Article 40

The Unification of All Sacred Geometries
&
Its Implication for Particle Physics

Part 3

by

Stephen M. Phillips, Ph.D.

Flat 3, 32 Surrey Road South. Bournemouth. Dorset BH4 9BP. England.

Website: http://www.smphillips.mysite.com
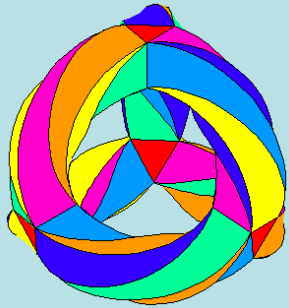
Figure 1

The quartic equation:

$$x^3y + y^3z + z^3x = 0$$

was studied by the mathematician Felix Klein. He showed that its Riemann surface is mapped onto itself (hence "automorphisms") by 168 analytic transformations. These symmetries are mapped onto a surface of genus 3, i.e., the 3-torus. In fact, this is the maximum number of symmetries for a surface of this genus. This Riemann surface can be represented in the hyperbolic plane by the Klein Configuration. For the {7,3} tiling, it has 168 red, hyperbolic triangles denoting the 168 automorphisms and 168 blue triangles denoting the 168 anti-automorphisms. The 168 red or blue triangles are grouped 24 to each of seven sectors, forming 24 heptagons. Three heptagons meet at each of its 56 vertices. Each sector has two slices. Each of the 14 slices shown numbered has 12 red triangles and 12 blue triangles. The 168 automorphisms belong to the group PSL(2,7). The 168 automorphisms and 168 anti-automorphisms belong to the double-cover group SL(2,7).

Figure 1

14×12 = 168
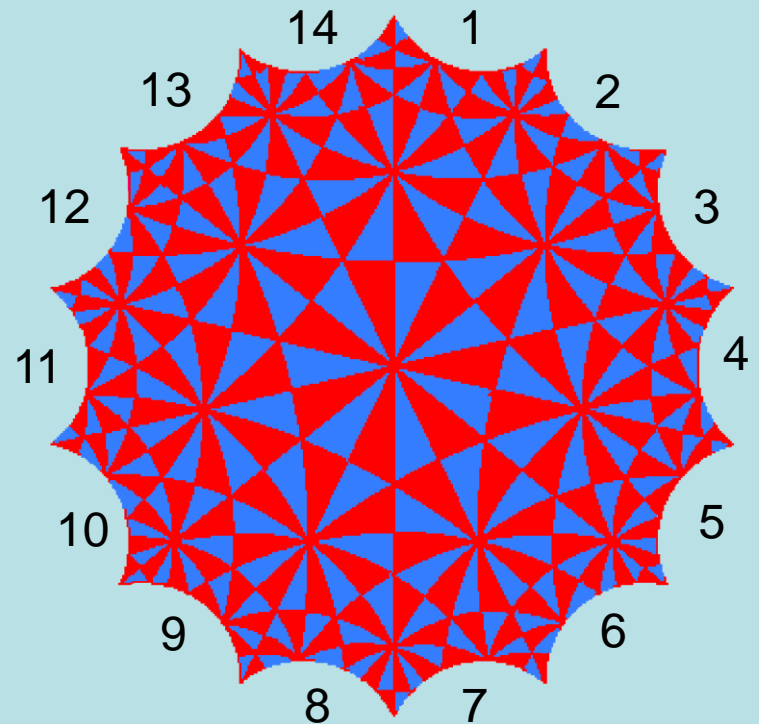

3-torus

168 (▲) =

Klein quartic:

$$x^3y + y^3z + z^3x = 0$$

{7,3} mapping of the 168 symmetries of the Klein quartic

The Klein configuration is the mapping on the hyperbolic plane of the 168 automorphisms (red hyperbolic triangles) of the Klein quartic. The 168 blue triangles denote its 168 anti-automorphisms.
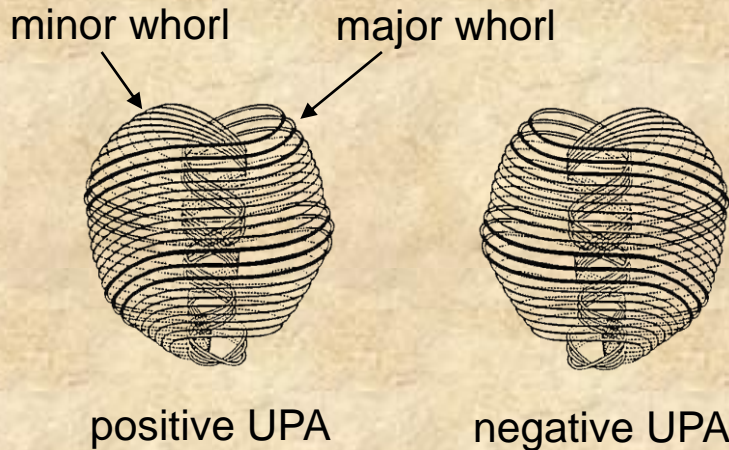
Figure 2

In their book *Occult Chemistry*, published in 1908, Annie Besant and C.W. Leadbeater recorded their observations of what they believed were atoms of all the elements. The basic unit of matter, or "ultimate physical atom" (UPA), has two varieties, one the mirror image of the other. In the positive type, the ten closed curves, or "whorls," spiral clockwise 2½ times around the axis about which the particle spins, then wind back 2½ times in a narrower helix back to the top of the particle. Each of the ten helices is a closed curve with 1680 turns. There are 336 turns in each of the five revolutions of each whorl, that is, 168 turns in a half-revolution. The ten whorls make 3360 turns in each revolution. The author proved that these particles are constituents of up and down quarks and has interpreted them as $E_8 \times E_8'$ heterotic superstrings.

Figure 2

Annie Besant
(1847-1933)

C.W. Leadbeater
(1854-1934)

minor whorl   major whorl

positive UPA   negative UPA

The two types of UPAs ('ultimate physical atoms') making up matter are mirror images of each other. They are $E_8 \times E_8$ heterotic superstring constituents of quarks. They consist of 10 separate, never touching, closed curves, or 'whorls,' each winding 1680 times around a torus. In the positive UPA, the whorls spiral clockwise; in the negative UPA, they spiral anticlockwise. Each curve makes five revolutions but does NOT form a knot either with itself or with other curves. The twisting of the three major whorls at the bottom of the UPA depicted in the 3rd edition of Besant's & Leadbeater's book "Occult Chemistry" was a diagrammatic error, as it would imply that the 1st and 3rd major whorls are the single edge of a Mobius strip, i.e., that there are 9 (not 10) curves, contrary to what they stated. Instead, the three major whorls form what is, topologically speaking, a 'link' with the seven minor whorls.
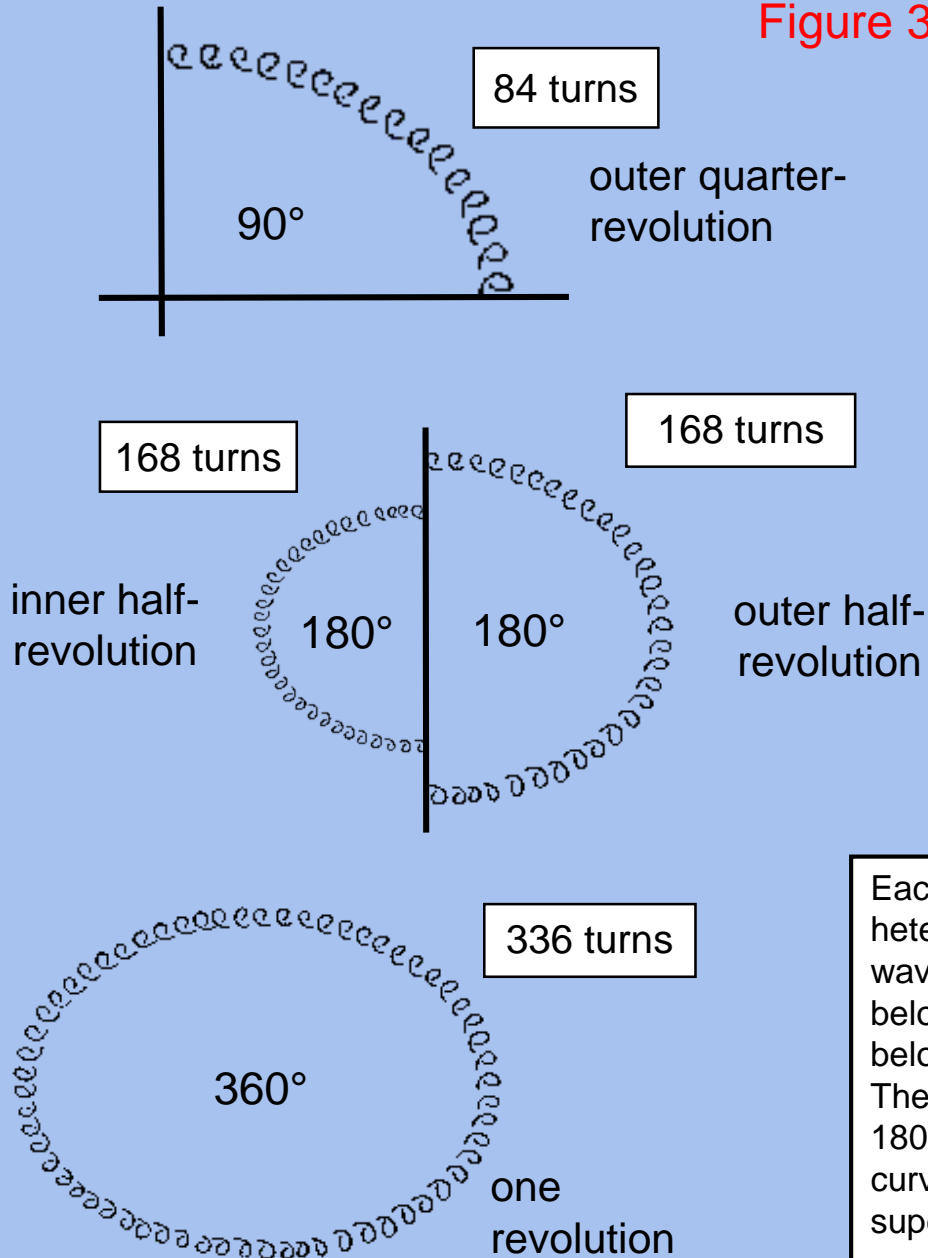
A whorl is a helical coil with 1680 turns. It winds 336 times in each of five revolutions around the spin axis of the positive or negative UPA.
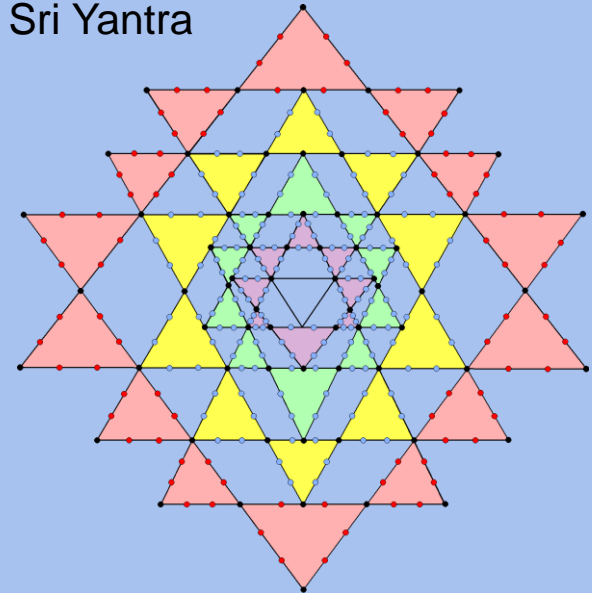
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Figure 3

The numbers 84, 168 and 336 appear in the Sri Yantra. Its 42 triangles have 84 vertices (black yods), 84 (red) hexagonal yods on the sides of its 14 outermost triangles and 168 (blue) hexagonal yods on the sides of the 28 triangles in its first three layers. 336 yods therefore line its 42 triangles, delineating its *shape*. 168 yods mark out each half. The Sri Yantra therefore represents the number of turns in one revolution of each helical whorl of the UPA. If one assigns the number 10 (the Pythagorean Decad symbolized by the tetractys) to each yod, the Sri Yantra generates the number 3360. This is the number of turns in one revolution of all ten whorls of the UPA. If one assigns the number value 50 of the Godname ELOHIM to each yod, the Sri Yantra generates the 16800 turns in all ten whorls of the UPA, a heterotic superstring. Created out of the number value of ELOHIM, the Godname of the third member of the Supernal Triad, the Sri Yantra generates the *form* of the fundamental subatomic particle making up the atoms of the elements. The Godname YAHWEH of the second member of the Supernal Triad with number value 26 prescribes the 26 dimensions of space-time from which the 10-dimensional superstring emerges.

