The 7 Layers of Vertices in the Disdyakis Triacontahedron Encode the 206 Bones of the Human Skeleton, the Superstring Symmetry Groups E₈ & E₈×E₈ and the Superstring Structural Parameters 168, 336, 840 & 1680

by

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Abstract

Earlier articles gathered evidence that the disdyakis triacontahedron is the polyhedral counterpart of the inner form of the Kabbalistic Tree of Life. The 60 vertices between two diametrically opposite A vertices are arranged in seven layers as two 8-sided polygons, two 10-sided polygons, two 6-sided polygons and one 12-sided polygon. Some vertices form 15 Golden Rectangles. The regular counterparts of these four types of polygons in the inner Tree of Life are shown to encode the 126 paired bones of the appendicular skeleton and the 34 single bones and 46 paired bones of the human axial skeleton. These enfolded polygons are defined by 137 geometrical elements, this being the integer defining the fine structure constant known to physicists. The seven polygons formed by vertices of the disdyakis triacontahedron have 240 geometrical elements surrounding their centres. This corresponds to the 240 hexagonal yods surrounding the centres of the seven separate polygons of the inner Tree of Life. They represent, respectively, the polyhedral and polygonal encoding of the 240 roots of the superstring gauge symmetry group E₈ and seven of its eight simple roots, the eighth one corresponding to the apex of the disdyakis triacontahedron directly above the seven centres. The first two polygons have 72 elements and the last five have 168 elements, correlating with the 72 roots of E₆ and the 168 other roots of E₈. The four enfolded polygons that correspond to the four types of polygons and encode the human skeleton have 168 yods (the superstring structural parameter) outside their root edge on the edges of their tetractys sectors. The 168:168 division of geometrical elements belonging to the first three and last three polygons corresponds to the same division of yods outside the root edge in the inner Tree of Life that are not vertices of polygons. The seven layers of polygons display the same geometrical pattern as that shown by the I Ching table, demonstrating the holistic character of the disdyakis triacontahedron. The pattern of geometrical elements in its faces and interior conforms to the pattern of numbers in Plato's Lambda Tetractys, whose ratios define the intervals of the Pythagorean musical scale. This is exhibited also by the seven polygons, which encode geometrical information embodied in the polyhedron.

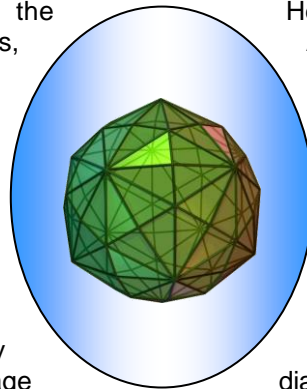Table 1. The number values of the Sephiroth in the four Worlds.

	SEPHIRAH	GODNAME	ARCHANGEL	ORDER OF ANGELS	MUNDANE CHAKRA
1	Kether (Crown) **620**	EHYEH (I am) **21**	Metatron (Angel of the Presence) **314**	Chaioth ha Qadesh (Holy Living Creatures) **833**	Rashith ha Gilgalim First Swirlings. (Primum Mobile) **636**
2	Chokmah (Wisdom) **73**	YAHVEH, YAH (The Lord) **26, 15**	Raziel (Herald of the Deity) **248**	Auphanim (Wheels) **187**	Masloth (The Sphere of the Zodiac) **140**
3	Binah (Understanding) **67**	ELOHIM (God in multiplicity) **50**	Tzaphkiel (Contemplation of God) **311**	Aralim (Thrones) **282**	Shabathai Rest. (Saturn) **317**
	Daath (Knowledge) **474**				
4	Chesed (Mercy) **72**	EL (God) **31**	Tzadkiel (Benevolence of God) **62**	Chasmalim (Shining Ones) **428**	Tzadekh Righteousness. (Jupiter) **194**
5	Geburah (Severity) **216**	ELOHA (The Almighty) **36**	Samael (Severity of God) **131**	Seraphim (Fiery Serpents) **630**	Madim Vehement Strength. (Mars) **95**
6	Tiphareth (Beauty) **1081**	YAHVEH ELOHIM (God the Creator) **76**	Michael (Like unto God) **101**	Malachim (Kings) **140**	Shemesh The Solar Light. (Sun) **640**
7	Netzach (Victory) **148**	YAHVEH SABAOTH (Lord of Hosts) **129**	Haniel (Grace of God) **97**	Tarshishim or Elohim **1260**	Nogah Glittering Splendour. (Venus) **64**
8	Hod (Glory) **15**	ELOHIM SABAOTH (God of Hosts) **153**	Raphael (Divine Physician) **311**	Beni Elohim (Sons of God) **112**	Kokab The Stellar Light. (Mercury) **48**
9	Yesod (Foundation) **80**	SHADDAI EL CHAI (Almighty Living God) **49, 363**	Gabriel (Strong Man of God) **246**	Cherubim (The Strong) **272**	Levanah The Lunar Flame. (Moon) **87**
10	Malkuth (Kingdom) **496**	ADONAI MELEKH (The Lord and King) **65, 155**	Sandalphon (Manifest Messiah) **280**	Ashim (Souls of Fire) **351**	Cholem Yesodoth The Breaker of the Foundations. The Elements. (Earth) **168**

The Sephiroth exist in the four Worlds of Atziluth, Beriah, Yetzirah and Assiyah. Corresponding to them are the Godnames, Archangels, Order of Angels and Mundane Chakras (their physical manifestation). This table gives their number values obtained by the ancient practice of gematria, wherein a number is assigned to each letter of the alphabet, thereby giving a number value to a word that is the sum of the numbers of its letters.

1. The seven layers of vertices in the disdyakis triacontahedron

Articles 22-30 provided evidence that the disdyakis triacontahedron (Fig. 1) — the Catalan solid with the most faces — is the polyhedral version of the polygonal, inner Tree of Life. It has **62** vertices, 180 edges and 120 triangular faces, that is, 362 geometrical elements in its faces. Therefore, 360 elements surround the central axis joining any two diametrically opposite vertices. Because it *is* the polyhedral counterpart of the inner form of the Tree of Life, it constitutes sacred geometry *par excellence*. This means that the geometrical (indeed, *all* mathematical) properties of this polyhedron are prescribed by the gematraic number values of the Hebrew names of the ten Sephiroth of the Tree of Life, their Hebrew Godnames, Archangelic Names, Orders of Angels and Mundane Chakras. Table 1 lists the English versions of these names (their literal English translations appear within brackets) and the number values of all the Hebrew names. It indicates the remarkable fact that ELOHA,* the Godname of Geburah with a number value of **36**, prescribes the shape of the polyhedron because the number of geometrical elements surrounding its axis is 360, which is the number of yods in **36** tetractyses. The Godname EL ("God") of Chesed with number value **31** also prescribes it because every one of **31** vertices in one-half of the polyhedron has its mirror image diametrically opposite it. Remarkably, the gematraic number value of *Tzadkiel*, the Archangel of Chesed, is **62** — the number of vertices in the disdyakis triacontahedron. Other ways in which the Godnames, etc define its properties were discussed in previous articles. This article will explain how they characterise the layering of vertices.

Figure 1. The disdyakis triacontahedron

The **62** vertices are of three types: 12 B vertices of an icosahedron, 20 C vertices of a dodecahedron (these form the 32 vertices of a rhombic triacontahedron (Fig. 2)) and 20 A vertices created by raising the centres of the Golden Rhombic faces of the latter polyhedron. Table 2 displays the x-, y- & z-coordinates
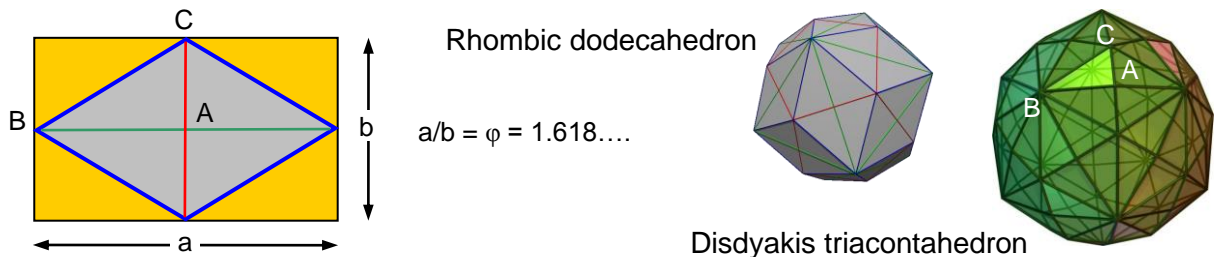

Rhombic dodecahedron

$a/b = \varphi = 1.618\ldots$

Disdyakis triacontahedron

Figure 2. The ratio of the longer diagonal (a) to the smaller diagonal (b) of the Golden Rhombus is φ, the Golden Ratio. It is enclosed by the Golden Rectangle.

of the **62** vertices of the disdyakis triacontahedron. All coordinate values belong to the set $(0, \pm\varphi, \pm\varphi^2, \pm\varphi^3, \pm2\varphi^2)$, where $\varphi = 1.6180\ldots$ is the Golden Ratio. The topmost vertex A is at a height $z = 2\varphi^2$ above the XY plane. Eight vertices are at a height φ^3, 10 vertices are at a height φ^2 and six vertices are at a height φ. Counterparts of these three layers of vertices lie below the XY plane, which contains 12 vertices. The vertices of the polyhedron are arranged in seven layers. In Fig. 2, each layer is assigned one of the seven colours in white light. Vertex 1 is the apex of the polyhedron and vertex **62** is its nadir. The dark line divides the first **31** vertices from their mirror images. Vertices are ordered so that the nth vertex at (a,b,c) is the mirror image of the (63–n)th vertex at (–a,–b,–c).

