Fig. 7) and the disdyakis triacontahedron (Fig. 8) — all holistic structures embodying the divine archetypes. It occurs in the 192 lines and broken lines making up the **64** trigrams in each half of the I Ching table as the 24 Yang/Yin lines in the eight diagonal trigrams and the **168** Yang/Yin lines in the 56 off-diagonal trigrams (Fig. 23).

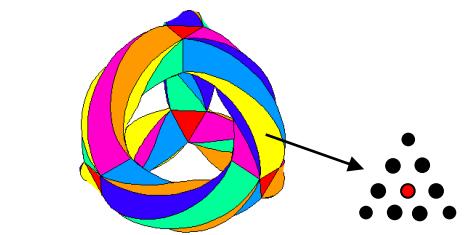
There are 384 harmonics up to the perfect fifth of the ninth octave. As this is the 61st note, all the Pythagorean notes up to this note can be assigned to the 61 yods in a

decagon whose sectors are turned into tetractyses (Fig. 24). The 30 overtones can be assigned to the 30 yods on the boundary of the decagon and the 30 fractional tone ratios can be assigned to the 30 internal yods surrounding the central yod denoting the tonic 1. The system of 384 harmonics — the musical counterpart of the 384 yods in the

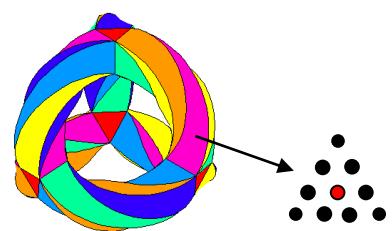


Starting from 1, the central tonic, increasing fractional tone ratios are ordered in a clockwise spiral.

Overtones are arranged clockwise on the boundary.



192 yods (●) on 84 sides of 56 tetractyses



192 yods (●) on 84 sides of 56 tetractyses

Figure 24. The last of the Pythagorean tone ratios that can be assigned to the 61 yods in a decagon is 384 (shown encircled). This is the number of yods needed to construct a 3-torus and its turned inside-out version.

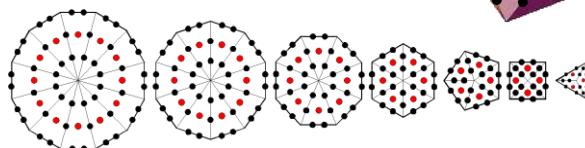
pair of 3-tori — constitute a holistic system determined by the decagon, which is the symbol of the number 10, the Decad, called “All Perfect” by the Pythagoreans.

As the eight notes of a musical scale have 27 intervals below the octave, the 44 notes of the seven musical scales have ($7 \times 27 = 189$) intervals below the octave. They include

48 (●) hexagonal yods at centres of tetractyses

192 (●) hexagonal yods on 96 sides

Total = 240 hexagonal yods



48 (●) hexagonal yods at centres of tetractyses

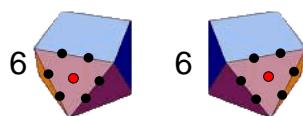
192 (●) hexagonal yods on sides

Total = 240 hexagonal yods

48 (●) hexagonal yods at centres of tetractyses

192 (●) hexagonal yods on 96 sides

Total = 240 hexagonal yods



48 (●) hexagonal yods at centres of tetractyses

192 (●) hexagonal yods on 96 sides

Total = 240 hexagonal yods

Figure 25. The (240+240) hexagonal yods in the two sets of seven separate polygons are the counterpart of the 240 yods on the 96 sides of the six square antiprisms and their counterpart in the turned inside-out version of the {3,7} tessellated 3-torus.

21 notes that are the second, third and fourth notes of each scale, leaving **168** intervals. Including the tonic, the octave and the unit interval between a note and itself, there are 192 intervals. They comprise 24 intervals (tonic, octave, unit interval and the **21** notes) and **168** intervals made up of the **21** complements of the **21** notes and 147 intervals between notes above the tonic (**21** in each scale). We see that both the seven musical scales and the Pythagorean scale display the same **24:168** division found for the inner Tree of Life, the I Ching table, the Sri Yantra, the disdyakis triacontahedron, the 3-torus and the 192 harmonics up to the perfect fifth of the eighth octave.

The eight triangular faces of the four triangular prisms consist of eight sets of three vertices. They correspond to the eight upper trigrams in the diagonal of the I Ching table. The **168** hexagonal yods on the edges of the 10 prisms and antiprisms are arranged as 56 sets of two interlaced triangular arrays of three yods shared between adjoining tetractyses, that is, as 28 independent sets of two arrays. They correspond to the 28 hexagrams with **168** Yang/Yin lines above the diagonal. The $(24+168=192)$ yods