Figure 3

84 turns

90°

outer quarter-revolution

168 turns

168 turns

inner half-revolution

180°   180°

outer half-revolution

336 turns

360°

one revolution

84 (●) ⎤
84 (●) ⎬ 336
168 (○) ⎦

3-d Sri Yantra

Each of the ten closed curves (whorls) of the $E_8 \times E_8$ heterotic superstring has 1680 circularly polarised waves or oscillations running along it. 840 waves belong to the outer 2½ revolutions and 840 waves belong to the 2½ inner revolutions around its spin axis. There are 84 waves in a 90° twist, 168 waves in an 180° twist and 336 waves in one revolution of each curve. These structural parameters of the heterotic superstring are embodied in the 3-d Sri Yantra.
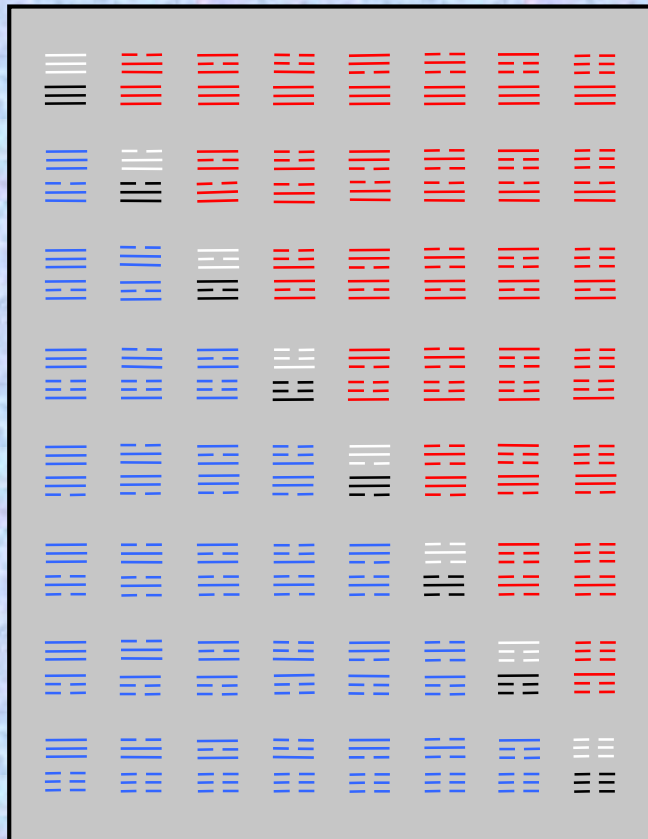
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Figure 4

The I Ching table, used for thousands of years by the Chinese for purposes of divination, is an 8×8 array of 64 hexagrams, each a pair of trigrams. There are eight hexagrams in its diagonal and 28 hexagrams (56 trigrams with 168 lines and broken lines) on either side of it. The table is equivalent to the Klein Configuration because the 168 red lines and broken lines of the 56 trigrams on one side of the diagonal correspond to the 168 red hyperbolic triangles denoting the 168 automorphisms, whilst the 168 blue lines and broken lines of the 56 trigrams on the other side of the diagonal correspond to the 168 blue hyperbolic triangles denoting the 168 anti-automorphisms.
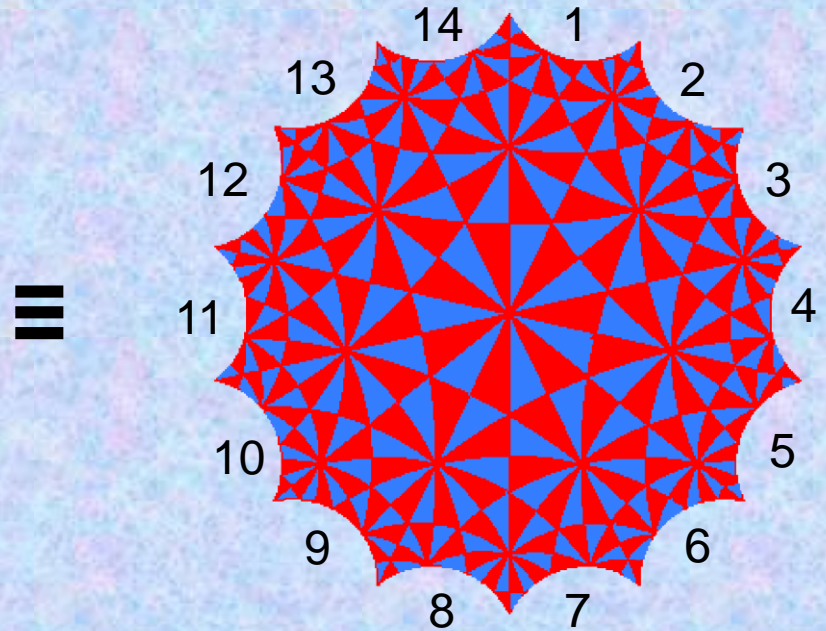
Figure 4

I Ching table

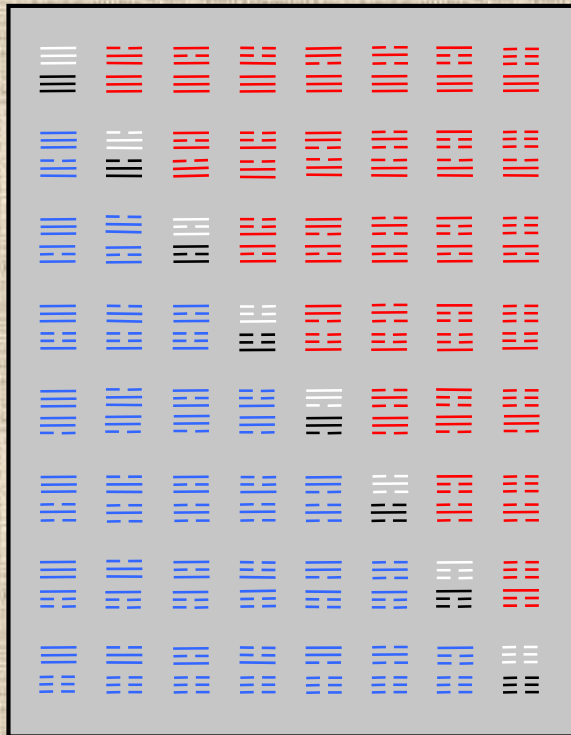Klein configuration

168 — & - -
168 — & - -

168 ▲ (automorphism)
168 ▲ (anti-automorphism)

9

Figure 5

In the {3,7} tiling of the Riemann surface of the Klein quartic, 56 triangles with 168 vertices tessellate completely the hyperbolic plane. The vertices denote the 168 automorphisms of the Klein quartic. The triangles correspond in the I Ching table to the 56 red trigrams on one side of the diagonal with 168 lines and broken lines, each one corresponding to an automorphism of this equation well-known to mathematicians.

Figure 5

The correspondence between the (168+168) lines/broken lines of the (56+56) off-diagonal trigrams and the (168+168) symmetries of the Klein quartic

{3,7} tiling

56 triangles with 168 vertices are needed to tessellate the 3-torus.

56×3 = 168

Poincaré dual

{7,3} tiling

24 heptagons with 168 vertices are needed to tessellate the 3-torus.

24×7 = 168

56 trigrams   56 trigrams

56×3 = 168
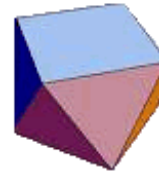
168
anti-automorphisms

168
automorphisms

11

Figure 6

The {3,7} tiling onto the 3-torus of the 168 automorphisms of the Klein quartic represented by the 168 vertices of the 56 triangles of the Klein configuration requires seven colours to colour the 56 triangles so that no two triangles sharing an edge have the same colour. The 3-torus may be deformed into a tetrahedral structure formed by sticking the 12 square faces of six square antiprisms onto the 12 square faces of four triangular prisms and then twisting each antiprism. This creates 56 triangles (eight triangular faces of the four prisms + 48 triangular faces of the six square antiprisms). The 56 triangular faces define the 56 vertices of seven cubes. As a cube has 24 different rotations, the 168 automorphisms of the Klein quartic correspond to seven copies of 24 symmetries of the octahedral group. Its 168 anti-automorphisms are represented by the 168 vertices of the 56 triangular faces of the tetrahedral structure after it has been turned inside out. These triangles, too, define the 56 vertices of seven cubes that generate seven copies of the 24 symmetries/inversions of the octahedral group. These make up the 168 anti-autmorphisms of the Klein quartic.

Figure 6

4×

triangular prism
(6 vertices, 3 square
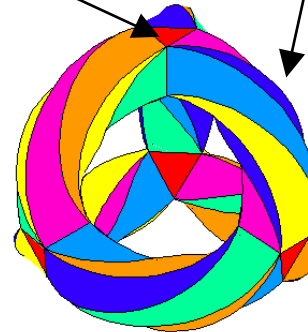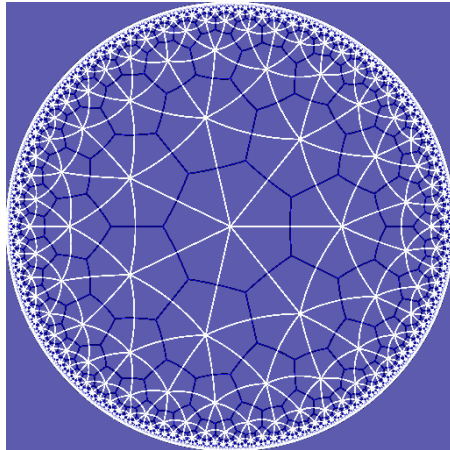faces, 2 triangular faces)

6×

square antiprism
(8 vertices, 2 square
faces, 8 triangular faces)

4×2=8 triangles

6×8=48 triangles

Attach a triangular prism to the 4
vertices of a tetrahedron. Then stick
each square face of an antiprism
onto a square face of each triangular
prism, giving it a twist.