Whether viewed along the X-, Y- or Z-axes, the polyhedron has seven layers of vertices between the A vertices at its apex and nadir. Counting from the top, the numbers of vertices in the layers are:

1. 8 (4A, 2B & 2C);
2. 10 (4A, 2B & 4C);
3. 6 (4A, 2C);
4. 12 (4A, 4B & 4C);
5. 6 (4A, 2C);
6. 10 (4A, 2B & 4C)
7. 8 (4A, 2B & 2C).

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* Godnames will be written in upper case and Archangelic Names, Angelic Names & Mundane Chakras will be italicized.

| Vertex	Type	X	Y	Z
1	A	0	0	$2\varphi^2$
2	B	φ^2	0	φ^3
3	A	φ	φ^2	φ^3
4	C	0	φ	φ^3
5	A	$-\varphi$	φ^2	φ^3
6	B	$-\varphi^2$	0	φ^3
7	A	$-\varphi$	$-\varphi^2$	φ^3
8	C	0	$-\varphi$	φ^3
9	A	φ	$-\varphi^2$	φ^3
10	A	φ^3	φ	φ^2
11	C	φ^2	φ^2	φ^2
12	B	0	φ^3	φ^2
13	C	$-\varphi^2$	φ^2	φ^2
14	A	$-\varphi^3$	φ	φ^2
15	A	$-\varphi^3$	$-\varphi$	φ^2
16	C	$-\varphi^2$	$-\varphi^2$	φ^2
17	B	0	$-\varphi^3$	φ^2
18	C	φ^2	$-\varphi^2$	φ^2
19	A	φ^3	$-\varphi$	φ^2
20	C	φ^3	0	φ
21	A	φ^2	φ^3	φ
22	A	$-\varphi^2$	φ^3	φ
23	C	$-\varphi^3$	0	φ
24	A	$-\varphi^2$	$-\varphi^3$	φ
25	A	φ^2	$-\varphi^3$	φ
26	A	$2\varphi^2$	0	0
27	B	φ^3	φ^2	0
28	C	φ	φ^3	0
29	A	0	$2\varphi^2$	0
30	C	$-\varphi$	φ^3	0
31	B	$-\varphi^3$	φ^2	0
32	B	φ^3	$-\varphi^2$	0
33	C	φ	$-\varphi^3$	0
34	A	0	$-2\varphi^2$	0
35	C	$-\varphi$	$-\varphi^3$	0
36	B	$-\varphi^3$	$-\varphi^2$	0
37	A	$-2\varphi^2$	0	0
38	A	$-\varphi^2$	φ^3	$-\varphi$
39	A	φ^2	φ^3	$-\varphi$
40	C	φ^3	0	$-\varphi$
41	A	φ^2	$-\varphi^3$	$-\varphi$
42	A	$-\varphi^2$	$-\varphi^3$	$-\varphi$
43	C	$-\varphi^3$	0	$-\varphi$
44	A	$-\varphi^3$	φ	$-\varphi^2$
45	C	$-\varphi^2$	φ^2	$-\varphi^2$
46	B	0	φ^3	$-\varphi^2$
47	C	φ^2	φ^2	$-\varphi^2$
48	A	φ^3	φ	$-\varphi^2$
49	A	φ^3	$-\varphi$	$-\varphi^2$
50	C	φ^2	$-\varphi^2$	$-\varphi^2$
51	B	0	$-\varphi^3$	$-\varphi^2$
52	C	$-\varphi^2$	$-\varphi^2$	$-\varphi^2$
53	A	$-\varphi^3$	$-\varphi$	$-\varphi^2$
54	A	$-\varphi$	φ^2	$-\varphi^3$
55	C	0	φ	$-\varphi^3$
56	A	φ	φ^2	$-\varphi^3$
57	B	φ^2	0	$-\varphi^3$
58	A	φ	$-\varphi^2$	$-\varphi^3$
59	C	0	$-\varphi$	$-\varphi^3$
60	A	$-\varphi$	$-\varphi^2$	$-\varphi^3$
61	B	$-\varphi^2$	0	$-\varphi^3$
62	A	0	0	$-2\varphi^2$

8

10

6

12

6

10

8
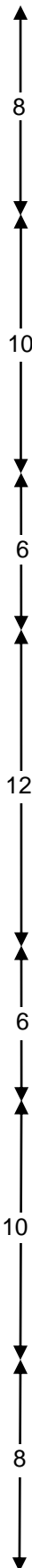
Table 2

The coordinates of the **62** vertices of the disdyakis triacontahedron arranged according to their height (z coordinate). Vertices in a section with the same colour have the same height. The number of vertices within each such plane is indicated within an arrow. Vertex 1 is the apex of the polyhedron and vertex **62** is its nadir. The 60 vertices between them are arranged in seven sheets. The pattern of **31** vertices below the thick line is the mirror image of the pattern of **31** vertices above it.
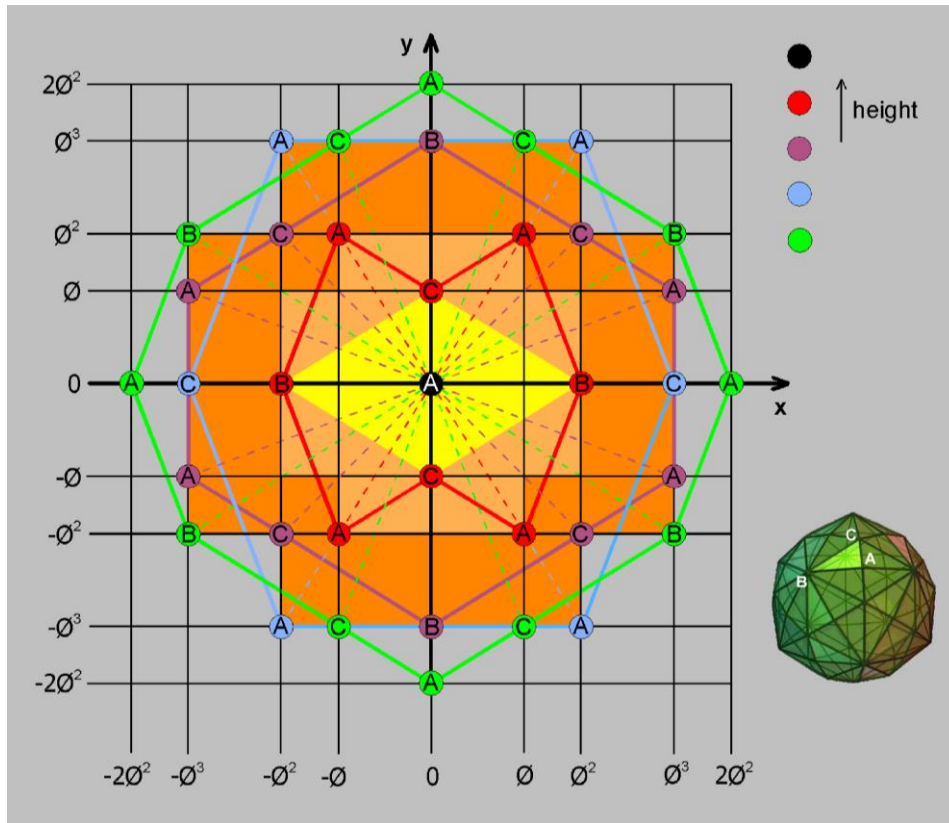
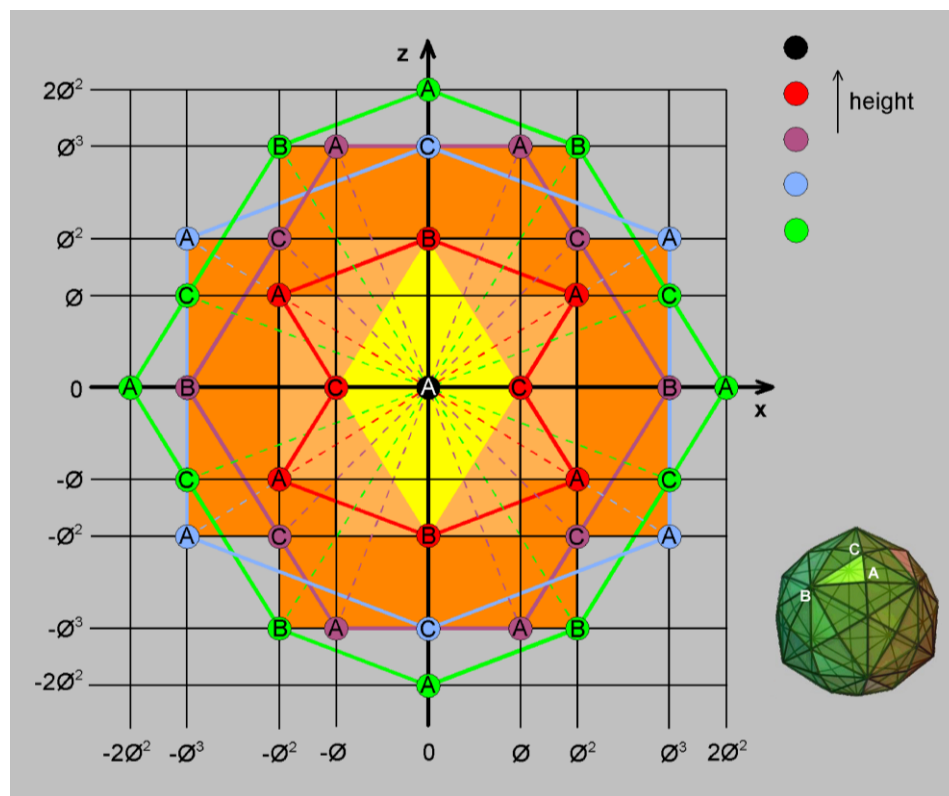Figure 3. Projection of the vertices of the disdyakis triacontahedron onto the XY plane.