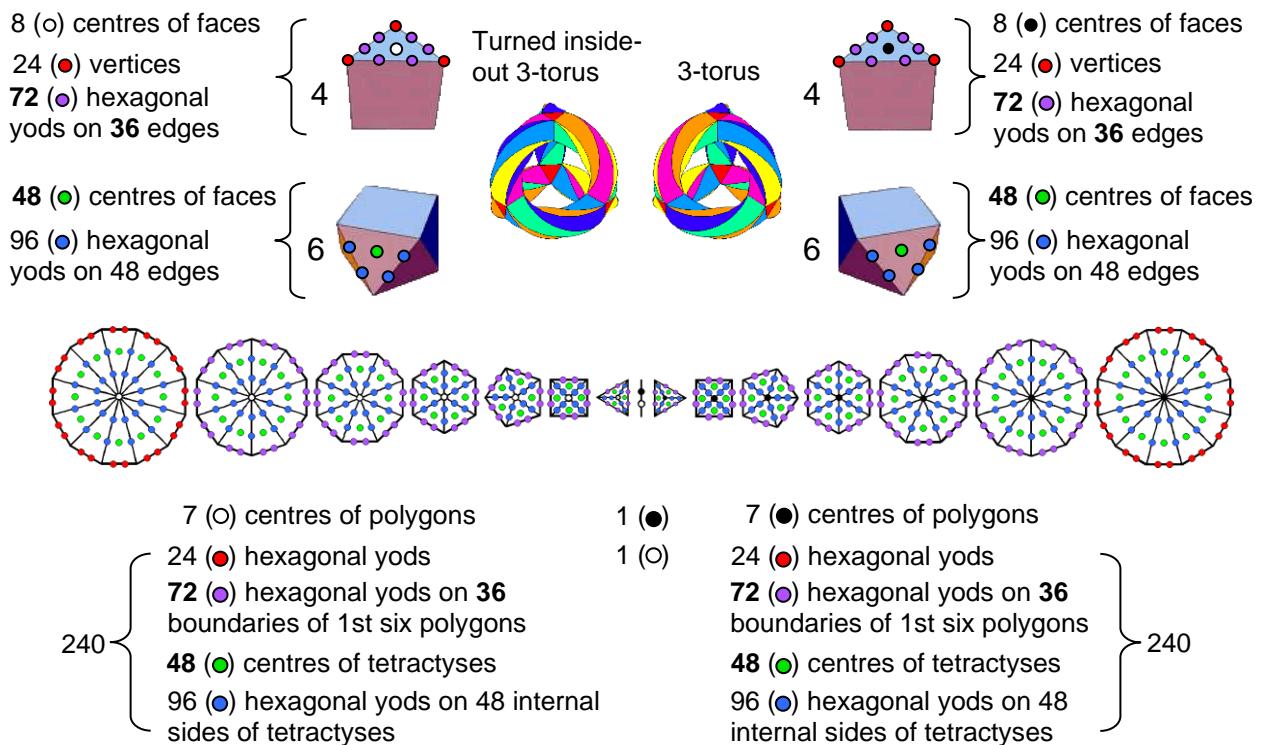


Figure 26. The 240 roots and 8 simple roots of the superstring gauge symmetry group E_8 are geometrically symbolized, respectively, by the 240 hexagonal yods and by the 8 yods that are either centres of polygons or one of the two hexagonal yods on the root edge. The $\{3,7\}$ tessellation of the 56 triangles of the Klein Configuration on a 3-torus exhibits an analogous pattern when the triangles are turned into tetractyses.

on the sides of the 56 tetractyses making up the turned-inside out version of the 3-torus corresponds to the 192 Yang/Yin lines in the lower, diagonal half of the table. The two diagonal halves of the I Ching table symbolize the 3-torus and its mirror image, just as the two sets of the first six enfolded polygons with $(24+24)$ corners outside their root edge and **(168+168)** yods do.

According to Table 2, the 144 triangles of the six separate square antiprisms have 96 vertices and 240 sides when their **48** triangular faces are each divided into three triangles. There are 240 vertices & triangles. This 240:240 division displayed by their

geometrical composition has its counterpart in the two sets of seven separate polygons making up the inner form of the Tree of Life (Fig. 25). Each set has 240 hexagonal yods made up of 96 hexagonal yods on their sides and 144 hexagonal yods in their interiors. The 240 hexagonal yods in one set correspond to the 240 sides and the 240 hexagonal yods in the other set correspond to the 240 vertices & triangles. The 480 hexagonal yods signify geometrical degrees of freedom needed to construct the six arms of the 3-torus from points, lines and triangles. There are 480 hexagonal yods on the 240 sides of the 144 tetractyses making up the six separate square antiprisms. This property further demonstrates their Tree of Life basis.

In the context of superstring theory, the gauge symmetry group E_8 has 240 roots and the symmetry group $E_8 \times E_8$ of heterotic superstrings has 480 roots. These (and the gauge bosons associated with them) are symbolized by the 240 pairs of hexagonal yods. The pairing of hexagonal yods on each edge of the 144 tetractyses reflects the direct product nature of the superstring symmetry group. It is evidence that $E_8 \times E_8$ heterotic superstrings, not $SO(32)$ heterotic superstrings, are the basic constituents of matter.

With faces of the six antiprisms regarded as single tetractyses, Table 2 shows that they are made up of 240 hexagonal yods that comprise **48** hexagonal yods at the centres of the **48** tetractyses and 192 hexagonal yods on their 96 sides. They correspond to the **48** hexagonal yods at the centres of the **48** tetractyses in the seven separate polygons and

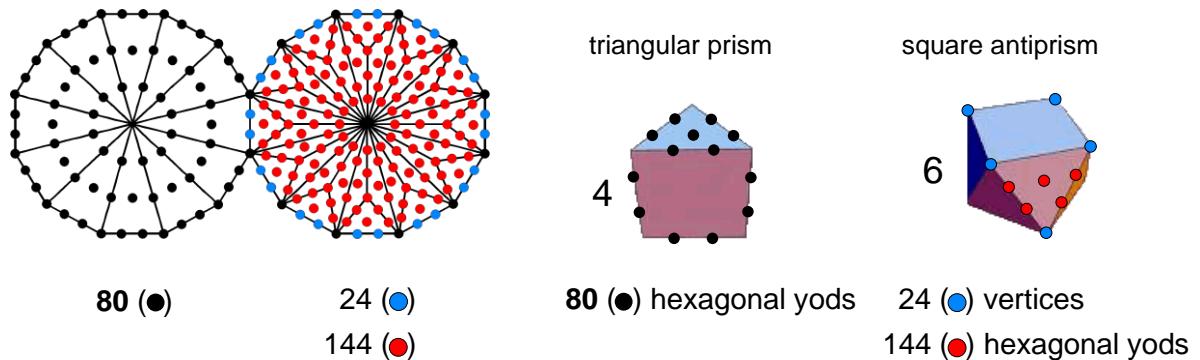


Figure 27. **248** yods surround the centres of two joined dodecagons when the sectors of one are turned into tetractyses and the sectors of the other are each turned into three tetractyses. The **80** yods either in one dodecagon or at the vertices of the other correspond to the **80** hexagonal yods in the four triangular prisms. The **168** yods other than corners in the other dodecagon correspond to the **168** yods in the six square antiprisms.