8 triangular faces of
4 triangular prisms

+

8×6=48 triangular faces
of 6 square antiprisms

=

56 triangles

Klein configuration

7-colour tiling of the Klein
configuration on the 3-torus

$\left\{ 3,7 \right\}$ 56 triangles

$\left\{ 7,3 \right\}$ 24 heptagons

→ 168 automorphisms

24 rotations of tetrahedron×7-
fold rotational symmetry of
Klein configuration = 168
symmetries of PSL(2,7)

The 336 symmetries
consist of 7 copies of the
(24+24=48) symmetries of
the octahedral symmetry
group shown by the cube.
This is because the 56
triangular faces define the
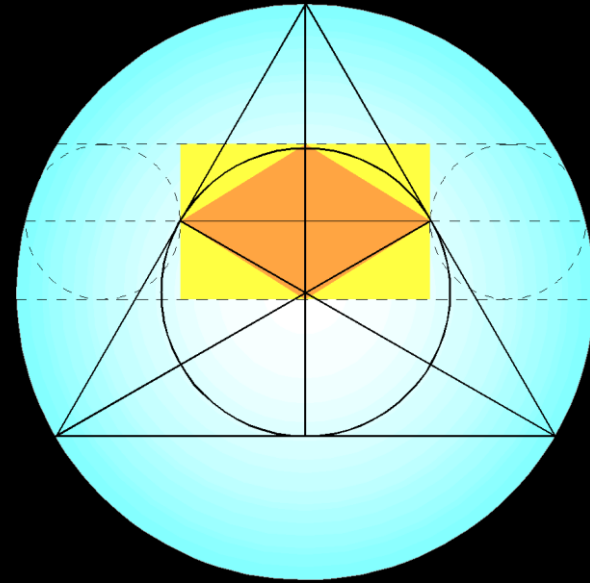56 vertices of 7 cubes.

The mirror reflections of the 56 triangles with 168 vertices
define the 168 anti-automorphisms of the Klein quartic.

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Figure 7

The 168 automorphisms of the Klein quartic belong to PSL(2,7). This group is isomorphic to PSL(3,2), whose 168 symmetries are displayed by the Fano plane, the simplest projective plane. Implicit in its geometry is a Golden Rectangle, the ratio of whose adjacent sides is the Golden Ratio φ = 1.6180..., and a Golden Rhombus, the lengths of whose diagonals are in the same proportion. The 60 edges of a rhombic triacontahedron form 30 Golden Rhombi that are the bases of pyramids with four faces, creating the 120 faces of the disdyakis triacontahedron. This shows how the ideal, aesthetically pleasing proportion of the Golden Ratio, used throughout history by artists and architects, manifests in the form of the disdyakis triacontahedron. Beauty is exhibited in the shape, as well as in the mathematical properties, of the polyhedral version of the inner Tree of Life.

Figure 7

Disdyakis triacontahedron with a
Golden Rhombus formed by edges

Fano plane with a Golden Rectangle
and a Golden Rhombus

The Fano plane has the 168 symmetries of SL(2,3), which is isomorphic
to PSL(2,7), the group of symmetries of the Klein quartic. The Golden
Rhombic shape of the faces of the rhombic triacontahedron underlying
the disdyakis triacontahedron is implicit in the geometry of the Fano
plane, which represents the algebra of the octonions.

Figure 8

A continuous, logical link connects the (2,3) torus knot to the quantitative description given in 1908 by Annie Besant and C.W. Leadbeater of the form of the heterotic superstring constituent of up and down quarks.

Figure 8

(2,3) torus knot $\longrightarrow$

$\downarrow$

$\longleftarrow$ (2,3) torus knot winds on 3-torus

$\downarrow$

3-torus is Riemann surface of Klein quartic with 168 symmetries of PSL(2,7) & 336 symmetries of SL(2,7)

$\downarrow$

PSL(2,7) is isomorphic to SL(3,2)

$\downarrow$

Fano plane has symmetry group SL(3,2) of order 168

$\downarrow$

Fano plane represents octonion algebra

$\downarrow$

Octonion algebra is isomorphic to E_8 Lie algebra

$\downarrow$

E_8 is gauge symmetry group of unified, superstring force

$\downarrow$

$E_8 \times E_8$ heterotic superstring is UPA

$\downarrow$

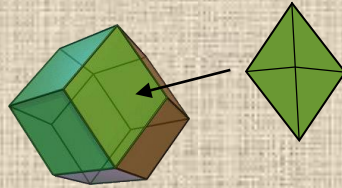Helical whorl of the UPA winds168 times on torus in half a revolution

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Figure 9

The polyhedral inner form of the Tree of Life is the 144 Polyhedron, which has 74 vertices, 216 edges and 144 faces. It generates another polyhedron with 62 vertices, 180 edges and 120 triangular faces. The latter is the disdyakis triacontahedron and the former is the disdyakis dodecahedron with tetrahedra added to its 48 faces. Their (24+24=48) peaks correspond to the 24 rotational symmetries of the octahedral group and the 24 rotations/inversions. This is a subgroup of SL(2,7), and seven copies of it define the 336 symmetries of SL(2,7), the double-cover group of PSL(2,7), which is isomorphic to SL(2,7)/Z_2 and describes the 168 automorphisms of the Klein quartic. This symmetry manifests in the 168 edges of the disdyakis triacontahedron above and below the equator that is perpendicular to the axis joining two diametrically opposite A vertices of this polyhedron. The centre of PSL(2,7) is Z_3, the cyclic group of order 3 that is isomorphic to the three primitive roots of 1, namely, 1, $\exp(2\pi i/3)$ & $\exp(4\pi i/3)$. The fact that the dimension of SL(2,7) is the number of edges above and below the equator of the disdyakis triacontahedron is evidence that PSL(2,7) is embodied in this polyhedron as a fundamental symmetry.
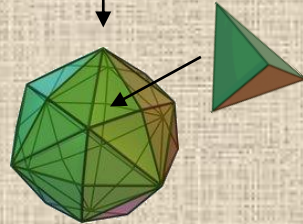
The algebra of the seven unit imaginary octonions is represented by the Fano plane, which has the symmetry of SL(3,2), a group that is isomorphic to PSL(2,7). It can alternatively be represented by assigning them to the vertices and centre of an octahedron. This suggests a connection between the disdyakis triacontahedron and the octonions. It is to be expected because the octonions form a basis of E_8, the superstring gauge symmetry group (see Figs. 7 & 8).
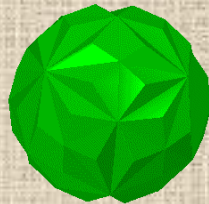
Figure 9

Rhombic dodecahedron
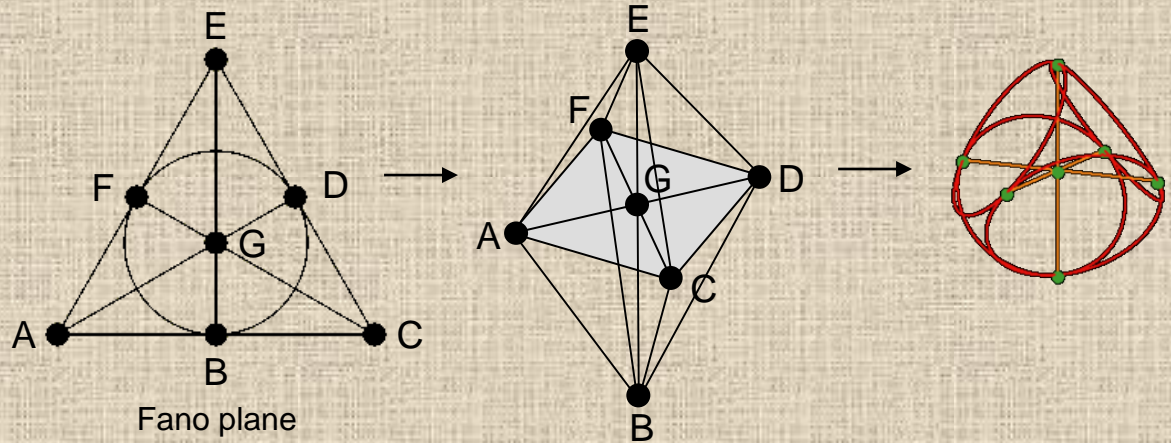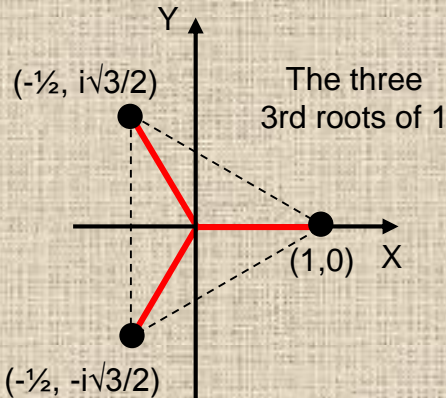14 vertices
24 edges
12 faces

Disdyakis dodecahedron
26 vertices
72 edges
48 faces

144 Polyhedron
74 vertices
216 edges
144 faces

48 of the 74 vertices of the 144 Polyhedron are peaks of tetrahedra that can be stuck on the 48 faces of a disdyakis dodecahedron. As the latter are the faces of rhombic pyramids attached to the 12 faces of a rhombic dodecahedron, they form 12 sets of four faces, that is, 24 faces and their 24 mirror images. The corresponding peaks of the tetrahedra correspond to the (24+24=48) symmetries of the octahedral group. This is a subgroup of SL(2,7) and seven copies of it define the 336 symmetries of SL(2,7), the double-cover group of PSL(2,7) ~ SL(2,7)/Z_2 describing the 168 automorphisms of the Klein quartic. They manifest in the disdyakis triacontahedron as its 168 edges above and below the equatorial plane parallel to its 7 sheets of vertices. The octahedral group is primary because an octahedron and its centre can represent the Fano plane defining the multiplication of the unit imaginary octonions and having the 168 symmetries of SL(3,2), which is isomorphic to PSL(2,7). The 168 symmetries of PSL(2,7) comprise (8×6=48) rotations by (1/7-6/7) turns of the central 8 heptagons in their {7,3} tessellation on the hyperbolic plane. The centre of PSL(2,7) is Z_3, the cyclic group of order 3 that is isomorphic to the three primitive 3rd roots of 1: 1, exp(2πi/3) & exp(4πi/3).