Figure 4. Projection of the vertices of the disdyakis triacontahedron onto the XZ plane.

5

The four uppermost layers have **36** vertices made up of 16 A vertices, eight B vertices and 12 C vertices. The three lowest layers have 24 vertices comprising 12 A vertices, four B vertices and eight C vertices. Figure 3 shows the projection of the uppermost four layers onto the XY plane, looking downwards along the Z-axis. The apex is shown as a black circle. The uppermost layer of eight vertices is in red, the next layer of 10 vertices is in violet, the third layer of six vertices is in blue and the fourth layer of vertices lying in the XY plane is in green. The black gridlines mark distances $\pm\varphi$, $\pm\varphi$, $\pm\varphi^2$ & $\pm2\varphi^2$. Four A vertices in the first layer are at the corners of a Golden Rectangle, as are four A vertices in the third layer, although the size of the rectangle is different. Similarly, four B vertices in the fourth layer lie at the corners of the same sized Golden Rectangle rotated by 90°. No four vertices in the second layer form a Golden Rectangle. The seven layers therefore contain five Golden Rectangles. Because of the mirror symmetry of the disdyakis triacontahedron, there is a vertex below every one of those shown in Fig. 3 that belongs to one of the three layers beneath the XY plane.

Figure 4 displays the projection of the vertices onto the XZ plane as viewed in the positive direction of the Y-axis. The distribution of vertices is the same as that seen along the Z-axis after rotating the disdyakis triacontahedron through 90°. The projection of vertices onto the YZ plane as viewed along the positive X-axis is identical to the projection onto the XY plane shown in Fig. 3. Hence, (3×5=**15**) Golden Rectangles are formed by vertices in the three perpendicular sets of seven layers totalling **21** layers. This shows how the Godname EHYEH of Kether with number value **21** and the Godname YAH with number value **15** prescribe the orthogonal layering of vertices in the disdyakis triacontahedron.

2. Encoding of the human skeleton in the seven layers

According to Kabbalah, Adam Kadmon, or Heavenly Man, is the universal prototype for all holistic systems, whether a subatomic particle or a human being. Genesis 1, verse 27, states: "So God created man in his own image, in the image of God created he him; male and female created he them." This 'image' is the Tree of Life. It is the cosmic blueprint governing matter, man and the universe. As the representation of God, it manifests in the four Worlds of Atziluth (Archetypal World), Beriah (Creative World), Yetzirah (Psychological World) and Assiyah (Physical World). Godnames, Archangels, Angels and Mundane Chakras are the Sephiroth of the four overlapping Trees of Life mapping these worlds, which, of course, are not literally places but functional divisions of the Divine Life. A human being at the Assiyatic level is the body composed of bones, flesh and vital organs. The Tiphareth of the body viewed as the lowest of the four Trees of Life is the central nervous system, a highly complex organisation of nerves that allow two-way communication between the brain (representing Kether) and all the parts of the body. Its Malkuth is the skeleton, which in the adult is composed of 206 bones. They form two groups: the **80** bones of the axial skeleton (bones of the head, vertebral column, ribs and breastbone or sternum) and the 126 bones of the appendicular skeleton (the upper and lower limbs and their attachments, or girdles) (Fig. 5).

Figure 5. The axial skeleton (purple) and the appendicular skeleton (mauve).

Some bones occur singly, and others exist in pairs, with one located in the left side of the body and the other belonging to its right side. Listed below are the numbers of single bones and pairs of bones:

	Single	Pairs
Axial skeleton (80 bones)		
Skull:	6	11
Hyoid:	1	
Vertebral column:	**26**	
Thoracic cage:	1	12
Subtotal:	34	23
Appendicular skeleton (126 bones)		
Pectoral girdles:		2
Upper extremity:		30
Pelvic girdle:		1
Lower extremity:		30
Subtotal:	0	63
Total:	34	86

The human skeleton is composed of 34 bones that occur singly and 172 bones that appear in pairs. The

80 bones of the axial skeleton consist of 34 single bones and 46 bones that occur in pairs. The 126 bones of the appendicular skeleton comprise 63 pairs of bones.

It was shown in Articles 32[1] and 33[2] that the axial skeleton is the manifestation of the 'trunk' of the Tree of Life. This is the downward sequence of point, line, triangle and tetrahedron — what mathematicians call the first four 'simplexes.' With its triangles constructed from three tetractyses, the trunk is made of 80 yods, each one denoting a bone of the axial skeleton. The rest of what is the lowest of the four Trees mapping the human body — what may be called its 'branches' — has 171 yods, 126 of which denote the 126 bones of the appendicular skeleton. The 34 types of single bones are encoded in the inner Tree of

1
4
19
56

19
61
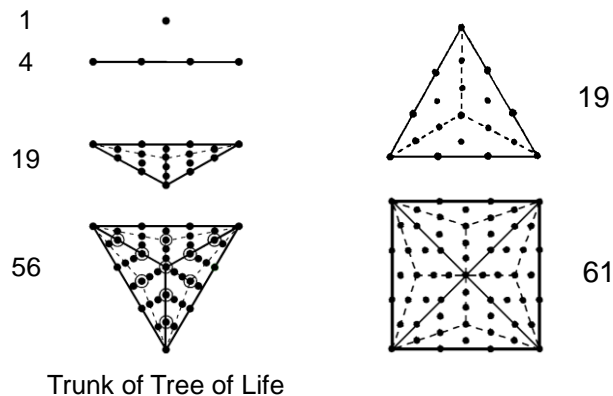
Figure 6. The triangle & square determine the number of bones in the axial skeleton represented by the **80** yods in the trunk of the Tree of Life because they have the same number of yods.

Trunk of Tree of Life

Life as the 34 yods in its pair of triangles. The 23 types of pairs of bones are encoded as the 23 pairs of yods in the pair of squares. With their sectors transformed into three tetractyses, the pair of joined triangles and the pair of joined squares contain 206 yods — the number of bones in the human skeleton, whilst a separate triangle and a square contain **80** yods when similarly transformed, which is the number of bones in the axial skeleton (Fig. 6).