their 192 hexagonal yods on the sides of these tetractyses (Fig. 26). The two sets of polygons correspond to the 3-torus and its turned inside-out version. Figure 25 shows the detailed correspondence between the two sets of polygons and the two 3-tori. The eight yods consisting of the seven centres of polygons and one of the two hexagonal yods in the separate root edge correspond to the yods at the centres of the eight triangular faces of the four triangular prisms. The 12 hexagonal yods lying on the boundary of the dodecagon are the counterpart of the 24 vertices of these prisms. The **72** hexagonal yods on the **36** sides of the first six polygons correspond to the **72** hexagonal yods on the **36** edges of these prisms. The hexagonal yods at the centres of the **48** tetractyses in the seven polygons correspond to the hexagonal yods at the centres of the **48** tetractyses in the faces of the six square antiprisms. The 96 hexagonal yods on the **48** interior sides of tetractyses of the seven polygons correspond to the 96 hexagonal yods on the **48** edges of these antiprisms. There is complete isomorphism

between the two geometries — one the representation in the Euclidean plane of the archetypal Tree of Life pattern and one its representation in the hyperbolic plane.

Suppose that the sectors of one dodecagon in the inner form of the Tree of Life are transformed into single tetractyses (Type A dodecagon) and that the sectors of the other one are each turned into three tetractyses (Type B dodecagon). **248** yods surround their centres (Fig. 27). As a holistic system, the pair of dodecagons embody the parameters of superstring physics — that is, the physical world of Malkuth. This is demonstrated by the fact that **280** yods outside the root edge other than yods at centres of tetractyses surround the centres of the two joined, Type B dodecagons. **280** is the number value of *Sandalphon*, Archangel of Malkuth. One Type B dodecagon has **168** yods other than corners surrounding its centre. **168** is the number value of *Cholem Yesodoth*, Mundane Chakra of Malkuth.

According to Table 3, the six square antiprisms have **168** yods. This is the number of yods other than its corners that surround the centre of the Type B dodecagon. The remaining **80** yods correspond to the **80** hexagonal yods in the four triangular prisms. The isomorphism found between the two tori and the two sets of seven separate polygons exists also for the two dodecagons when their sectors are turned into either one or three tetractyses. Furthermore, the distinction between the four ‘corners’ and the six ‘edges’ of the tetrahedron into which the 3-torus can be deformed defines the same **80:168** division of the number **248** found for the lowest five overlapping Trees of Life (Fig. 15) and for the 3-torus, which has **80** vertices and centres of tetractyses and **168** hexagonal yods on their sides, as well as **80** vertices and **168** tetractyses when the triangular faces are turned into three tetractyses.

According to Table 1, **80** is the number value of Yesod, which means “foundation.” Just as the lowest Tree of a set of overlapping Trees of Life has **80** yods and is their foundation, so the four triangular prisms with **80** hexagonal yods can be regarded as the

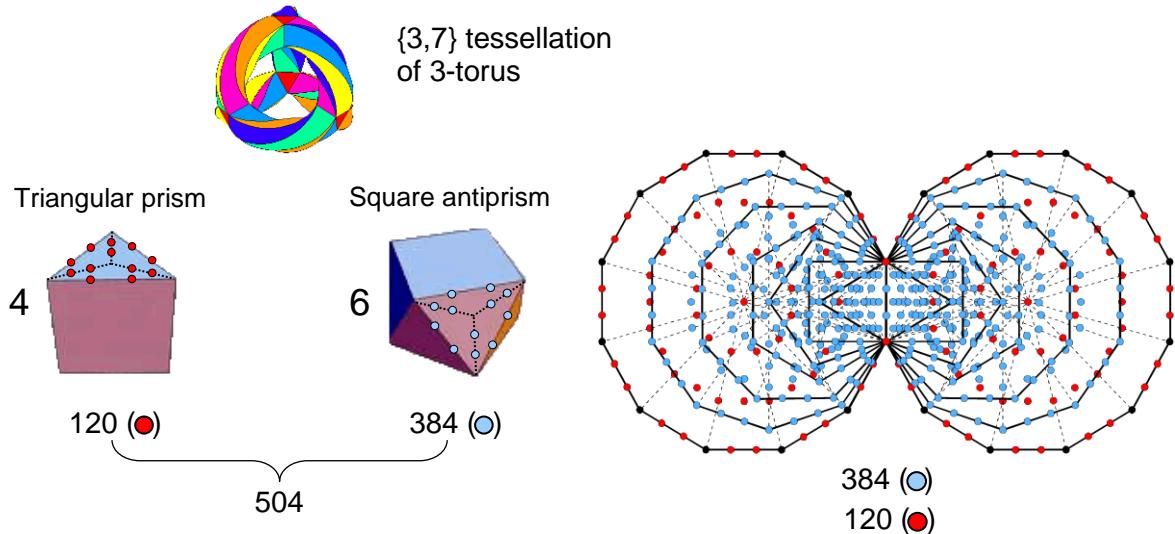


Figure 28. The inner Tree of Life has 504 yods other than the 20 (black) corners of the two dodecagons outside their shared edge. This is the number of hexagonal yods on the sides of tetractyses making up the prisms and antiprisms in the 3-torus. The 120 yods in the prisms correspond to the 120 yods in the two dodecagons other than their 20 corners. The 384 yods in the antiprisms correspond to the 384 yods in the two sets of six enfolded polygons.

foundation of the 3-torus because they are the ‘vertices,’ so to speak, of the tetrahedron into which the 3-torus can be deformed. According to Table 2, the ten separate prisms

and antiprisms have (**80+240=320**) hexagonal yods. The 3-torus and its version turned inside-out have **640** hexagonal yods. **640** is the number value of *Shemesh* (“the Solar Light”), the Mundane Chakra of Tiphareth. The eight triangular prisms in the 3-torus and its turned-inside out version with single tetractyses as triangular faces have **48** vertices, whilst the six antiprisms have **48** faces. **48** is the number value of *Kokab* (“The Stellar Light”), the Mundane Chakra of Hod. With three tetractyses as their faces, they have 64 vertices. **64** is the number value of *Nogah* (“Glittering Splendour”), the Mundane Chakra of Netzach. The 3-torus and its turned inside version need **496** yods to construct their triangular faces from tetractyses. Table 3 indicates that the 3-torus with each face of a prism and antiprism divided into three triangles consists of **248** vertices and triangles. The tessellated 3-torus and its turned inside-out version have **496** vertices and sides, as well as **496** yods in the case where their faces are single tetractyses. **496** is the number value of Malkuth. They have **112** faces. **112** is the number value of *Beni Elohim*, the order of angels assigned to Hod.