The three
3rd roots of 1

(-½, i√3/2)

(-½, -i√3/2)

(1,0)

Y

X

Fano plane

E

F   D

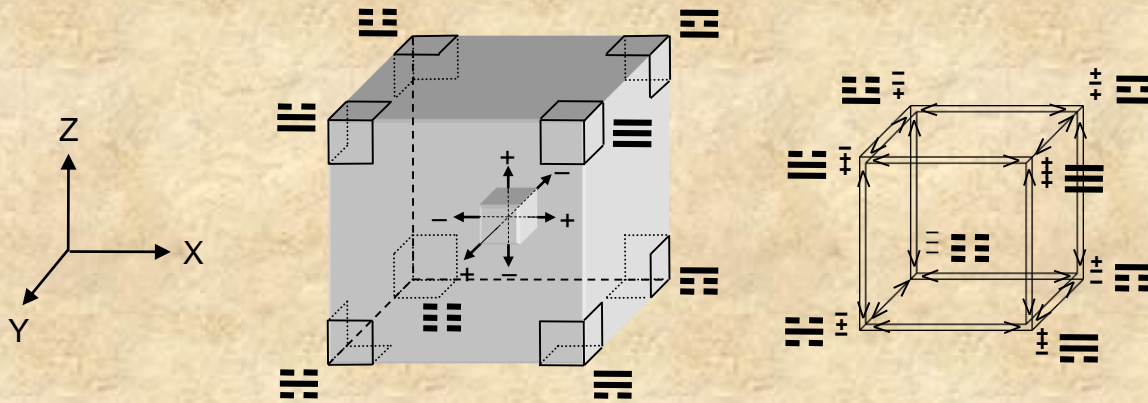G

A   B   C

E

F

G   D

A

C

B

The 7 lines of the Fano plane representing the seven 3-tuples of unit imaginary octonions become the 3 diagonals of the octahedron plus the 4 circles which circumscribe 4 of its 8 faces.
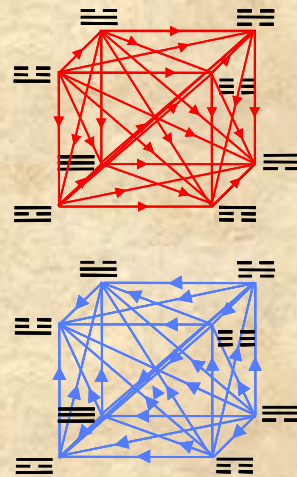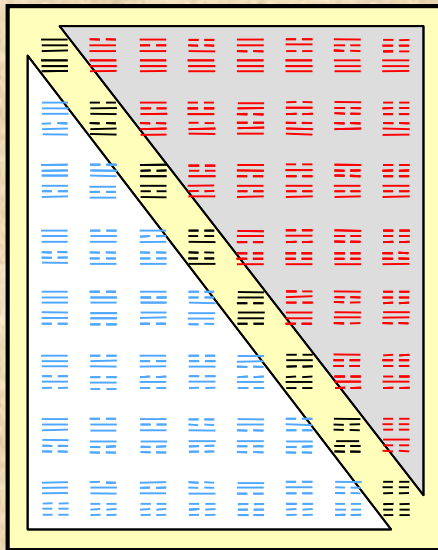
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Figure 10

If a Yang (unbroken) line of a trigram denotes a face of a cube orientated along one of the three perpendicular, positive directions and a Yin (broken) line denotes a face facing a negative direction, the eight trigrams symbolize the sets of three perpendicular faces that intersect at the eight corners of a cube. As a pair of different trigrams, a hexagram defines the straight line joining two corners of a cube. The pairs of identical trigrams in the eight hexagrams in the diagonal of the I Ching table define the eight corners themselves. The ordering of pairs of trigrams signifies the directions of the lines regarded as arrows. This means that the hexagrams below the diagonal signify arrows connecting different corners that point in the opposite direction to those connecting corners signified by hexagrams above the diagonal. The 28 hexagrams above the diagonal have 168 lines/broken lines denoting the 168 faces (84 positive, 84 negative) defined by the endpoints of the 28 lines joining different corners. Each combination of three faces appears seven times, creating seven copies of each corner, i.e., 56 copies of the eight corners, totalling 64 corners. These are the counterparts of the 56 hyperbolic triangles with 168 vertices in the {3,7} tiling of the automorphisms of the Klein quartic on the hyperbolic plane, each triangle in fact defining one of the corners of seven cubes. The 56 hyperbolic triangles mapped onto a 3-torus turned inside-out, whose 168 vertices denote the 168 anti-automorphisms of the Klein quartic, similarly correspond to the 56 trigrams with 168 lines/broken lines in the 28 hexagrams below the diagonal, each trigram defining a corner of seven copies of a cube - the seven cubes whose 56 corners are defined by these 56 triangles. The Yang/Yin nature of each line/broken in the I Ching table signifies whether the faces of the cube are orientated towards the positive or negative directions of each perpendicular axis. These 168 bi-polar degrees of freedom manifest in the disdyakis triacontahedron as the 168 edges other than the 12 edges in its equatorial plane perpendicular to an axis joining two diametrically opposite A vertices. Regarded as arrows that can point in two opposite directions, the edges of the polyhedron point in 168 directions above the equatorial plane and 168 directions below it. The 168 pairs of oppositely directed edges are the counterpart of the 168 automorphisms and the 168 anti-automorphisms of the Klein quartic and the 168 lines and 168 broken lines in the 56 off-diagonal hexagrams of the I Ching table.

Figure 10

The eight trigrams define the eight corners of a cube because their three yin/yang lines correspond to the three orthogonal faces (positive or negative) that intersect at these corners.

That the full octahedral group with 48 elements is relevant to the understanding of the formation of the disdyakis triacontahedron from the '48 beams of light' is indicated by the fact that they emanate from the centres of the 48 faces of the disdyakis dodecahedron, the Catalan solid that displays the symmetries of the octahedral group. Furthermore, the 168 edges of the disdyakis triacontahedron above and below its equator correspond to the 168 Yang/Yin lines in the 28 hexagrams on either side of the diagonal of the I Ching table. The eight trigrams define the corners of a cube, each line or broken line denoting a 'positive' or 'negative' face. Their pairing into the 28 hexagrams defines the 28 straight lines joining pairs of its corners. The ordering of their pairing signifies the direction of the joining of pairs of corners by arrows. The eight diagonal hexagrams with 48 lines/broken lines denote the eight corners. Their 12 Yang lines denote the 12 positive faces and the 12 Yin lines denote the 12 negative faces needed to define the 8 corners.

8 diagonal hexagrams → 8 corners
28 upper, off-diagonal hexagrams → 28 directed lines →
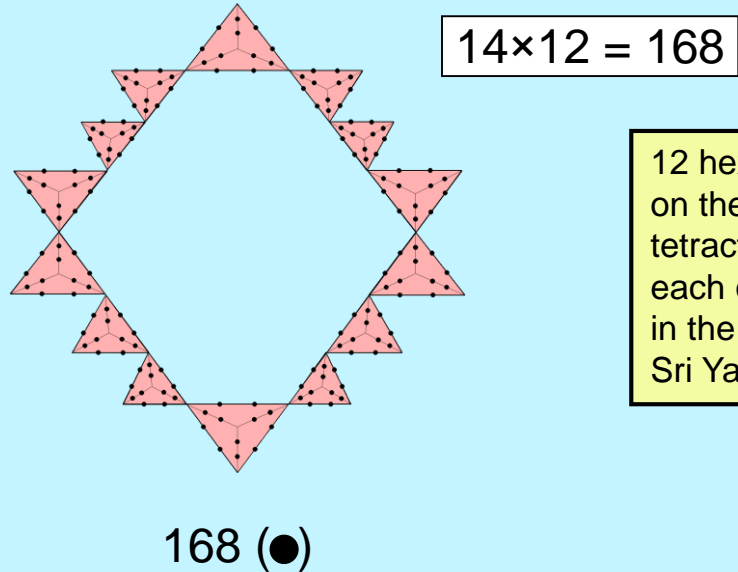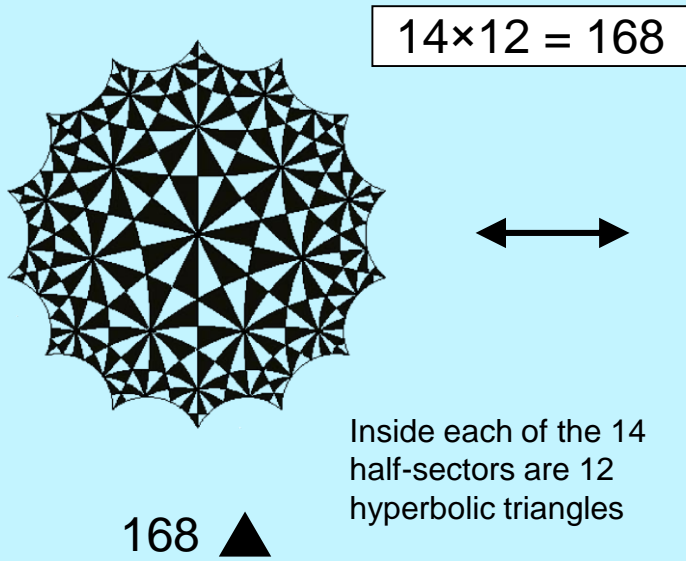28 lower, off-diagonal hexagrams → 28 oppositely directed lines ←

The ordered pairing of trigrams into hexagrams signifies the joining of the corners of a cube by arrows. The diagonal hexagrams correspond to the corners themselves.
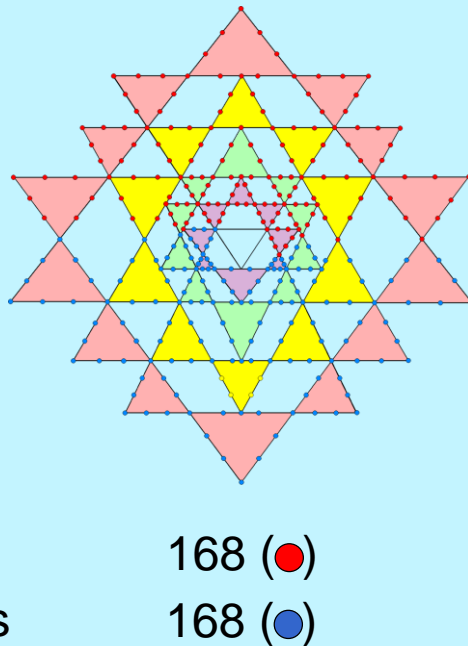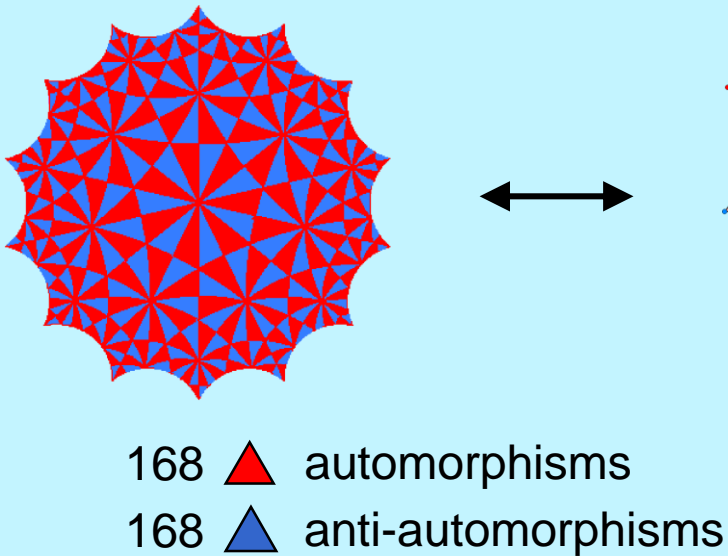
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Figure 11

In its {7,3} tiling, the Klein configuration (KC) is divided into seven identical sectors. A sector is composed of two half-sectors, each with 12 hyperbolic triangles, so that KC is divided into 14 sectors with 168 triangles. Compare this with the 14 triangles in the fourth layer of the Sri Yantra. When each triangle is constructed from three tetractyses, there are 12 hexagonal yods on their six sides. This means that (14×12=168) hexagonal yods line on the (14×6=84) sides of the (14×3=42) tetractyses comprising the 14 triangles. KC is isomorphic in this sense to the last layer of triangles of the Sri Yantra. To establish such isomorphism, however, we need to consider only each triangle as a tetractys, for 168 yods symbolizing the 168 automorphisms line the 63 sides of the 21 triangles in one half of the Sri Yantra and 168 yods symbolizing the 168 anti-automorphisms line the 63 sides of the 21 triangles in its other half. Unlike, however, in the case of the disdyakis triacontahedron, one half of the Sri Yantra is *not* the exact mirror image of its other half.

Figure 11

14×12 = 168

14×12 = 168

12 hexagonal yods lie on the 6 sides of the 3 tetractyses making up each of the 14 triangles in the 4th layer of the Sri Yantra.