Let us return to the seven layers of polygons in the disdyakis triacontahedron. They are of four types: an 8-sided polygon, a 10-sided polygon, a 6-sided polygon and a 12-sided polygon. They correspond in the

Figure 7. The polygonal inner Tree of Life.

polygonal Tree of Life (Fig. 7) to the octagon, decagon, hexagon and dodecagon, although the polygons formed by the layers of vertices are not regular. It is trivial to prove that the number of yods in an n-sided polygon (regular or irregular) whose n triangular sectors are tetractyses with four yods on each side is 6n+1. The hexagon (n=6) has 37 yods, of which (37–4=33) yods are outside the root edge,[*] the octagon (n=8) has **49** yods (45 outside the root edge), the decagon (n=10) has 61 yods (57 outside the root edge) and the dodecagon (n=12) has **73** yods (69 outside the root edge). The hexagon and octagon have (33+45=78) yods outside the root edge. Two yods of the latter are associated with one set of seven polygons and the two other yods of the root edge are associated with the other set of seven polygons (Fig. 8). (78+2=**80**) yods are associated with the hexagon and octagon on each side of the root edge. They consist of (33+1=34) yods associated with each hexagon and (45+1=46) yods associated with each octagon. They symbolize, respectively, the 34 single bones of the axial skeleton and its 46 bones that exist as pairs, one on the left-hand side of the body and
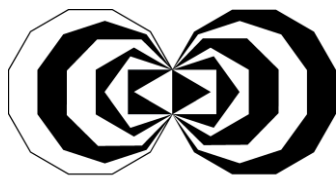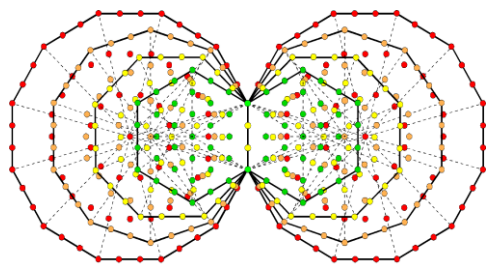

Figure 8. Encoding of the 206 bones of the human skeleton in the last four polygons of the inner Tree of Life.

34 (●) → 34 singles bones of the axial skeleton
46 (●) → 46 paired bones of the axial skeleton
126 (● & ●) → 126 paired bones of the appendicular skeleton

the other on the right-hand side. The (57+69=126) yods of the decagon and dodecagon symbolize the 126 bones of the appendicular skeleton.

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[*] The root edge is the vertical edge shared by all the 14 polygons of the inner Tree of Life.

Layers 1 and 3 of the disdyakis triacontahedron have 14 vertices arranged as an 8-sided polygon and as a 6-sided polygon. They correspond in the inner Tree of Life to, respectively, the octagon and the hexagon associated with which are **80** yods symbolizing the **80** bones of the axial skeleton. Layers 2 and 4 have 22 vertices arranged as a 10-sided polygon and as a 12-sided polygon. Their counterparts in the inner Tree of Life are the decagon and the dodecagon, which have 126 yods outside the root edge symbolizing the 126 bones of the appendicular skeleton. The odd-numbered layers therefore determine the axial skeleton and the even-numbered layers determine the appendicular skeleton. Layers 5, 6 & 7 are the respective mirror images of layers 3, 2 &1.

Of the 264 yods in the seven enfolded polygons, the 206 yods associated with the last four polygons symbolize bones, leaving 58 yods in the first three enfolded polygons. The last (4+4) enfolded polygons have 58 corners, showing how this number defines the last four polygons in two different ways. Eight of these corners coincide with Sephiroth of the lowest tree mapping the human body (Fig. 9), leaving **50** corners that are unshared with it. This shows how the Divine Name ELOHIM with number value **50** prescribes the last (4+4) polygons and therefore the number of bones in the human skeleton. The 70 triangular sectors of the last (4+4) enfolded polygons have 66 corners, where 66 is the **65**th integer after 1. This shows how ADONAI, the Godname of Malkuth with number value **65**, prescribes these polygons embodying information about the human body — the Malkuth aspect of Adam Kadmon. 33 such corners are associated with each set of the last four polygons. The centre of the hexagon is a corner of the triangle and the centre of the decagon is a corner of the pentagon. This leaves (33–2=**31**) corners of the
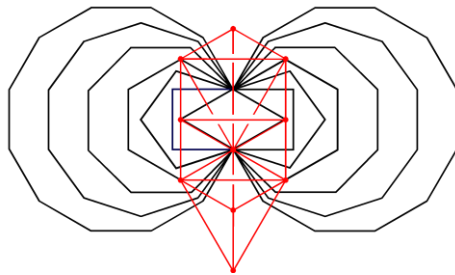

Figure 9. Eight Sephiroth are located at corners of polygons.

35 sectors of the last four polygons that are *intrinsic* to them alone because they do not belong to the first three polygons. EL, the Godname of Chesed with number value **31**, therefore prescribes the last four polygons of the inner Tree of Life that encode the bone composition of the human skeleton.

The 35 sectors of the last four enfolded polygons have 34 corners and 69 sides, totalling 138 geometrical elements. The number of elements associated with each set of four polygons is 137. This integer defines the fine-structure constant $e^2/\hbar c \approx 1/137.036$, making it perhaps the most important number in physics because its reciprocal measures the strength of the coupling of the electromagnetic field to electric charges and thus the size of atoms and the energy scale of their chemical bonding. 137 is the 33rd prime number. As mentioned earlier, 33 corners of sectors are associated with the last four polygons, where 33 = 1! + 2! + 3! + 4!, i.e., the sum of the numbers of permutations of 10 objects arranged in the four rows of a tetractys. Of the 34 corners, eight are either centres of polygons, shared with the Tree of Life or both. This leaves **26** corners, showing how YAHWEH, the Godname of Chokmah with number value **26** prescribes the last four enfolded polygons. **26** = 1 + 3 + 7 + **15**, where '1' is the number of

Table 3. Geometrical composition of the seven polygons (single triangles as sectors).

		n	Number of vertices n+1	Number of edges 2n	Number of triangles n	Total 4n+1
1		8	8+1	16	8	32+1
2		10	10+1	20	10	40+1
	Subtotal	18	18+2	**36**	18	**72**+2
3		6	6+1	12	6	24+1
4		12	12+1	24	12	**48**+1
	Subtotal	**36**	**36**+4	**72**	**36**	144+4
5		6	6+1	12	6	24+1
6		10	10+1	20	10	40+1
7		8	8+1	16	8	32+1
	Total	60	60+7	120	60	240+7

combinations of one object, '3' is the number of combinations of two objects, '7' is the number of combinations of three objects and '**15**' is the number of combinations of four objects. The

Number of geometrical elements surrounding axis

apex (A vertex) → ●

n	4n+1
8	32+1
10	40+1
6	24+1
12	**48**+1
6	24+1
10	40+1
8	32+1

Total = 240 + 7

axis

A

A

disdyakis
triacontahedron

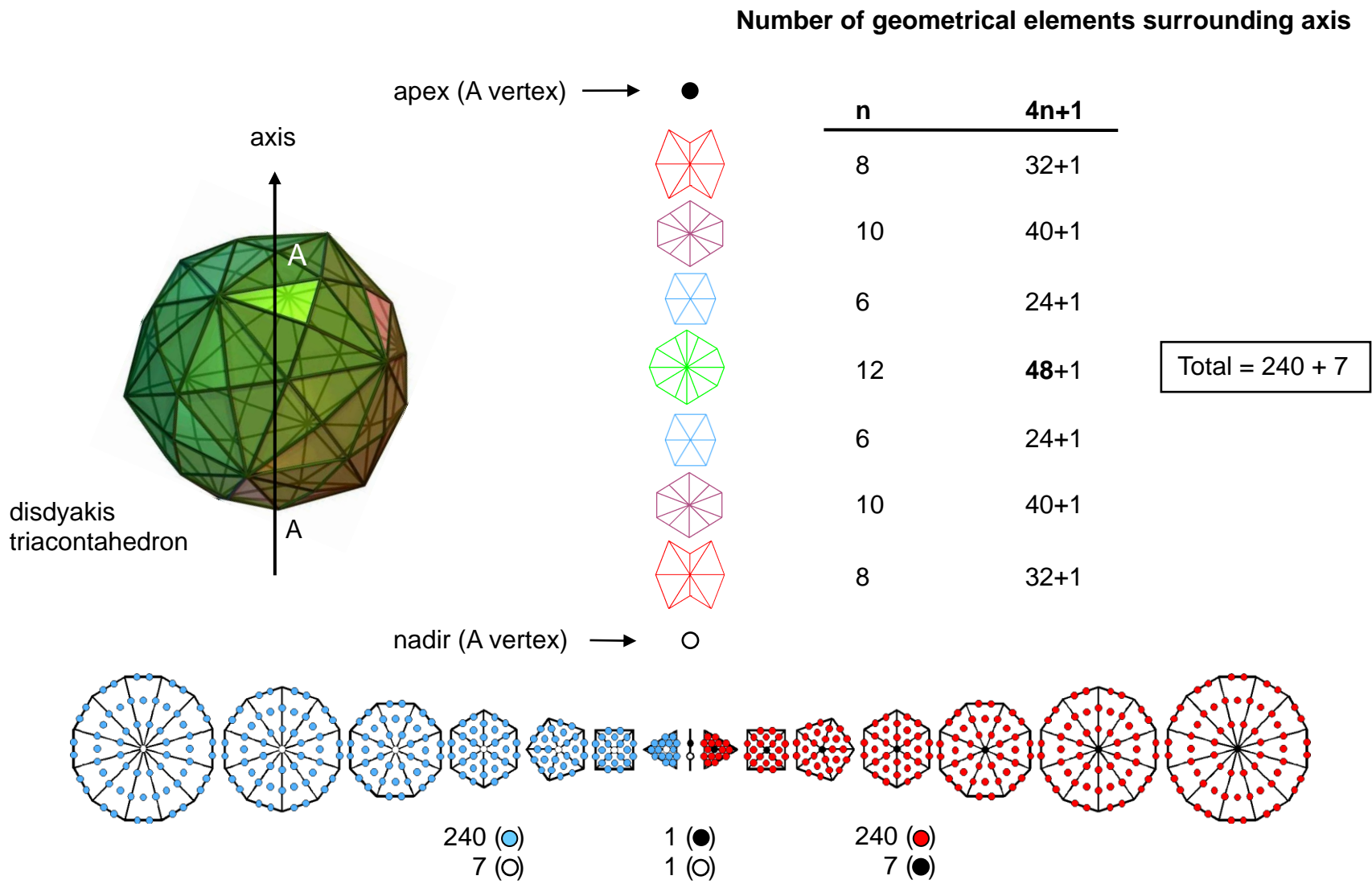nadir (A vertex) → ○

240 (○)　　1 (●)　　240 (●)
7 (○)　　1 (○)　　7 (●)