Table 3 indicates that there are 584 yods on the sides of the **168** tetractyses making up the prisms and antiprisms. Suppose that these **168** triangles are divided into their 504 sectors and each sector turned into a tetractys. This adds **168** vertices to the **80** vertices already present, so that there are now **248** vertices — the same as the number of yods

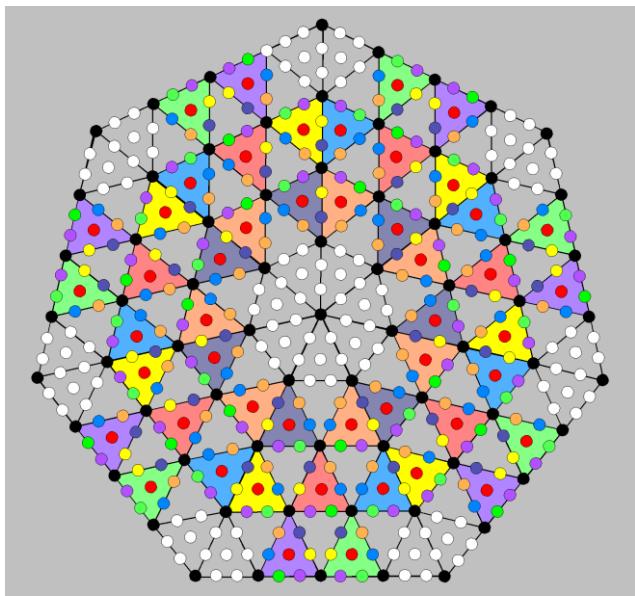


Figure 29. Surrounding the centre of the heptagon are 504 yods when its sectors are turned into 2nd-order tetractyses. This is the number of hexagonal yods on the sides of the tetractyses making up both the Sri Yantra and the {3,7} tessellation of the 3-torus.

in the prisms and antiprisms assembled in the 3-torus when each of their faces is a tetractys. The conversion of each of the **168** triangles adds seven yods on sides of tetractyses. There are ($7 \times 168 + 584 = 1760$) yods on the 756 sides of the 504 tetractyses. ($1760 - 80 = 1680$) yods on these sides surround the vertices and centres of the original 56 triangles used in the {3,7} tessellation of the **168** automorphisms of the Klein quartic. This higher-order conversion thus leads to the simultaneous appearance of the superstring structural parameter 1680 and the dimension **248** of the superstring gauge symmetry group E₈ as properties of the {3,7} tessellation of the 3-torus. This cannot be due to chance. Instead, it confirms that the object described over a century ago by Annie Besant and C.W. Leadbeater as the basic constituent of matter is, indeed, a superstring.

The counterpart of this property in the disdyakis triacontahedron is the 1680 vertices, sides & triangles that surround its axis when the triangles in its interior formed by edges and its centre are each divided into three sectors.⁹

As discussed in Section 4, 504 hexagonal yods lie on the 252 sides of the **168** tetractyses formed when the 56 triangles in the {3,7} tessellation of the 3-torus are each divided into three tetractyses. This is the number of hexagonal yods lying on the 252 sides of the 126 tetractyses making up the Sri Yantra when each of the 42 triangles surrounding its central one is divided into three tetractyses (Fig. 19). It is further confirmation that this tessellation of the automorphisms of the Klein quartic is a manifestation of the mathematical archetypes embodied in the Sri Yantra, Tree of Life and all other examples of sacred geometry. This is also demonstrated by the fact that there are 504 yods in the inner form of the Tree of Life other than the 20 corners of the dodecagons outside their shared edge. That this is not coincidental is shown by the fact that the two dodecagons have 120 yods other than these corners and the two hexagonal yods in the shared edge, whilst the two sets of the first six polygons have 384 yods (Fig. 28). They correspond to the 120 hexagonal yods on the sides of the tetractyses making up the four triangular prisms and to the 384 hexagonal yods on the sides of the tetractyses making up the six square antiprisms. This 120:384 division is another Tree of Life pattern.

Table 3 shows that there are **248** yods in the tessellated 3-torus with single tetractyses as each triangle and 752 yods with three tetractyses as each triangle. 504 yods are added by the higher-order transformation. Nine hexagonal yods are added to each of the 56 primary triangles ($9 \times 56 = 504$). They comprise **168** hexagonal yods at centres of the **168** tetractyses and 336 hexagonal yods on their **168** sides. In the Sri Yantra, the former correspond to the **168** hexagonal yods on sides of the 14 triangles in the fourth layer and the latter correspond to the 336 hexagonal yods on sides of the 28 triangles in the first three layers. Such correlation is strong evidence of the holistic nature of the {3,7} tessellation of the 3-torus and the relevance to superstring theory of the Klein quartic.

The heptagon has 504 yods surrounding its centre when its seven sectors are turned into 2nd-order tetractyses (Fig. 29). As illustrated earlier, all holistic systems are seven-fold in their physical manifestation. 43 yods — the number of triangles in the Sri Yantra — are needed to construct the heptagon from 1st-order-tetractyses. 504 yods are needed to construct it from 2nd-order tetractyses. Notice that the 70 tetractyses in this heptagon have 70 vertices surrounding its centre. The sum of the first 70 odd integers after 1 is 5040:

$$71^2 - 1 = 3 + 5 + 7 + \dots + 141 = 5040.$$

This is the number of yods in 504 tetractyses. If the 70 odd integers after 1 are assigned to the 70 vertices, we generate the same number as the number of yods obtained by replacing each of the 504 yods by tetractyses. There is always a harmony between number and sacred geometry. Figure 14 shows that three of the 10 whorls in the unit of matter described by Besant & Leadbeater are thicker than the seven other ones. As each whorl is a helix with 1680 turns, these three ‘major whorls’ have ($3 \times 1680 = 5040$) helical turns. As the microscopic manifestation of the Tree of Life and tetractys paradigm, the three major whorls correspond to the yods at the corners of the tetractys, which symbolize in Kabbalah the Supernal Triad of Kether, Chokmah and Binah and in Hinduism the trimûrti of Shiva, Brahma and Vishnu — the triple Godhead.