Inside each of the 14 half-sectors are 12 hyperbolic triangles
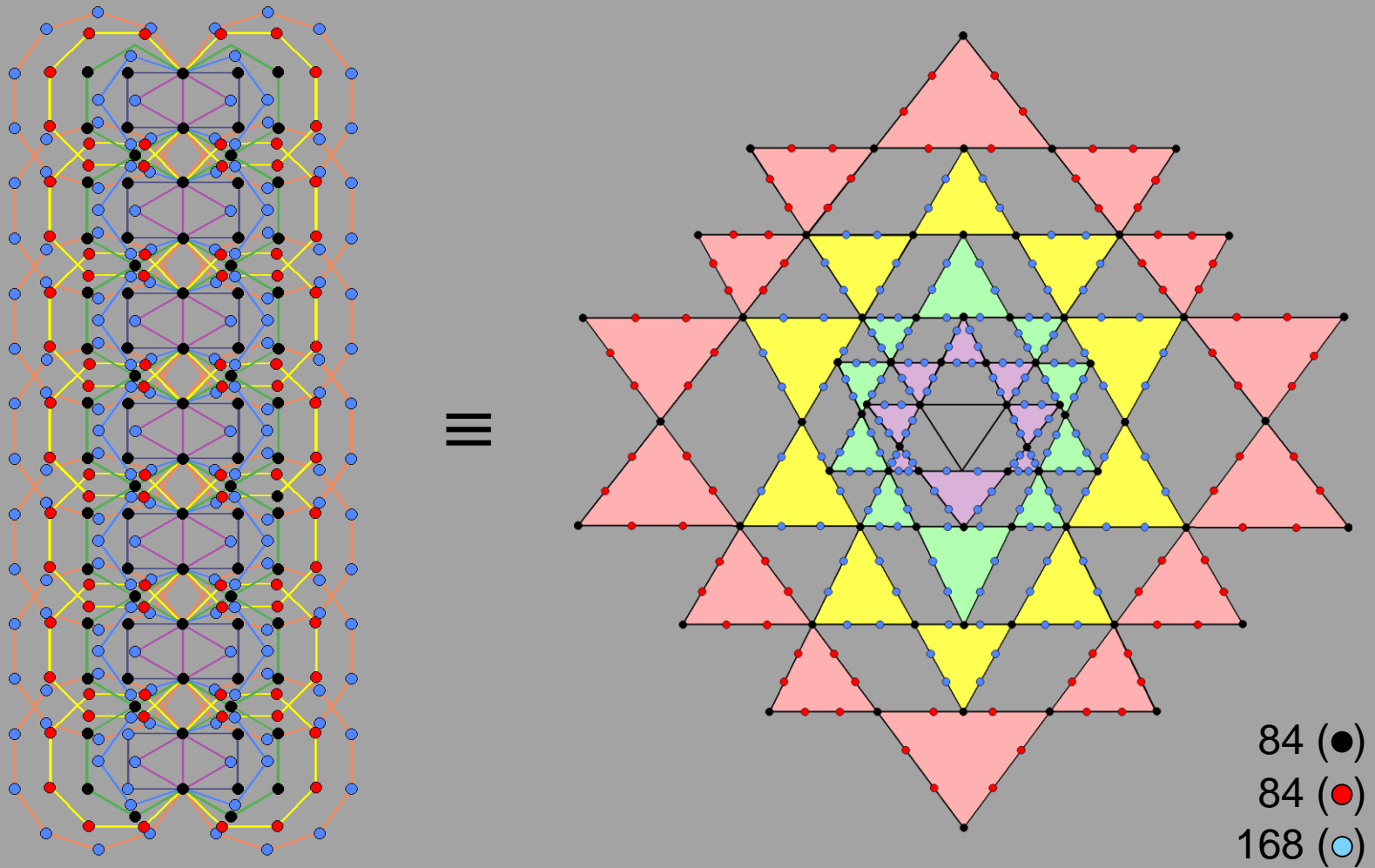
168 ▲

168 (●)

The isomorphism between the Klein configuration and the Sri Yantra. The higher order transformation of the 4th layer of triangles is equivalent to the lower-order transformation of *all* 4 layers.

168 ▲ automorphisms

168 ▲ anti-automorphisms

168 (●)

168 (●)

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Figure 12

It was shown in Part 2 (pp. 21, 22) that the Sri Yantra and the lowest seven overlapping Trees of Life are equivalent. Here is displayed the isomorphism between the Sri Yantra and the inner form of seven Trees of Life – or rather, the first six polygons enfolded in each tree, as these two sets of polygons constitute a Tree of Life pattern in themselves. There are 168 corners associated with the 42 polygons enfolded in seven overlapping Trees of Life on each side of its central pillar. They correspond to the 168 yods lying on sides of the 21 triangles in each half of the Sri Yantra. The 42 (black) vertices associated with each set of 21 triangles correspond to the 42 (black) corners associated with the squares and hexagons in each set of six polygons. The 42 (red) hexagonal yods on sides of each set of seven triangles in the fourth layer correspond to the 42 (red) corners of the octagons outside their shared root edge in each set of polygons. The 84 (blue) hexagonal yods on the 42 sides of the 14 triangles in each half of the first three layers correspond to the 84 (blue) corners of the triangle, pentagon & decagon in each set of polygons outside their root edges.

Figure 12

84 (●)
84 (●)
168 (○)

The Sri Yantra is isomorphic to the inner form of 7 Trees of Life.

Figure 13

Of the 384 yods in the tetractyses forming the lowest seven Trees of Life up to the top of the seventh tree, 48 (red) yods are in the lowest tree up to the level of Chesed, the first Sephirah of Construction. The counterpart of this in the Sri Yantra is that there are 48 (red) yods which are either centres of triangles or hexagonal yods on sides of the central triangle. The 336 (black) yods on the sides of the 42 triangles correspond to the 336 (black) yods in the lowest seven trees above Chesed of the lowest tree.

Figure 13

(a circle denotes a yod behind another one)

The 48 red yods that are either centres of tetractyses or hexagonal yods in the innermost triangle correspond to the 48 red yods up to Chesed of the lowest Tree of Life.
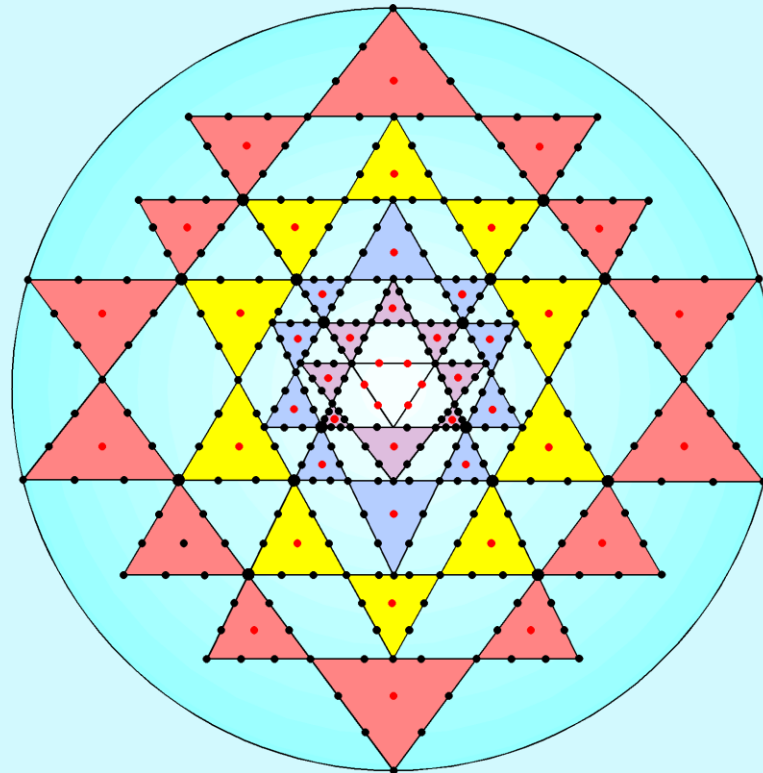
≡

Chesed of 1st tree

48 (●)
336 (●)
Total = 384

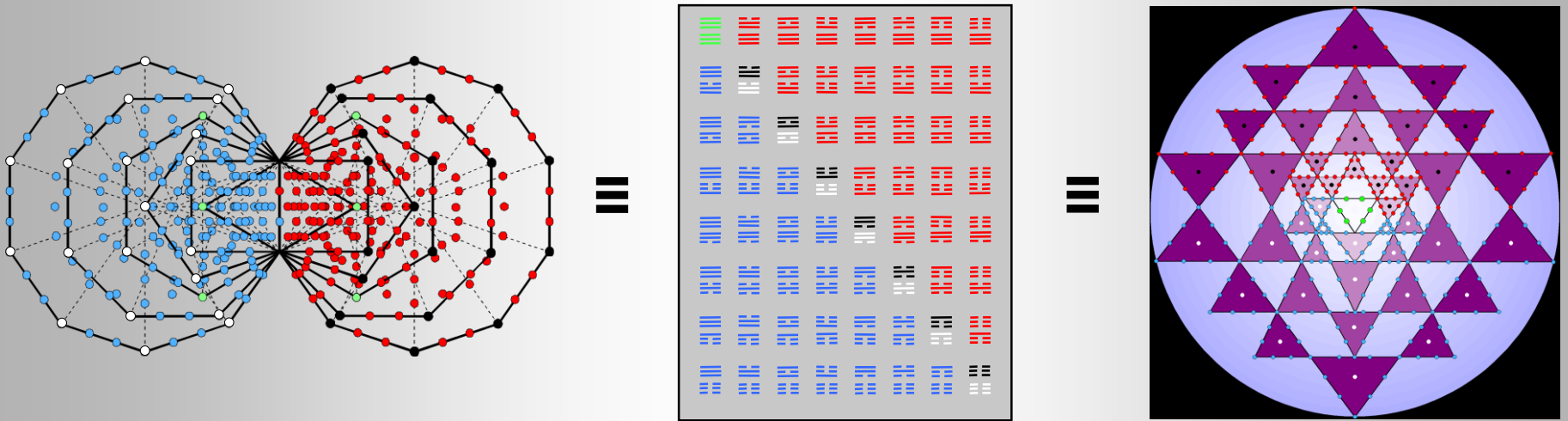**The Sri Yantra is isomorphic to the outer form of 7 Trees of Life.**

Figure 14

This displays the corresponding features of the inner Tree of Life, the I Ching table and the Sri Yantra as isomorphic representations of holistic systems.

1. The six green yods either at the external corners of the triangles or at the topmost and lowest corners of the hexagons coincide with Sephiroth. They correspond to the six lines of the pair of Heaven trigrams in the top left-hand corner of the table and to the six green hexagonal yods on the sides of the central triangle of the Sri Yantra.

2. The 21 black yods denoting corners of one set of the first six polygons that lie outside their shared edge correspond to the 21 black lines/broken lines in the upper trigrams of the seven remaining diagonal hexagrams and to the black centres of the 21 triangles in one half of the Sri Yantra. The 21 white yods denoting external corners of the other set of polygons correspond to the 21 white lines/broken lines in the lower trigrams of the seven hexagrams on the diagonal of the table and to the white centres of the 21 triangles in the other half of the Sri Yantra.