Figure 10. Encoding of the (248+248) roots of $E_8 \times E_8$ in the polygonal and polyhedral forms of the Tree of Life.

permutational and combinatorial properties of ten objects arranged in the form of a tetractys are therefore connected to the shapes of the last four of the seven enfolded polygons that make up the inner Tree of Life. When regarded as separate polygons, the last four polygons have (6+8+10+12=**36**) corners, where **36** is the number value of ELOHA, the Godname of Geburah. Their **36** sectors have (**36**+4=40) corners, where

$$40 = \begin{matrix} & & 4 & & \\ & 4 & & 4 & \\ 4 & & 4 & & 4 \\ 4 & 4 & & 4 & 4 \end{matrix}.$$

This demonstrates the basic role of the Tetrad (4) in defining properties of any geometrical object that conforms to the universal pattern of the Tree of Life — in this case the 40 corners of the **36** sectors of the first four layers of vertices of its polyhedral form — the disdyakis triacontahedron.

3. Geometrical composition of the seven polygons

Table 3 tabulates the numbers of vertices, edges and triangles making up each polygon in the seven layers of the disdyakis triacontahedron. The seven polygons comprise **67** vertices, where **67** is the number value of Binah. This is very appropriate in view of the meaning of the Sephirah Binah as embodying the archetype of *form*, for this number does, indeed, quantify the shapes of the seven
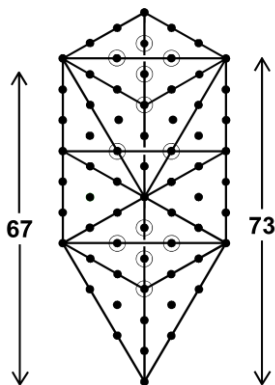

Figure 11. The number value **73** of Chokmah is the number of yods up to the level of Chokmah in the 1-tree when its 19 triangles are turned into tetractyses. The number value **67** of Binah is the number of yods below Binah.

polygons when constructed from tetractyses. In fact, when the 19 triangles of the 1-tree are converted into tetractyses, there are **67** yods below the horizontal path joining Binah to Chokmah (Fig. 11). They include the seven yods that coincide with the seven Sephiroth of Construction, leaving 60 other yods. This 60:7 division in the 1-tree constructed from tetractyses parallels the division of vertices in the seven polygons into 60 vertices that surround their seven centres. The correspondence is striking evidence of the Tree of Life nature of the disdyakis triacontahedron.

According to Table 3, there are 247 geometrical elements in the seven polygons. 240 of these elements surround the seven centres. This important property, the superstring significance of which will be discussed later, corresponds in the polygonal form of the inner Tree of Life to the 240 hexagonal yods of
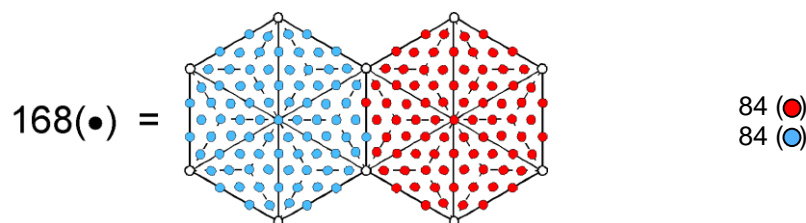

Figure 12. Two adjoined hexagons with their sectors divided into three tetractyses contain **168** yods other than their corners.

the seven separate polygons that surround their seven centres (Fig. 10. The second subtotal row in the table above for the four layers of vertices making up half of the disdyakis triacontahedron shows that they comprise **148** elements. **148** is the number value of Netzach, the *fourth* Sephirah of Construction. 144 of these elements surround the centres of the four polygons, where $144 = 12^2 = 12$th Fibonacci number. **36** of them are vertices of polygons, showing how the Godname ELOHA with number value **36** prescribes

the shapes of the four polygons, just as we saw earlier that it does for their separate, regular counterparts in the inner Tree of Life. The four polygons have **72** edges and **36** triangles, where **72** is the number value of Chesed, the *fourth* Sephirah from the top of the Tree of Life. Including the apex of the disdyakis triacontahedron (an A vertex), there are 41 vertices from this 'North pole' to the equatorial plane containing the 12 vertices. 41 is the **21**st odd integer, showing how EHYEH, the Godname of Kether with number value **21**, prescribes the number of vertices in half of the disdyakis triacontahedron when the
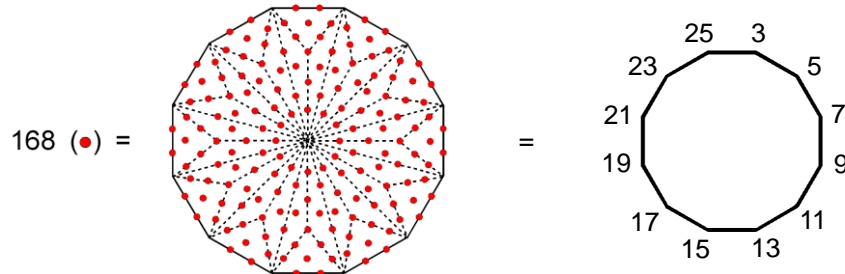
Figure 13. Division of each of the 12 sectors of a dodecagon into three tetractyses generates **168** more yods. **168** is the sum of the 12 odd integers after 1 that can be assigned to its 12 corners.

polygons of vertices in its layers are constructed from tetractyses.

The two uppermost layers are made up of an 8-sided polygon and a 10-sided polygon. They contain **36** edges. **72** geometrical elements surround their two centres. The five lowest layers are made up of an 8-sided polygon, a 10-sided polygon, two 6-sided polygons and a 12-sided polygon. They have 47 vertices, where 47 is the **15**th prime number, showing how the Godname YAH of Chokmah with number value **15** prescribes the polygons in the five lowest layers. 42 of these vertices surround their five centres, where 42 is the **21**st even integer, showing how EHYEH, Godname of Kether with number value **21**, prescribes these five polygons. They have 84 edges, where $84 = 1^2 + 3^2 + 5^2 + 7^2$, and 42 triangles. They also have (42+42=84) vertices and triangles surrounding their centres. The lowest five polygons therefore have (84+84=**168**) geometrical elements, where **168** is the number value of the Mundane Chakra of Malkuth. *This division of the number of elements into two sets of 84 is highly significant.* It has been encountered in previous articles such as:

1. Article 15,[3] which discussed the 84 permutations of pairs and triplets of octonions belonging to the seven triples of octonions (e_i, e_{i+1}, e_{i+3}) and the 84 permutations of their inverses;
2. Article 18,[4] which discussed the I Ching table of **64** hexagrams and the 28 hexagrams on either side of its diagonal. Each hexagram is a pair of trigrams, so that each set of 28 off-diagonal hexagrams has (28×2×3=**168**) lines and broken lines. Each set comprises 84 lines and 84 broken lines;
3. Article 21,[5] which discussed the 84 hyperbolic triangles in the two types of seven similar slices of the Klein Configuration, the diagram whose **168** hyperbolic triangles represent the **168** automorphisms of the equation known to mathematicians as the Klein Quartic;
4. Article 16,[6] which discussed, firstly, the 84 repetitions of the Pythagorean-valued, rising intervals between the notes of the seven types of musical scales and their 84 falling intervals (pp. 17-18), and, secondly, the 84 yods belonging to the first four enfolded polygons enfolded in each half of the inner Tree of Life that either are not centres of polygons or do not coincide with Sephiroth (pp. 23–24).

Such correspondences demonstrate powerfully that the lowest five layers of the disdyakis triacontahedron form a Tree of Life pattern in themselves. This could have already been anticipated, because the number **168** is the structural parameter of the superstring — the manifestation in the subatomic world of the Tree of Life blueprint. Indeed, this is why the number **168** *is* the number value of the Mundane Chakra of Malkuth, the Sephirah of the Tree of Life denoting the physical universe of space and time.