The Cosmic Tree of Life¹⁰maps all levels of reality, including the **26**-dimensional space-time continuum. It consists of 91 overlapping Trees of Life with 550 Sephirothic levels (Fig. 30). The lowest seven Trees of Life with 47 Sephirothic levels map space-time. The 503 levels above the top of the seventh Tree span all superphysical levels of consciousness. Hence, the top of the seventh Tree marking the first of the 25 spatial

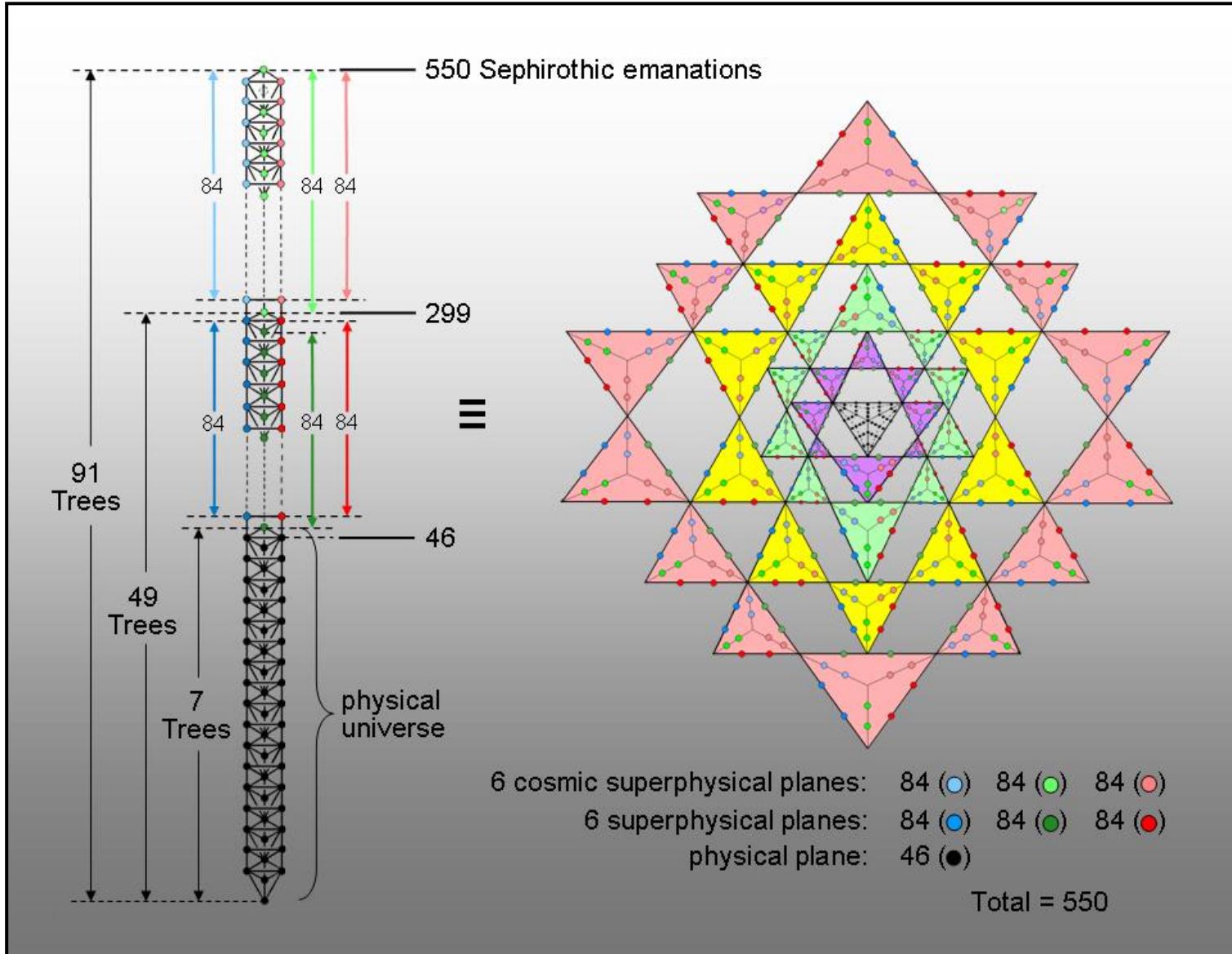
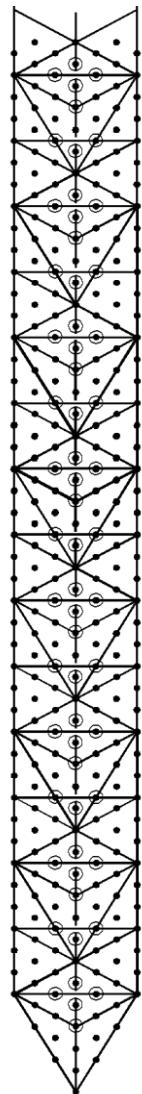
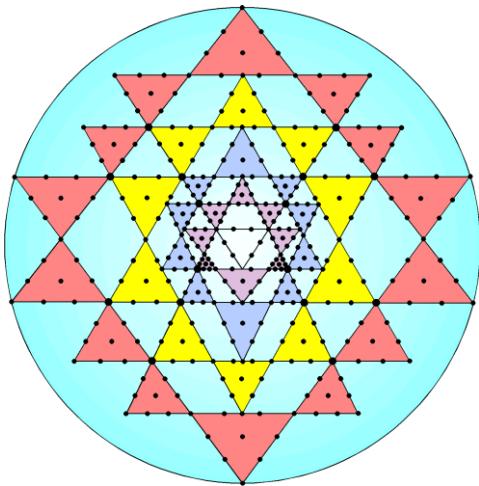


Figure 30. The 42 triangles of the Sri Yantra denote the 42 superphysical subplanes and 42 cosmic superphysical subplanes mapped by the 84 Trees above the lowest seven Trees, whose counterpart is the central triangle.