3. The 168 red yods in one set of six polygons that are not corners correspond to the 168 red lines/broken lines in the 28 off-diagonal hexagrams of half of the table and to the 168 red yods on the 63 sides of the 21 triangles in one half of the Sri Yantra. The 168 blue yods in the other set of six polygons that are not corners correspond to the 168 blue lines/broken lines in the 28 off-diagonal hexagrams of the other half of the table and to the 168 blue yods on the 63 sides of the 21 triangles in the other half of the Sri Yantra.

Figure 14

6 (●)

168 (○) 21 (●) 168 (●)

21 (○)

Total = 384

6 ─

168 ─ & ─ ─ 21 ─ & ─ ─ 168 ─ & ─ ─

21 ─ & ─ ─

Total = 384

6 (●)

168 (○) 21 (●) 168 (●)

21 (○)

Total = 384

**The isomorphism between the inner Tree
of Life, the I Ching & the Sri Yantra.**

Figure 15

192 yods are associated with each set of the first six enfolded polygons of the inner Tree of Life. They are intrinsic to that set because none of them is shared with the six polygons enfolded in the next higher Tree of Life, which also have 192 such yods. The 192 yods associated with each set of six polygons correspond to the 192 lines/broken lines in each half of the I Ching table.

Figure 15

Of the 193 yods associated with each set of 6 enfolded polygons, one yod (the topmost corner of the hexagon) is shared with the polygons enfolded in the next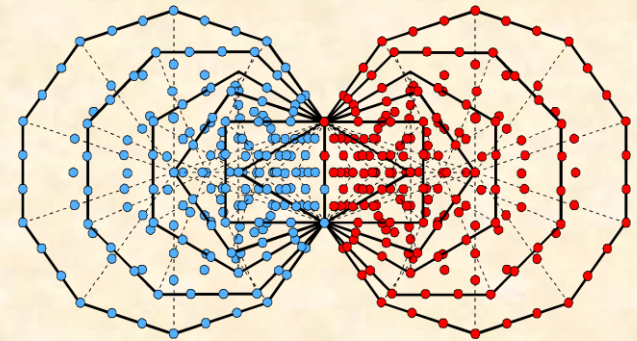 higher Tree of Life because it coincides with the lowest corner of the hexagon in this set. Therefore, 192 yods are intrinsic to each set of polygons. Successive sets of (6+6) polygons require 384 yods. The number 384 therefore characterizes a holistic system (in this case the two sets of 6 enfolded polygons).

2×384 =

384 =

192 (○)   192 (●)

384 yods are intrinsic to each inner Tree of Life.

31

Figure 16

Considered as tetractyses, the 21 triangles in one half of the Sri Yantra have 21 black centres and 21 white pairs of vertices, i.e., 21 triplets of one white yod and two black yods. They also have 21 purple triplets of hexagonal yods and 21 brown triplets of hexagonal yods, each pair of triplets forming a Star of David. The 189 yods associated with the 21 tetractyses in each half of the Sri Yantra comprise three sets of 21 triplets of yods, i.e., 63 triplets.

The 21 centres of tetractyses in each half of the Sri Yantra comprise 12 centres of the tetractyses in the third and fourth layers and nine centres of tetractyses in the first and second layers. The significance of this will be discussed on pages 41 & 42.

21:

Half of Sri Yantra

42 vertices + 21 centres

21 tetractyses have:
21 purple triplets
21 brown triplets

Half of Sri Yantra

189 yods $= 21 \quad 21 \times 2 \quad 21 \times 3 \quad 21 \times 3$

$= \quad 63 \quad + \quad 63 \quad + \quad 63$

$21 (\bullet) \rightarrow$ 12 centres of 3rd & 4th layers
9 centres of 1st & 2nd layers

The 189 yods in each half of the Sri Yantra comprise 63 triplets of yods.

Figure 17

Selection of successive notes of the Pythagorean scale as the starting note (tonic) generates seven possible musical scales. These sequences of intervals (T) and leimmas (L), the counterpart of the modern semitone, define the musical modes of the Roman Catholic Church. The four "authentic modes" are the Dorian mode (D scale), the Phrygian mode (E scale), the Lydian mode (F scale) & the Mixolydian mode (G scale). They have four "hypo" modes separated by the interval of a perfect fourth from their authentic counterparts – the Hypodorian (A scale), the Hypophrygian (B scale), the Hypolydian (C scale) & the Hypomixolydian (D scale). In terms of notes, the lattermost is identical to the Dorian, differing in the choice of the 'dominant' and 'reciting note.'

The table displays the 'tone ratios' of the notes in the seven musical scales. These are the frequencies or pitches of the notes relative to that of the tonic. The Pythagorean musical scale is the C scale (Hypolydian mode). Tone ratios of notes belonging to this scale are written in black; non-Pythagorean tone ratios are written in red. The seven scales have 26 Pythagorean notes between the tonic and the octave, showing how YAHWEH, the Godname of Chokmah with number value 26, prescribes the seven musical scales. Their notes have 14 different tone ratios, i.e., there are 12 distinct types of notes between the tonic and octave. The sequence of 14 notes is split into the first seven, primary notes and their seven "complements" (notes whose tone ratios are the interval between its corresponding primary note and the octave). They comprise the eight notes of the Pythagorean scale and six non-Pythagorean notes. The 12 types of notes between the tonic and octave comprise six Pythagorean notes and six non-Pythagorean notes.

Sequences of intervals in the 7 musical scales $T = 9/8,\quad L = 256/243$

7.	T T L T T T L	Hypolydian (mode 6, C scale)
6.	L T T L T T T	Hypophrygian (mode 4, B scale)
5.	T L T T L T T	Hypodorian (mode 2, A scale)
4.	T T L T T L T	Mixolydian (mode 7, G scale)
3.	T T T L T T L	Lydian (mode 5, F scale)
2.	L T T T L T T	Phrygian (mode 3, E scale)
1.	T L T T T L T	Dorian (mode 1, D scale)

Figure 17

Musical scale	Tone ratio						
D scale	1	9/8	32/27	4/3	3/2	27/16	16/9	2
E scale	1	256/243	32/27	4/3	3/2	128/81	16//9	2
F scale	1	9/8	81/64	729/512	3/2	27/16	243/128	2
G scale	1	9/8	81/64	4/3	3/2	27/16	16/9	2
A scale	1	9/8	32/27	4/3	3/2	128/81	16/9	2
B scale	1	256/243	32/27	4/3	1024/729	128/81	16/9	2
C scale	1	9/8	81/64	4/3	3/2	27/16	243/128	2

Notes   **Complements of notes**

C   D   E   F   G   A   B   C'
1 256/243 9/8 32/27 81/64 4/3 1024/729 | 729/512 3/2 128/81 27/16 16/9 243/128 2

Figure 18

There are 28 intervals between the eight notes of a musical scale. Excluding the octave, there are (7×27=189) intervals between the notes of the seven varieties of scales. They all have the values of the tone ratios of the 12 types of notes between the tonic and the octave found in the seven scales. The second table lists the number of intervals of each type. The number in the brackets next to each one is the number of times the *note* with that tone ratio appears in the scales. Some of the notes and intervals can be paired with their complements. Their numbers are listed in the third table, together with the numbers of intervals without complements. There are 21 notes and 42 intervals that have complements. They form two sets of 63 intervals, one set paired with its complementary set, creating 63 pairs. There are 63 unpaired intervals. The 189 intervals therefore divide into three sets of 63 intervals, that is, 63 triplets, each triplet comprising a pair of notes or intervals and an unpaired interval. They comprise 21 notes made of the second, third & fourth notes of the seven scales and 168 intervals made up of the 21 complements of these notes, 42 intervals and their 42 complements and 63 unpaired intervals.

Types of intervals in the 7 musical scales

1	2	1×2 = 2
256/243	243/128	256/243×243/128 = 2
9/8	16/9	9/8×16/9 = 2
32/27	27/16	32/27×27/16 = 2
81/64	128/81	81/64×128/81 = 2
4/3	3/2	4/3×3/2 = 2
1024/729	729/512	1024/729×729/512 = 2

Interval	Number	Interval	Number
256/243	14 (2)	243/128	4 (2)
9/8	35 (5)	16/9	10 (5)
32/27	24 (4)	27/16	12 (4)
81/64	18 (3)	128/81	9 (3)
4/3	30 (6)	3/2	24 (6)
1024/729	4 (1)	729/512	5 (1)
Total = 125 (21)		Total = 64 (21)	

Figure 18

Number of paired notes+intervals

Interval	Number	Interval	Number
256/243	2+2	243/128	2+2
9/8	5+5	16/9	5+5
32/27	4+8	27/16	4+8
81/64	3+6	128/81	3+6
4/3	6+18	3/2	6+18
1024/729	1+3	729/512	1+3
Total = 21+42=63		Total = 21+42=63	

Number of unpaired intervals

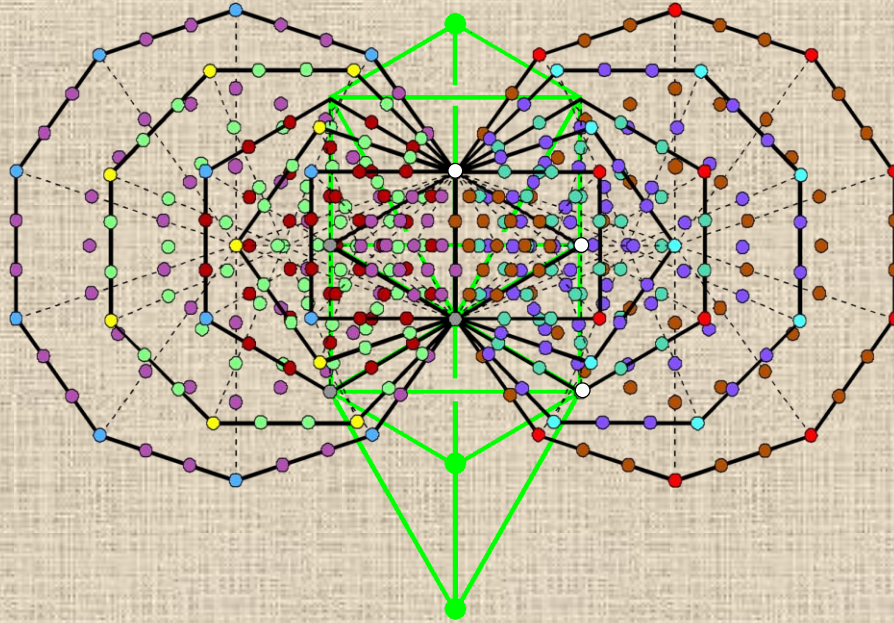| Interval	Number	Interval	Number
256/243	10	243/128	0
9/8	25	16/9	0
32/27	12	27/16	0
81/64	9	128/81	0
4/3	6	3/2	0
1024/729	0	729/512	1
Total = 62		Total = 1	

63

21   168

Figure 19

Between the notes of the seven scales, there are 189 rising intervals below the octave. They also have 189 falling intervals (this is the interval between the lower note and the higher note. It is the reciprocal of the corresponding rising interval). So there are 378 rising and falling intervals other than the octave. As the seven scales constitute a holistic system, their patterns of intervals are encoded in the inner form of the Tree of Life. The two sets of the first six enfolded polygons have 384 yods that are intrinsic to them. Six of them coincide with Sephiroth or Daath, leaving 378 yods that do not belong to the outer form of the Tree of Life. The 189 yods in one set of polygons denote rising intervals. Their mirror images in the other set denote falling intervals.