The 84:84 division of geometrical elements in the last five polygons of vertices, which start with the 6-sided polygon, has its counterpart in the pair of joined hexagons belonging to the polygonal form of the inner Tree of Life. **168** more yods (84 per hexagon) are needed to transform their sectors into three tetractyses (Fig. 12). Furthermore, it is remarkable that the 12-sided polygon in the middle of the seven layers of vertices requires **168** more yods to construct its sectors from three tetractyses. Figure 13 shows this property for the dodecagon. The fact that the sides of the 12-sided polygon are unequal in length is irrelevant. As $13^2 - 1 = $ **168** $= 3 + 5 + 7 + \ldots + 25$, assigning the first 12 odd integers after 1 to the vertices

of this central 12-sided polygon generates the number **168** as their sum. This is an example of the beautiful coherence between number and sacred geometry. It also illustrates the Tetrad Principle because the four-fold sequence of 8-sided, 10-sided, 6-sided and 12-sided polygons ends with one that embodies a number of universal significance, namely, the very number **168** that defines the form of an $E_8{\times}E_8$ heterotic superstring because each of its 10 curves makes **168** circularly polarized oscillations during one half of a revolution around its spin axis. It manifests in the disdyakis triacontahedron as its **168** edges above and below its equator that give it a 3-dimensional shape.

4. The seven polygons encode the roots of E₈.

The seven polygons have 240 geometrical elements surrounding their centres, of which **72** belong to the uppermost two layers and **168** belong to the five lowest layers. This **72**:**168** division appears in the structure of the non-zero roots of the superstring gauge symmetry group E_8. The roots of the E_8 algebra can be described in terms of eight orthonormal unit vectors $\{u_i\}$. Eight zero roots correspond to points at the centre of the root diagram and 240 non-zero roots all have length $\sqrt{2}$. They are given by:

$$\pm u_i \pm u_j \qquad\qquad (i, j = 1, 2, \ldots, 8)$$

and

$$\tfrac{1}{2}(\pm u_1, \pm u_2, \ldots \pm u_8) \qquad \text{(even number of +'s)}$$

Their explicit forms as 8-tuples and their numbers are listed below:

(1, 1, 0, 0, 0, 0, 0, 0) and all permutations. Number = $\begin{bmatrix}8\\2\end{bmatrix}$ = 28;

(-1, -1, 0, 0, 0, 0, 0, 0) and all permutations. Number = $\begin{bmatrix}8\\2\end{bmatrix}$ = 28;

(1, -1, 0, 0, 0, 0, 0, 0) and all permutations. Number = $2{\times}\begin{bmatrix}8\\2\end{bmatrix}$ = 56; $\qquad$ **168**

(-½, -½, ½, ½, ½, ½, ½, ½) and all permutations. Number = $\begin{bmatrix}8\\2\end{bmatrix}$ = 28;

(-½, -½, -½, -½, -½, -½, ½, ½) and all permutations. Number = $\begin{bmatrix}8\\2\end{bmatrix}$ = 28;

(-½, -½, -½, -½, ½, ½, ½, ½) and all permutations. Number = $\begin{bmatrix}8\\4\end{bmatrix}$ = 70;

(½, ½, ½, ½, ½, ½, ½, ½). Number = 1; $\qquad$ **72**

(-½, -½, -½, -½, -½, -½, -½, -½). Number = 1.

The 240 non-zero roots of E_8 comprise **168** non-zero roots made up of four sets of 28 and one set of 56 and **72** non-zero roots comprising one set of 70 and two single ones. Remarkably, surrounding the seven centres of the polygons are as many geometrical elements (240) as there are non-zero roots of E_8, the first two polygons having as many elements (**72**) as there are non-zero roots of E_6, an exceptional subgroup of E_8 of much interest to physicists seeking to extend the Standard Model of particle physics using superstring theory. This provides highly convincing evidence that superstring physics is encoded in the pattern of vertices belonging to the seven layers of the disdyakis triacontahedron.

The 2:5 division of layers generating the **72**:**168** division of geometrical elements surrounding centres of polygons reflects the distinction in the Tree of Life between the two Sephiroth of Construction — Chesed and Geburah, which lie outside its Lower Face, and the remaining five Sephiroth of Construction that span it. The eight zero roots of E_8 consist of seven zero roots of one kind and one zero root of another kind. The former are denoted by the seven centres of the polygons and the latter is denoted by the topmost A vertex of the disdyakis triacontahedron (see Fig. 10).

The **72**:**168** division was encountered in Article 26[7] in discussion of the construction of the disdyakis triacontahedron from tetractyses. When its 180 internal triangles formed by its centre and each edge of the polyhedron are divided into three tetractyses and its 120 triangular faces are turned into single tetractyses, there are 1680 (=**168**×10) geometrical elements surrounding the axis that joins two diametrically opposite A vertices. When each of its 120 faces is constructed from three tetractyses, there are 2400 (=240×10) such elements, i.e., an extra 720 (=**72**×10) elements are added. What is being represented geometrically here are the root structures of the superstring gauge symmetry group E_8 and its exceptional subgroup E_6, for E_8 has **168** non-zero roots that do not belong to E_6, which has **72** such roots.

Every simple root of E_8 is associated with a Yang-Mills gauge charge — the source of a 10-dimensional Yang-Mills gauge field. The factor of 10 in these properties therefore encodes the information that each gauge field has ten space-time components. The layers of the disdyakis triacontahedron encode the number of 10-dimensional gauge fields defining the unified superstring force, whilst the various numbers of geometrical elements making up the interior and exterior of this polyhedron are the numbers of *components* of these fields. They represent the number of bits of information needed to define the gauge fields of the superstring in 10-dimensional space-time. In the context of the disdyakis triacontahedron, the additional 720 elements correspond to the (240×3=720) components of these fields in 3-dimensional space.

The **72**:**168** division of the non-zero roots of E_8 appears in the $E_8 \times E_8$ heterotic superstring as the distinction between the three major whorls and the seven minor whorls in the object described over a century ago by Annie Besant and C.W. Leadbeater[8] as being the basic unit of matter (Fig. 14). As the
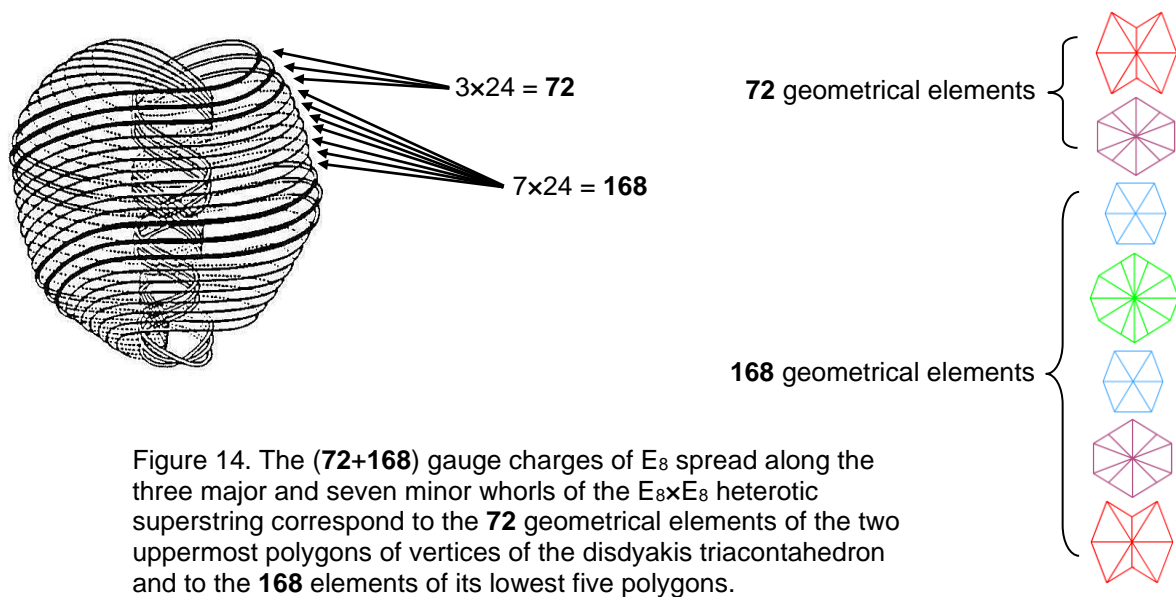

3×24 = **72**

7×24 = **168**

72 geometrical elements

168 geometrical elements

Figure 14. The (**72+168**) gauge charges of E_8 spread along the three major and seven minor whorls of the $E_8 \times E_8$ heterotic superstring correspond to the **72** geometrical elements of the two uppermost polygons of vertices of the disdyakis triacontahedron and to the **168** elements of its lowest five polygons.