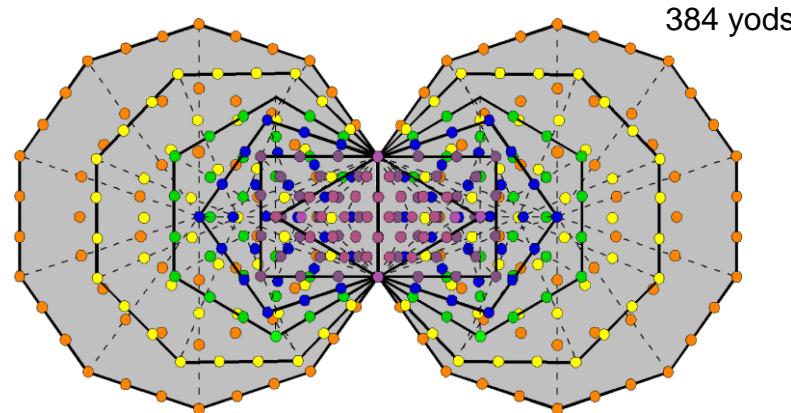


384 (●)

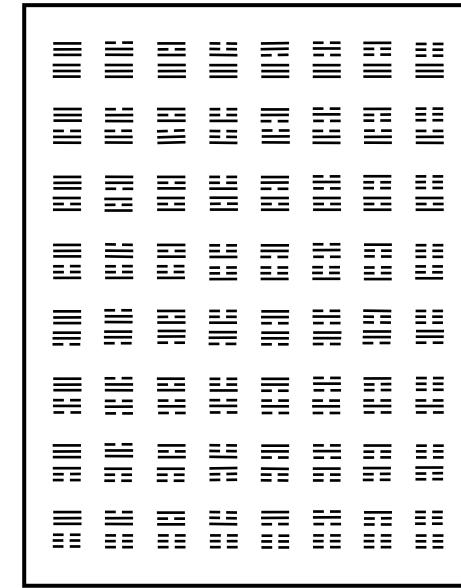


384 (●)

Sri Yantra

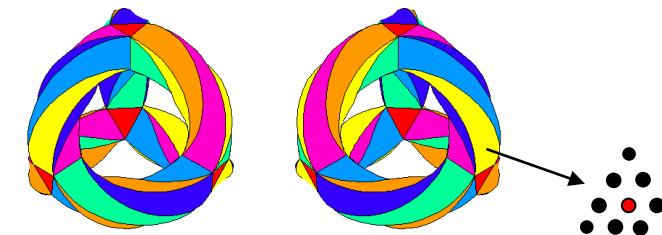


384 yods



384— & --

I Ching table



384 (●) yods

Figure 31. There are 384 yods up to the top of the seven Trees of Life mapping space-time. They correspond to the 384 yods in the Sri Yantra, the 384 lines/broken lines of the I Ching table, the 384 yods of the two sets of six enfolded polygons and the 384 yods on the sides of the **112** triangles in the tessellated 3-torus and its turned inside-out version. This proves the holistic nature of these systems, for the seven Trees constitute the Tree of Life representation of the seven Sephiroth of Construction.

dimensions¹¹ is the 504th from the top of the 91st Tree. The 504 hexagonal yods in the 42 triangles of the Sri Yantra when each is divided into three tetractyses symbolise the 504 Sephirothic levels down to the top of the lowest seven Trees mapping space-time. The 252 yods on the internal sides of tetractyses symbolize the 252 Sephirothic levels down to the top of the **49th** Tree that map the 42 subplanes of the six cosmic superphysical planes of consciousness. The 252 hexagonal yods on the sides of the 42 triangles symbolize the 252 Sephirothic levels from the top of the **49th** Tree to the top of the seventh Tree that map the 42 subplanes of the six superphysical planes of consciousness. The final 46 Sephiroth levels of the seven lowest Trees are symbolized by the 46 yods of the *central* triangle, which signifies the physical universe/plane.

There are 84 Sephirothic emanations on each pillar of the Cosmic Tree of Life spanning the 42 cosmic superphysical subplanes. There are also 84 emanations on each pillar spanning the 42 cosmic physical subplanes above the physical plane. These (3+3=6) sets of 84 emanations correspond to the three exterior and three interior sides of the tetractyses in each triangle in the Sri Yantra. Yods on exterior sides correspond to the emanations in the 42 superphysical subplanes of the cosmic physical plane. Yods on interior sides correspond to the emanations in the 42 subplanes of the six cosmic superphysical planes. Intuitively speaking, it seems reasonable to associate the hexagonal yods in the base of each outward-pointing triangle with emanations of the central Pillar of Equilibrium and to associate hexagonal yods on their inclined sides with emanations on the side pillars. Similarly, it is reasonable to associate the hexagonal yods on the sides of tetractyses converging to the tip of each triangle with emanations of the uppermost 42 Trees on the central pillar, the hexagonal yods on inclined internal sides being associated with the emanations on the side pillars.