As evidence that this is not a coincidence, the yod populations of pairs of polygons correlate exactly with the numbers of different classes of intervals. The 21 corners of polygons that are outside their shared edge and which do not coincide with Sephiroth of the Tree of Life symbolize the 21 notes made up of the second, third & fourth notes of each scale. Their mirror images in the other set of polygons denote the complements of these notes (not necessarily in the same scale, of course). The 21 corners comprise 12 corners of the square, hexagon & decagon, and nine corners of the pentagon & octagon. The 42 yods in the square & hexagon symbolize the 42 intervals that are not notes. The 63 yods in the triangle & decagon denote the 63 complements of the notes and intervals. The 63 yods in the pentagon & octagon denote the 63 intervals lacking a complement.

This is the Tree of Life basis of the seven types of musical scales as a holistic system.

Figure 19

384 =

shared with outer Tree of Life	3 ⚪(grey)		shared with outer Tree of Life	3 ⚪(white)
corners	12 🔵 + 9 🟡		corners	12 🔴 + 9 🔵(cyan)
square+hexagon	42 🔴(dark red)		square+hexagon	42 🟢(teal)
triangle+decagon	63 🟣		triangle+decagon	63 🟠
pentagon+octagon	63 🟢		pentagon+octagon	63 🟣
192 = 3 + (21+42) + 63 + 63			192 = 3 + (21+42) + 63 + 63	

Tree of Life representation of the three sets of 63 rising & falling intervals between the notes of the seven musical scales.
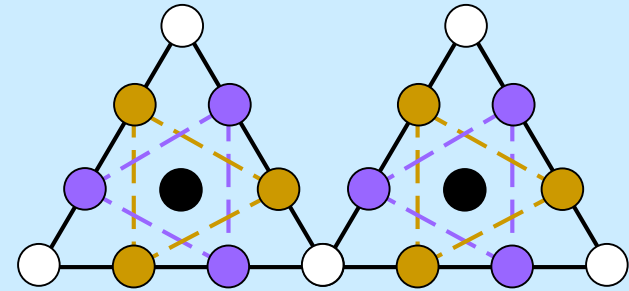
39

Figure 20

This correlation between the intervallic structure of the seven musical scales and the sacred geometry of the inner Tree of Life extends to the tetractys-transformed Sri Yantra as well. The 21 notes made up of the first three notes above the tonic in each scale determine the seven scales collectively because the 21 remaining notes below the octave are their complements and therefore are defined by them. They are symbolized as rising intervals by the yods at the centres of the 21 triangles/tetractyses in one half of the Sri Yantra, the centres of the 21 tetractyses in the other half denoting their corresponding 21 falling intervals (note: *not* their complements) . The tips of the 21 triangles denote the 21 complements of the notes. The tips of the 21 triangles in the other half of the Sri Yantra denote their falling interval counterparts. The 42 intervals and their complements are symbolized by the 42 hexagonal yods on each of the two sides of each triangle forming its tip. Their corresponding falling intervals are symbolized by the counterparts of these yods in the other half of the Sri Yantra. The 63 intervals without complements are symbolized by the 63 yods on the bases of the 21 tetractyses in one half, their falling interval counterparts being denoted by the 63 yods on the bases of the 21 tetractyses in the other half of the Sri Yantra.

The centres of the 21 tetractyses in either half of the Sri Yantra comprise the centres of the 12 tetractyses in the third and fourth layers and the centres of the nine tetractyses in the first and second layers. This 12:9 differentiation is the counterpart of the distinction between the 12 notes above the tonic made up of the second, third & fourth notes in the four authentic modes and the nine notes above the tonic made up of the second, third & fourth notes in the three plagal modes (the fourth, plagal mode is not a distinct octave species, the Hypomixolydian mode having the same sequence of notes as the Dorian). The uppermost first and second layers of the Sri Yantra therefore correlate with the three plagal modes that have different sets of notes, whilst its third and fourth layers correlate with the four authentic modes.

C scale: C   D   E   F   G   A   B   C'

                         3 notes   3 complements

7 scales: (7×3=21) notes   (7×3=21) complements

21 notes → 21 centres (●) of 21 tetractyses

21 complements of notes → 21 tips (○) of tetractyses

Figure 20

42 intervals + 42 complements → (42+42) hexagonal yods on
                                        sides of 21 tetractyses

63 unpaired intervals → 63 yods on bases of 21 tetractyses

4 Authentic modes have (4×3=12) notes   3 Plagal modes have (3×3=9) notes

21 (●)
    → 12 centres of triangles in 3rd & 4th layers
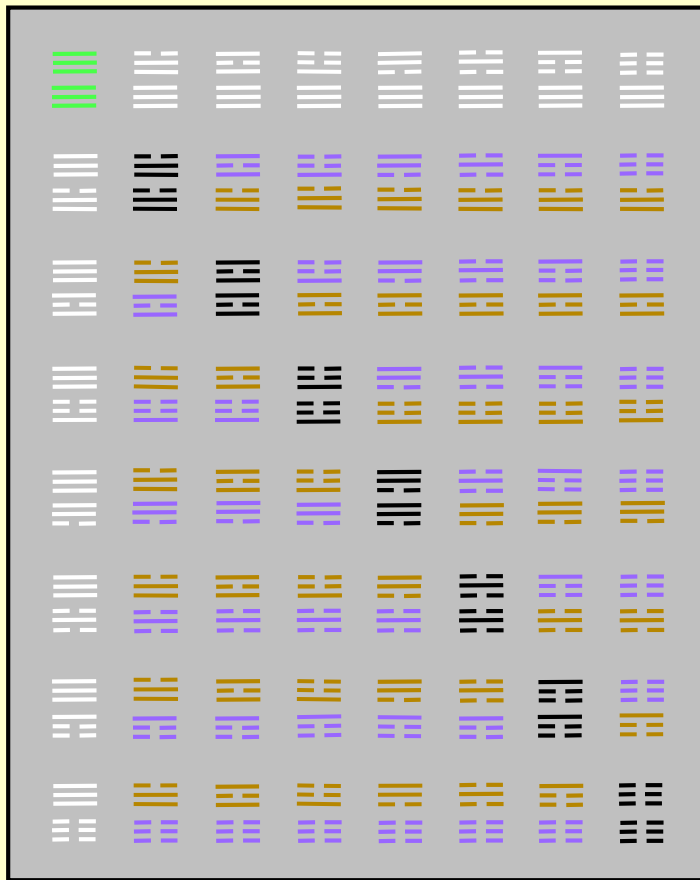   → 9 centres of triangles in 1st & 2nd layers

Therefore,

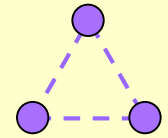   1st & 2nd layers of triangles → Plagal musical modes

    3rd & 4th layers of triangles → Authentic musical modes
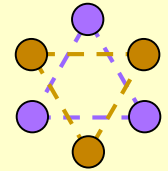
41

Figure 21

Arranged in a Star of David in each tetractys of the Sri Yantra, the two triads of hexagonal yods correspond to the two trigrams in each hexagram of the I Ching table. The six green, unbroken lines of the two Heaven trigrams at the upper left-hand corner of the table correspond to the six hexagonal yods arranged as two triangles in a Star of David in the central triangle (see Fig. 14). The (21+21) black lines/broken lines of the seven remaining hexagrams in the diagonal correspond to the centres of the two sets of 21 tetractyses. The 84 white lines/broken lines of the 14 hexagrams containing the Heaven trigram correspond to the 84 corners of the 42 tetractyses. Each category 'marks out', so to speak, the boundary of their respective systems. The 21 purple trigrams on one side of the diagonal correspond to the 21 triplets of purple, hexagonal yods in one half of the Sri Yantra. The 21 purple trigrams on the other side of the diagonal correspond to the 21 triplets of purple, hexagonal yods in the other half of the Sri Yantra. Similarly for the brown, hexagonal yods. The I Ching table is a non-geometrical version of the Sri Yantra.