manifestation in the space-time continuum of the Tree of Life blueprint, the major whorls correspond to the Supernal Triad of Kether, Chokmah and Binah, whilst the minor whorls correspond to the seven Sephiroth of Construction. Besant & Leadbeater connected them to the three "solar Logoi" and to the seven "Planetary Logoi:"

> "The Anu is a sun in miniature in its own universe of the inconceivably minute. Each of the seven whorls is connected with one of the Planetary Logoi, so that each Planetary Logos has a direct influence playing on the very matter of which all things are constructed. It may be supposed that the three conveying electricity, a differentiation of Fohat, are related to the Solar Logos."[9]

Although each whorl is a separate closed curve that never intersects itself or touches one another, this separateness is an illusion, existing only in ordinary 3-dimensional space. The curves are the 3-dimensional cross-sections of a hyper-dimensional object embedded in a higher, 15-dimensional space with a topology such that its cross-section is a set of ten closed curves topologically equivalent to circles, each one twisting 2½ times in an outer circuit and 2½ times in a tighter revolution around the spin axis of the spin-½ superstring. A 1-dimensional curve embedded in **26**-dimensional space-time has 24 transverse vibrational degrees of freedom. These generate generalised 24 Kaluza-Klein gauge fields that, on compactification of the 16 dimensions beyond superstring space-time, become 24 Yang-Mills gauge fields of E_8 existing in 10-dimensional space-time. Their charges are spread along each curve, 24 to each one. The three major whorls have **72** E_8 gauge charges and the seven minor whorls have **168** charges. The former are the gauge charges of E_6, an exceptional subgroup of E_8, and the latter are the remaining gauge charges of E_8. They correspond in the seven polygons of vertices of the disdyakis triacontahedron to the **72** geometrical elements surrounding the centres of the two uppermost polygons and to the **168** elements around the centres of the lowest five polygons. The first of these polygons has 24 elements, the second has 24 edges and 24 vertices and triangles, the third has 24 elements and the last two polygons have **72** elements. These sets of 24 elements are the polygonal counterpart of the 24 gauge charges

spread along each of the ten curves of the superstring.

Of the **72** elements in the two uppermost polygons, 18 elements are vertices of the disdyakis triacontahedron (eight A, four B & six C vertices). Hence, these polygons have (**72**–18=54) elements that are not polyhedral vertices. Of the **168** elements in the lowest five polygons, 42 elements are vertices of the disdyakis triacontahedron (20 A, eight B & 14 C vertices). This leaves (**168**–42=126) elements in the
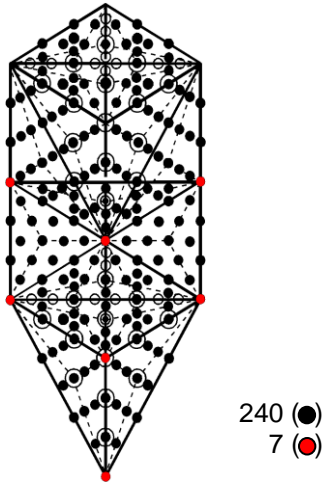

240 (●)
7 (●)

Figure 15. The 240 extra yods needed to construct the 1-tree from tetractyses symbolize the 240 roots of the superstring gauge symmetry group E_8. They correspond to the 240 geometrical elements defining the polygons in the seven layers of the disdyakis triacontahedron. The seven yods coinciding with the seven Sephiroth of Construction of the 1-tree symbolize seven of the eight simple roots of E_8. They correspond to the centres of the seven polygons.

lowest five polygons that are not polyhedral vertices. Including their five centres, there are (126+5=**131**) elements that are not polyhedral vertices. **131** is the number value of *Samael*, the Archangel of Geburah. All seven polygons have 180 edges and triangles. Including the seven centres of polygons, there are **187** geometrical elements that are not polyhedral vertices. This is the number value of *Auphanim*, the Order of Angels assigned to Chokmah.

The counterpart in the outer Tree of Life of the 240 geometrical elements surrounding the centres of the seven polygons is the 240 extra yods needed to construct the 1-tree when its 19 triangles are each divided into three tetractyses (Fig. 15). The counterpart of the centres of the seven polygons is the locations of the seven Sephiroth of Construction.

Suppose now that the 60 sectors of the seven polygons are each divided into three tetractyses instead of single ones. Table 4 lists the number of vertices, edges and triangles in each polygon.

Table 4. Geometrical composition of polygons (three triangles as sectors).

		n	Number of vertices 2n+1	Number of edges 5n	Number of triangles 3n	Total 10n+1
1		8	16+1	40	24	**80**+1
2		10	20+1	**50**	30	100+1
	Subtotal	18	**36**+2	90	54	180+2
3		6	12+1	30	18	60+1
4		12	24+1	60	**36**	120+1
	Subtotal	**36**	**72**+4	180	108	360+4
5		6	12+1	30	18	60+1
6		10	20+1	**50**	30	100+1
7		8	16+1	40	24	**80**+1
	Total	60	120+7	300	180	600+7

The seven polygons have 120 vertices surrounding their centres. This is a beautiful property because it indicates that the disdyakis triacontahedron has the same number of triangular faces as there are vertices in the 180 triangles making up the polygons in its seven layers of vertices. 60 of them are polyhedral ones, leaving **67** non-polyhedral vertices, where **67** is the number value of Binah. The numbers 120 and 60 conform to the Tetrad Principle[10] because

$$120 = 2^2 + 4^2 + 6^2 + 8^2 = 2^3 + 2^4 + 2^5 + 2^6$$

and
$$60 = 2^2 + 2^3 + 2^4 + 2^5.$$

The 8-sided polygon at the base of the disdyakis triacontahedron has **80** vertices, edges & triangles surrounding its centre. **80** is the number value of Yesod, the penultimate Sephirah. Its meaning — 'foundation' — is appropriate, given that this is the lowest polygon. The first two polygons have 36 vertices, 90 edges and 54 triangles surrounding their centres, which total 180 geometrical elements. The last five polygons have 84 vertices, 210 edges & 126 triangles surrounding their five centres, totalling 420 elements. With single triangles as sectors, the last five polygons have 84 edges. Once again, the heterotic superstring structural parameter 84 appears. The number of edges & triangles in the lowest five polygons = 210 + 126 = 336. This, too, is a structural parameter of the superstring, being the number of circularly polarized oscillations in one revolution of each whorl of the UPA/superstring. Both numbers are consistent with the Tetrad Principle governing Tree of Life parameters because

$$84 = 1^2 + 3^2 + 5^2 + 7^2$$

and

$$336 = 4 \times 84 = 2^2 \times 84 = 2^2(1^2 + 3^2 + 5^2 + 7^2)$$
$$= 2^2 + 6^2 + 10^2 + 14^2.$$

As

$$10 = 1 + 2 + 3 + 4$$

and

$$30 = 1^2 + 2^2 + 3^2 + 4^2,$$

the number of edges of the 180 triangles is

$$300 = 10 \times 30 = (1+2+3+4)(1^2+2^2+3^2+4^2).$$

The number of edges and triangles is

$$480 = 2 \times 240 = 2(2^4 + 2^5 + 2^6 + 2^7) = 2^5 + 2^6 + 2^7 + 2^8.$$
$$= 4 \times 120 = 2^2(2^2 + 4^2 + 6^2 + 8^2) = 4^2 + 8^2 + 12^2 + 16^2.$$

This property again illustrates the Tetrad Principle.

The number of geometrical elements surrounding the seven centres is 600, which has the tetractys representation:

$$600 = \begin{matrix} 2^2 \\ 2^2\ 2^2 \\ 2^2\ 2^2\ 2^2 \\ 2^2\ 2^2\ 2^2\ 2^2 \end{matrix} + \begin{matrix} 2^3 \\ 2^3\ 2^3 \\ 2^3\ 2^3\ 2^3 \\ 2^3\ 2^3\ 2^3\ 2^3 \end{matrix} + \begin{matrix} 2^4 \\ 2^4\ 2^4 \\ 2^4\ 2^4\ 2^4 \\ 2^4\ 2^4\ 2^4\ 2^4 \end{matrix} + \begin{matrix} 2^5 \\ 2^5\ 2^5 \\ 2^5\ 2^5\ 2^5 \\ 2^5\ 2^5\ 2^5\ 2^5 \end{matrix}.$$

Remarkably, this is the number of geometrical elements surrounding the axis of the disdyakis triacontahedron, for it has 60 vertices surrounding this axis, each joined internally to its centre as an edge of an internal triangle, 180 edges, each the edge of an internal triangle, and 120 triangles — a total of (60+60+180+180+120=600) elements.