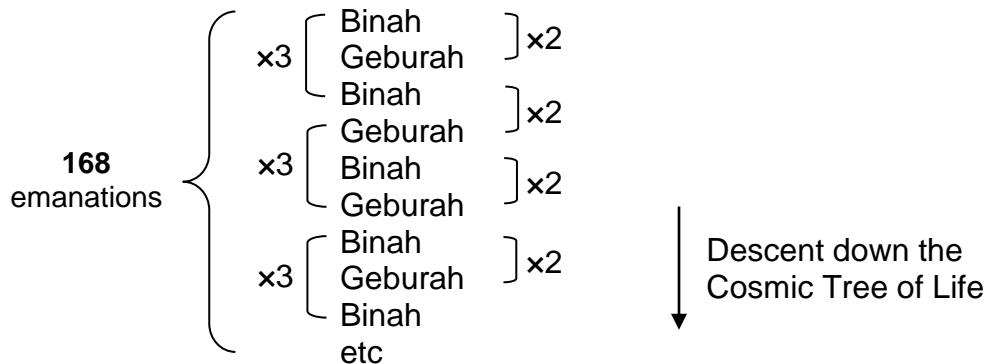
There is an alternative correlation between yods in the Sri Yantra and the emanations of the Cosmic Tree of Life. It is motivated by the fact that the 3-torus, when constructed from prisms and antiprisms, have **168** hexagonal yods on their 84 sides when their 56 triangular faces are tetractyses, and by the fact that a further 336 (=2×**168**) hexagonal yods are added on **168** more sides when these faces are each divided into three tetractyses. In other words, the 504 hexagonal yods on the 252 sides of **168** tetractyses bear the same **168**:2×**168** pattern that we see in the Cosmic Tree of Life for the **168** emanations on the central pillar down to the top of the seventh Tree and the 336 emanations above it on the two side pillars. This is no coincidence, for the {3,7} tessellation of the **168** automorphisms of the Klein quartic is a holistic system exhibiting the characteristics of the universal blueprint called the “Tree of Life.” A way of looking at the Sri Yantra that reproduces the **168**:2×**168** division in both the Cosmic Tree of Life and the tessellated 3-torus becomes apparent when we notice that the first three sets of triangles number 28 — twice the number in the fourth set. This means that the set of 14 triangles with 84 internal hexagonal yods and 84 external hexagonal yods corresponds, respectively, to the 84 emanations on the central pillar down to the top of the **49th** Tree and to the next 84 emanations on this pillar down to the top of the seventh Tree. The 28 triangles have (6×28=**168**) hexagonal yods on their sides and **168** hexagonal yods on interior sides of tetractyses. These 336 yods correspond to the **168** emanations on each side pillar down to the top of the seventh Tree and to the 336 hexagonal yods added by turning each triangle of the tessellated 3-torus into three tetractyses. The 1:2 proportion of triangles in the four layers of the Sri Yantra is the counterpart of the division of the three pillars of the Cosmic Tree of Life into the central Pillar of Equilibrium and the two side pillars — the Pillar of Mercy and the Pillar of Severity.

We see that the 504 hexagonal yods on the sides of the **168** tetractyses making up the 56 hyperbolic triangles of the {3,7} tessellation of the **168** automorphisms of the Klein

quartic have a connotation that goes beyond the physics of superstrings — or even pure mathematics. This is because the properties of holistic systems have different, but analogous, levels of meaning. The 384 yods on the sides of the **112** triangles in the pair of 3-tori are the counterpart of the 384 yods up to the top of the seventh Tree mapping space-time (Fig. 31). The **48** vertices of the triangles correspond to the **48** yods up to Chesed of the lowest Tree. The correlation once again furnishes strong evidence that the {3,7} tiling of the 3-torus with 56 hyperbolic triangles constitutes a holistic system of fundamental significance to particle physics.

The nature of the three branch points of the **168**-sheeted Riemann surface of the Klein quartic is analogous to the different ways of grouping the **168** Sephirothic emanations in each pillar of the 84 Trees of the Cosmic Tree of Life above the lowest seven Trees. The following correspondences exist:

1. The 84 sextactic points of the Klein quartic where the sheets join in twos correspond to the 84 pairs ($\times 2$) of Sephiroth on each pillar of the 84 Trees (Geburah-Binah, Tiphareth-Yesod or Chesed-Chokmah):



- The counterpart in the Sri Yantra of the **168** emanations on the Pillar of Equilibrium are the 84 pairs of hexagonal yods on external and internal sides of the 42 tetractyses composing the fourth layer of 14 triangles when each triangle is divided into three tetractyses (Fig. 30). The 42 external sides correspond to the 42 superphysical subplanes and the 42 internal sides correspond to the 42 cosmic superphysical subplanes. Similarly, the 84 pairs of hexagonal yods on external sides of the first three layers of 28 triangles correspond to the 84 pairs of emanations on the side pillars for the 42 superphysical subplanes, whilst the 84 pairs of hexagonal yods on internal sides of these triangles correspond to the 84 pairs of emanations on side pillars for the 42 cosmic superphysical subplanes. There must be 14 triangles in the last layer because each side of a triangle corresponds to one of the 42 superphysical subplanes, and $42/3 = 14$. The 336 hexagonal yods (**168** pairs) on the internal sides of the **168** tetractyses making up the tessellated 3-torus correspond to the 336 emanations on the side pillars of the Cosmic Tree of Life above the seventh Tree, **168** emanations on each one. The **168** hexagonal yods on the 84 sides of the 56 triangles correspond to the **168** emanations on the central Pillar of Equilibrium down to the top of the seventh Tree;
2. The 56 points that are points of contact of the Klein quartic with the 28 bitangents correspond to the 56 triplets ($\times 3$) of Sephiroth on each side pillar (the Pillar of Judgement is used as the example shown above). Notice that there are 28 triplets of one combination of Sephiroth and 28 triplets of another combination. This corresponds to the 28 bitangents, each with two points of contact. Their counterpart is the {3,7} tessellation of the 3-torus into 56 triangles with **168** vertices. The counterparts of the 28 bitangents in the I Ching table are the 28 hexagrams on each

side of its diagonal (Fig. 23), the **168** lines and broken lines in one diagonal half denoting the **168** automorphisms of the Klein quartic and the **168** lines and broken lines in the other half denoting its **168** antiautomorphisms. The two points of contact of the Klein quartic with each bitangent are symbolized by the pair of different trigrams in each off-diagonal hexagram. In the Sri Yantra, their counterparts are the 28 sets of two groups of three hexagonal yods on external and internal sides of the 42 tetractyses in the 14 outermost triangles. There are 28 groups of three hexagonal yods on internal sides corresponding to cosmic superphysical subplanes and 28 groups of three hexagonal yods on external edges corresponding to