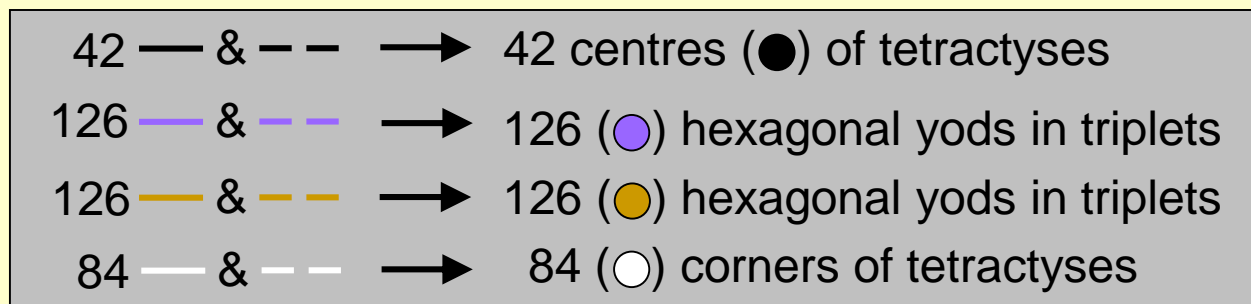Trigrams → triplets of hexagonal yods

Hexagrams → pairs of triplets of hexagonal yods

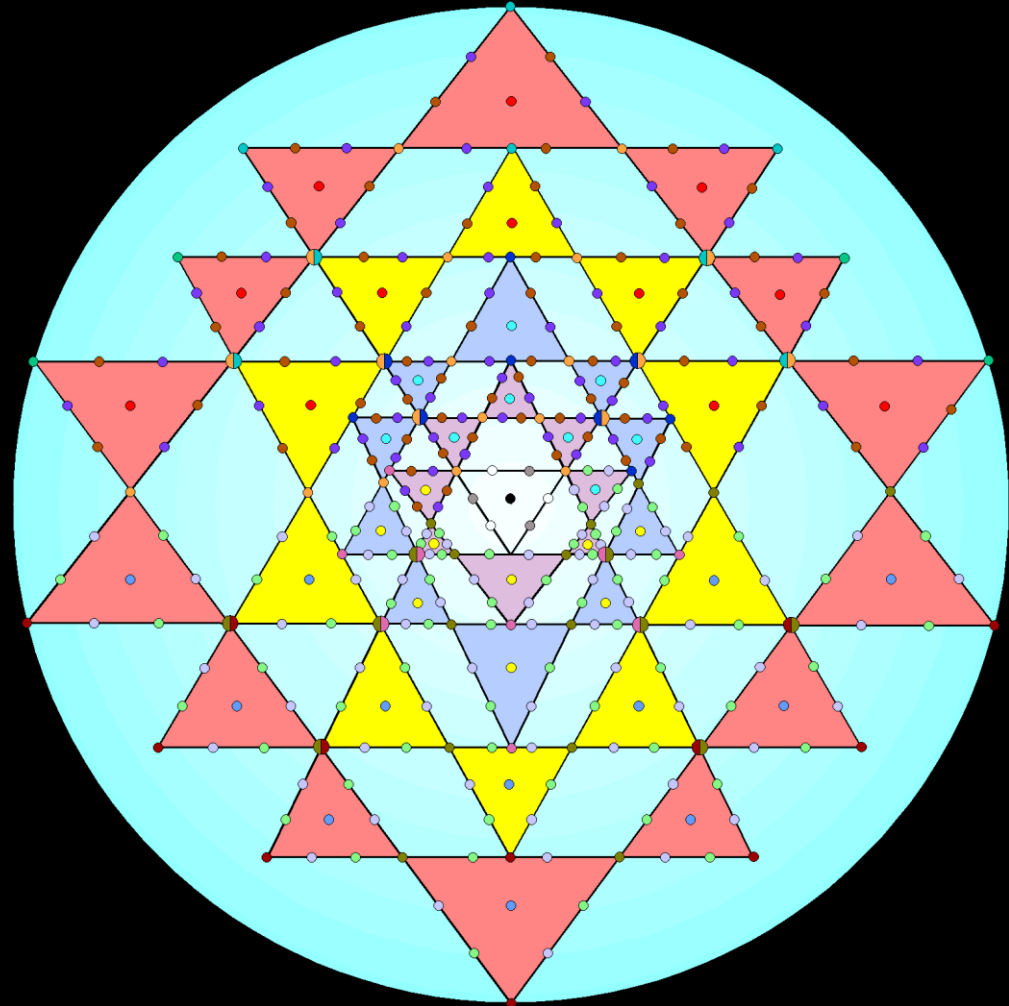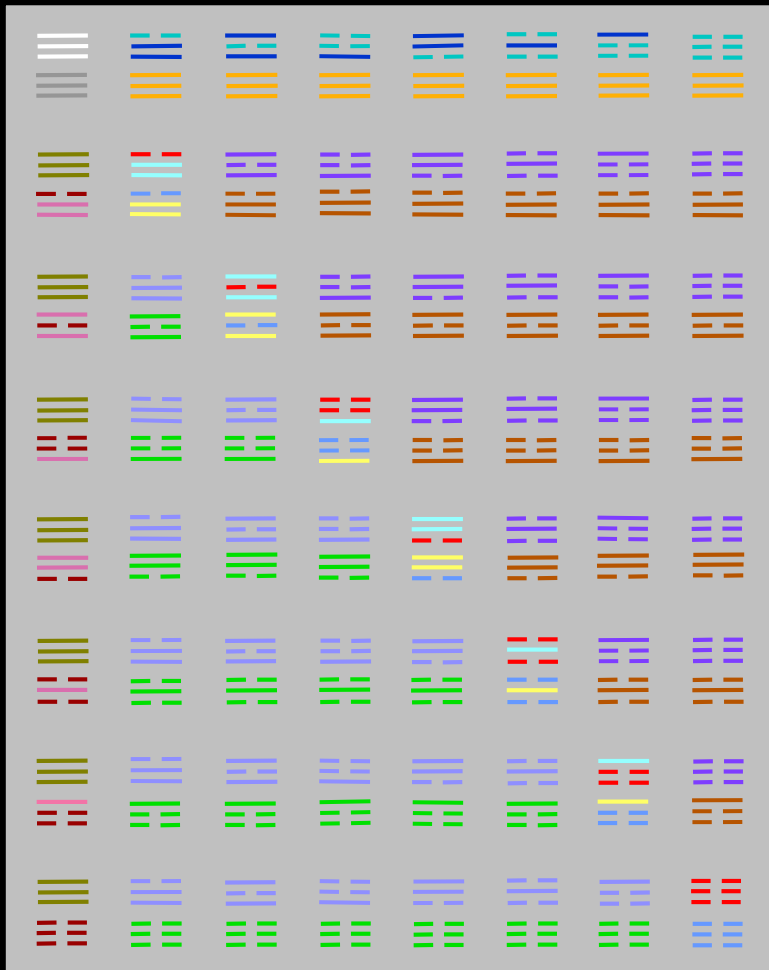**Correspondence of the Yin/Yang lines and the yods in the Sri Yantra**

Figure 21

42 —— & – – ⟶ 42 centres (●) of tetractyses

126 —— & – – ⟶ 126 (●) hexagonal yods in triplets

126 —— & – – ⟶ 126 (●) hexagonal yods in triplets

84 —— & – – ⟶ 84 (○) corners of tetractyses

Figure 22

Displayed here is the detailed correspondence between different types of trigrams in the I Ching table and the triplets of hexagonal yods in the tetractys-transformed triangles of the Sri Yantra. Lines/broken lines and their counterpart yods have the same colour.
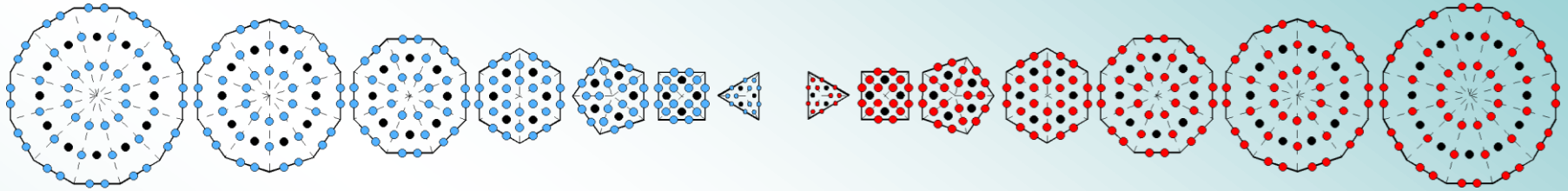
Figure 22

Correspondence between the I Ching table and the Sri Yantra.

Figure 23

The seven separate, regular polygons constituting the inner form of the Tree of Life comprise 295 yods when their 48 sectors are turned into tetractyses. In other words, starting with the seven polygons divided into their sectors, (295-48-7=240) more yods are needed to transform their sectors into tetractyses. There are 48 centres of tetractyses. This leaves (240-48=192) hexagonal yods arranged at the corners of six-sided polygons (i.e., at the tips of Stars of David). 192 yods are similarly arranged in the other set of polygons. In terms of the formal equivalence between the Tree of Life and the tetractys, the six yods at the corners of a six-sided polygon (it does not need to be a hexagon, of course) symbolize the six Sephiroth of Construction above Malkuth. These two sets of 192 hexagonal yods arranged at the corners of triangles are the Tree of Life counterpart of the 192 lines/broken lines grouped in trigrams in each half of the I Ching table. The 24 red lines/broken lines in the upper trigrams of the eight diagonal hexagrams correspond to the 24 red, hexagonal yods in one of the hexagons. The 24 blue lines/broken lines in the lower trigrams of these eight hexagrams correspond to the 24 blue, hexagonal yods in the hexagon belonging to the other set of polygons. As 24 = 4×3×2 and as each sector of a polygon contributes four of these hexagonal yods, the counterpart of the two sets of four diagonal trigrams are the four hexagonal yods per sector multiplied, firstly, twice (a hexagon is two equilateral triangles rotated by 60°) and then thrice (pairs of hexagonal yods are arranged at each corner of a triangle). The trigrams are expressing the three-fold, rotational symmetry of an equilateral triangle – a sector of a hexagon. The eight-fold nature of trigrams corresponds to the fact that there are eight hexagonal yods on the sides of a pair of adjacent sectors in the hexagon. These are basic because the rotation of each one by 120° about the centre of the hexagon generates all its remaining yods.
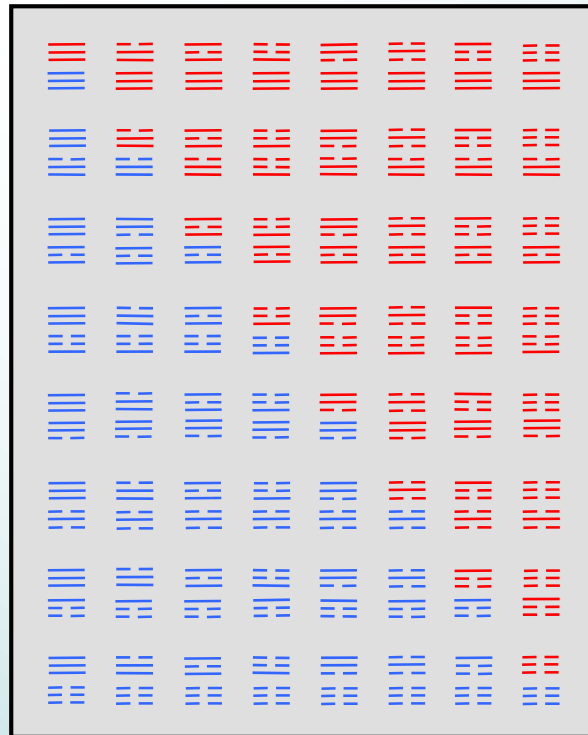
46

Figure 23

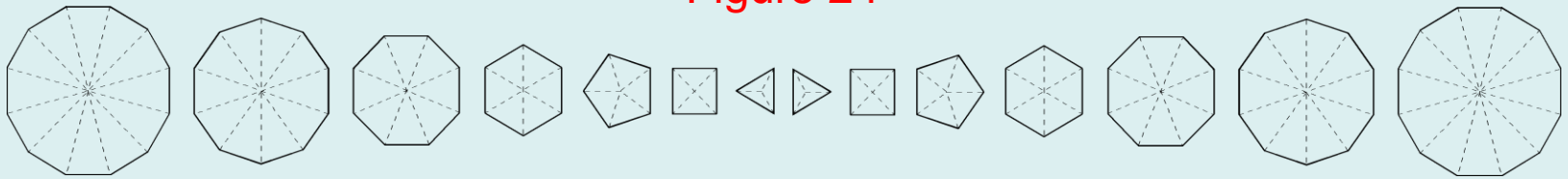192 (●) 192 (●)

192 — & ‑ ‑ 192 — & ‑ ‑

Correspondence between the I Ching table and the (7+7) separate polygons

47

Figure 24

Divided into their sectors, the seven separate polygons have 48 corners, 96 sides and 48 triangles surrounding their centres, i.e., 192 geometrical elements. They correspond to the 192 lines/broken lines in one diagonal half of the I Ching table. The 192 geometrical elements in the other set of seven polygons of the inner Tree of Life correspond to the 192 lines/broken lines in the other half of the table. The 12 lines and 12 broken lines in the upper trigrams of the eight diagonal hexagrams correspond to the 12 sides and 12 corners/triangles of the sectors of the hexagon. Similarly, the 12 lines and 12 broken lines in the lower trigrams correspond to the 12 sides and 12 corners/triangles of the sectors of the hexagon in the other set of polygons. The 384 lines and broken lines in the table express the number of geometrical elements needed to construct the two sets of seven polygons making up the inner form of the Tree of Life. They symbolize the independent 'bits of information' needed to build a holistic system.
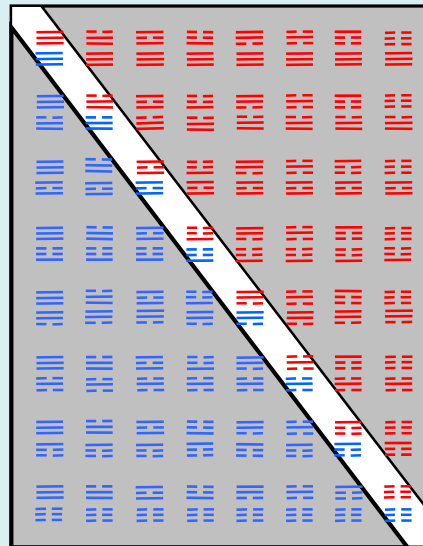
Figure 24

Number of corners =   48
Number of sides =   96
Number of triangles =   48
Total = 192

Number of corners =   48
Number of sides =   96
Number of triangles =   48
Total = 192

96 —
96 --
Total = 192

12 lines & 12 broken lines of lower 8 trigrams in diagonal

⟶ 12 sides & 12 corners/triangles of hexagon

96 —
96 --
Total = 192

12 lines & 12 broken lines of upper 8 trigrams in diagonal

⟶ 12 sides & 12 corners/triangles of hexagon

The 384 lines & broken lines of the I Ching table denote the 384 geometrical elements composing the 14 regular polygons of the inner Tree of Life

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