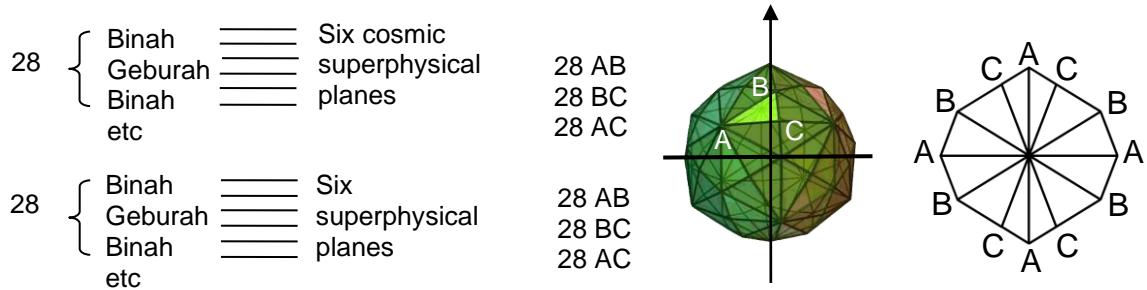


Figure 32. The 56 triplets of emanations on a side pillar of the Cosmic Tree of Life above the seventh Tree correspond to the 56 sets of three types of edges of the disdyakis triacontahedron above and below the equator perpendicular to the axis joining diametrically opposite A vertices. The 28 triplets in the cosmic superphysical planes and the 28 triplets in the superphysical planes correspond to the 28 sets of edges above and below the equator.

superphysical planes. As with the Cosmic Tree of Life, the **168** yods form 56 triplets, 28 triplets of one type and 28 of another type.

The counterpart in the disdyakis triacontahedron of the **168** sheets of the Riemann surface of the Klein quartic joining together in 56 sets of three is the following: the polyhedron (Fig. 32) has 180 edges made up of 56 sets of edges AB, BC & AC in the polyhedron above and below the equator formed by a 12-sided polygon with four AB edges, four BC edges & four AC edges. The 28 sets of three emanations on a side pillar of the Cosmic Tree of Life mapping the 42 subplanes of the six cosmic superphysical planes correspond in the polyhedron to the 28 sets of three edges above its equator. The 28 sets of three emanations on a side pillar for the 42 subplanes of the six superphysical planes correspond to the 28 sets of three edges below the equator;

3. The **168** sheets join in groups of seven to form the 24 Weirstrass points. The 84 Trees map the six cosmic superphysical planes, each of seven subplanes, and the six superphysical planes, each of seven subplanes. Each of the seven generic subplanes is repeated ($6+6=12$) times. A Tree of Life maps each subplane and there are two Sephiroth on the side pillar per Tree. The **168** emanations on a side pillar can be grouped into seven sets of ($12 \times 2 = 24$), each set being associated with a Sephirah of Construction. The 24 Weirstrass points in each of the seven groups correspond therefore to the 24 emanations on a side pillar associated with each generic subplane, that is, with each Sephirah of Construction. For example, the third subplane of each plane (counting from the lowest subplane) corresponds to Hod, the third Sephirah of Construction from Malkuth. This Sephirah has 12 subplane manifestations, each mapped by a Tree of Life with two emanations on a side pillar associated with it, i.e., associated with Hod are 24 emanations. Similarly, 24 emanations in the Cosmic Tree of Life are associated with each of the six other Sephiroth of Construction. Their mathematical counterpart is the {7,3} tessellation of

the 3-torus as 24 heptagons, each with seven vertices. In the {3,7} tessellation on the 3-torus, the 56 triangular faces define the 56 vertices of seven cubes, each having the 24 symmetries of the octahedral cube. The **168** symmetries of the Klein quartic belong to $\text{PSL}(2,7)$, which is of order **168**. It has seven subgroups of order 24, isomorphic to the octahedral group. In terms of the model of particle physics presented in Part 4 of Article 40,¹² each subgroup describes the 24 particles in each of the seven colour states. The clustering of the **168** sheets of the Riemann surface of the Klein quartic into 56 groups of three corresponds to the three generations of 56 types of particles.

The seven-fold nature of the **168** symmetries of the Klein quartic is the manifestation of the seven Sephiroth of Construction, which determine both the seven planes and the seven subplanes of each plane. The **168** automorphisms mapped on the 3-torus tessellated into 56 triangles correspond to the **168** emanations on one side of the Cosmic Tree of Life down to the top of the seventh Tree. The **168** antiautomorphisms mapped by the tessellated 3-torus turned inside out correspond to the **168** emanations on the opposite pillar, on which are located Sephiroth of opposite polarity. The positive and negative dualities embodied in the Pillars of Mercy and Judgement manifest in the projective duality of the 3-torus, which maps the **168** automorphisms, and its turned inside-out version, which maps its **168** antiautomorphisms. The two tori embody the 336 Sephirothic emanations of the side pillars down to the physical universe. *Cholem Yesodoth*, the Mundane Chakra of Malkuth, the Sephirah signifying this universe, has the number value **168** because it defines the location in the Cosmic Tree of Life of the physical plane mapped by its lowest seven Trees of Life as the number of emanations on the central Pillar of Equilibrium down to the top of the seventh Tree.

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- ⁷ The Klein quartic is fully analysed in the book "The Eightfold Way: The Beauty of Klein's Quartic Equation," edited by Silvio Levy, Cambridge University Press, 1999. This is downloadable as PDF files from: <http://www.msri.org/publications/books/Book35/index.html>.
- ⁸ Ref. 1, Part 4, pp. 38-47.
- ⁹ Ref. 6, p. 6.
- ¹⁰ Phillips, Stephen M. Article 5: "The superstring as microcosm of the spiritual macrocosm," <http://www.smphillips.mysite.com/Article05.pdf>, pp. 2, 3.
- ¹¹ Phillips, Stephen M. Article 2: "The physical plane and its relation to the UPA/superstring and space-time," <http://www.smphillips.mysite.com/Article02.pdf>, p. 6 et seq. Also Fig. 8 on p. 37.
- ¹² Ref. 8.