The Divine Name EL assigned to Chesed with number value **31** prescribes the seven layers of vertices because the total number of vertices is 127, which is the **31**st prime number. In view of the fact that there are seven layers, it is fitting that 127 is the *seventh* Mersenne prime number! EL prescribes the disdyakis triacontahedron also because it consists of **31** vertices and their mirror images. The Divine Name YAH assigned to Chokmah with number value **15** prescribes the vertex composition of the polygons because 120 vertices surround their centres and 120 is the **15**th triangular number:

$$120 = 1 + 2 + 3 + \ldots + \mathbf{15}.$$

Including the apex and nadir of the disdyakis triacontahedron, there are **129** vertices. This is the number value of YAHWEH SABAOTH, the Godname of Netzach, showing how the Godname of the *seventh* Sephirah prescribes the *seven* layers of vertices in the disdyakis triacontahedron.

We saw earlier that the 8-sided polygon at the base of the disdyakis triacontahedron comprises **80** vertices, edges & triangles when its sectors are divided into three triangles. With these triangles regarded as tetractyses, the polygon has 120 yods around its centre. In other words, it has as many yods around

its centre as all seven polygons have vertices surrounding their centres! When each sector is regarded as a tetractys, the 8-sided polygon has **49** yods. This shows how EL CHAI, Godname of Yesod with number value **49**, prescribes the seven polygons, which begin and end with this one.

Table 5. Yod composition of the seven polygons (single tetractys).

		n	Number of vertices n+1	Number of yods 6n+1	Number of hexagonal yods 5n
1		8	8+1	**48**+1	40
2		10	10+1	60+1	**50**
	Subtotal =	18	18+2	108+2	90
3		6	6+1	**36**+1	30
4		12	12+1	**72**+1	60
	Subtotal =	**36**	**36**+4	**216**+4	180
5		6	6+1	**36**+1	30
6		10	10+1	60+1	**50**
7		8	8+1	**48**+1	40
	Total =	60	60+7	360+7	300

Table 4 indicates that the four polygons in each half of the disdyakis triacontahedron have 360 geometrical elements surrounding their centres (including **36** polyhedral vertices), the fourth polygon having **36** triangles. This demonstrates how the Godname ELOHA assigned to Geburah with number value **36** prescribes the polygons formed by the 60 vertices of the disdyakis triacontahedron. They have **72** vertices (**36** polyhedral vertices, **36** non-polyhedral vertices). Including their four centres, the first four polygons have **76** vertices. This is the number value of YAHWEH ELOHIM, the Godname of Tiphareth,

1   2   3   4

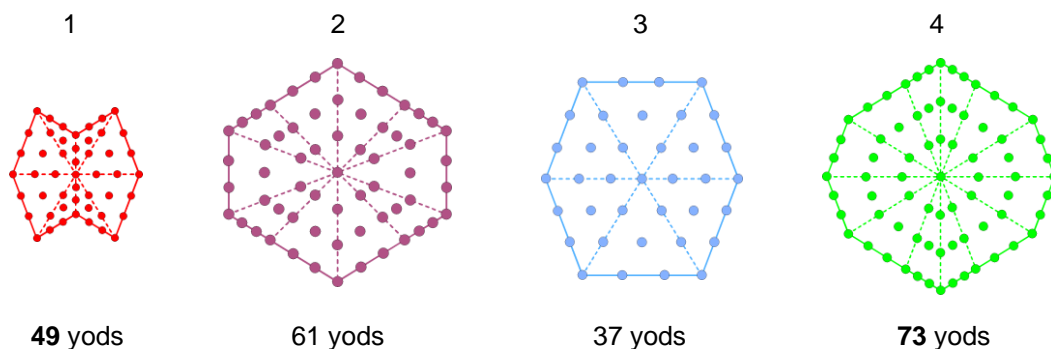**49** yods   61 yods   37 yods   **73** yods

Figure 16. The tetractys construction of the four polygons (not to scale).

showing how this Godname prescribes each half of the disdyakis triacontahedron.

The first four polygons have **36** internal vertices surrounding their centres and 180 edges in their tetractyses, that is, **216** vertices and edges. This is the number value of the Sephirah Geburah, whose Godname has the number value of **36**.

5. The yod composition of the seven polygons

Table 5 lists the yod populations of the seven polygons when their sectors are turned into tetractyses. Figure 16 displays the yods in the first four polygons. The polygon in the second layer has 10 vertices and therefore is 10-sided even though its boundary has six straight lines. Similarly, the polygon in the fourth layer is 12-sided because it has 12 vertices, even though it is bounded by eight straight lines. The seven polygons have **67** yods at the corners of their 60 tetractyses, the first two polygons have 20 yods at corners of their 18 tetractyses and the last five polygons have 47 yods at the corners of their 42 tetractyses. As 47 is the **15**th prime number, this shows how the Godname YAH with number value **15** prescribes the last five polygons. They have 210 (=**21**×10) hexagonal yods, showing how the Godname EHYEH with number value **21** prescribes the last five polygons. The first four polygons have **36** yods at corners of their **36** tetractyses surrounding their centres, showing how the Godname ELOHA of Geburah with number **36** prescribes these polygons. They have **216** yods surrounding their centres, where **216** is

the number value of Geburah. All seven polygons have 360 (=**36**×10) yods surrounding their centres, yet again demonstrating how ELOHA prescribes the seven polygons. The number of yods in the fourth polygon is **73**, which is the number value of Chokmah. It has **72** yods surrounding its centre. This is the number of Chesed, the Sephirah next below Chokmah on the right-hand Pillar of Mercy. The number values of Chokmah, Binah, Chesed and Geburah measure in a natural way different groups of yods in the seven polygons. EHYEH also prescribes the central layer of the disdyakis triacontahedron because **73** is the **21**st prime number.

There are 307 yods on the edges of the 60 tetractyses. ELOHIM SABAOTH with number value **153** prescribes the shapes of the polygons because 307 is the **153**rd odd integer after 1. Formally including the two A vertices above and below the sheets, 309 yods shape the nine layers of vertices. ADONAI
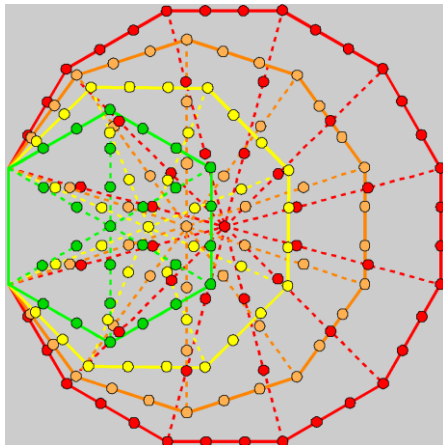

Figure 17. **168** yods outside the root edge lie on the edges of the tetractyses of the four polygons in the inner Tree of Life corresponding to the four types of polygons in the disdyakis triacontahedron.

MELEKH with number value **155** prescribes them as 309 is the **155**th odd integer after 1. 247 new yods are needed to delineate the edges of the tetractyses (240 hexagonal yods on their 120 edges and seven centres). We found earlier that they are composed of 247 geometrical elements, seven of which are their centres (see Table 1). Remarkably, the 240:7 division exists for *both* yods and geometrical elements.

Now consider the last four regular polygons enfolded in the inner Tree of Life that are the counterparts of the polygons in one-half of the disdyakis triacontahedron. Tabulated below is the number of yods in each polygon outside the root edge, the number of centres of their tetractyses and the number of yods on the edges of tetractyses:

Table 6. Yod composition of the last four enfolded, regular polygons.

Polygon	Number of yods	Number of yods outside root edge	Number of centres of tetractyses	Number of yods on edges of tetractyses
Octagon	**49**	**49** – 4 = 45	8	45 – 8 = 37 ⎤ 84
Decagon	61	61 – 4 = 57	10	57 – 10 = 47 ⎦
Hexagon	37	37 – 4 = 33	6	33 – 6 = 27 ⎤ 84
Dodecagon	**73**	**73** – 4 = 69	12	69 – 12 = 57 ⎦
				Total = **168**

There are **168** yods in the four polygons outside the root edge on the edges of their **36** tetractyses (Fig. 17). The number value of *Cholem Yesodoth*, the Mundane Chakra of Malkuth, defines the shapes of all the tetractyses that form the four polygons having counterparts in the disdyakis triacontahedron. It demonstrates the meaning of this Mundane Chakra: it expresses the form that the Tree of Life takes in the physical world. This form is the heterotic superstring and **168** is its structural parameter, as many previous articles have discussed. The first two polygons have the same number of yods (84) as the last two. Thus, there is the same 84:84 division as that which appears in the 84 edges and 84 vertices and triangles of the tetractyses in the last five polygons. This distinction manifests in the superstring as the

840 circularly polarised waves in the outer half of each of its 10 curves and the 840 oscillations in its inner half (Fig. 18).

We saw earlier that, associated with the first two regular polygons — the hexagon and the octagon — are **80** yods symbolizing the **80** bones of the axial skeleton, whilst the decagon and dodecagon has 126 yods symbolizing the 126 bones of the appendicular skeleton. Now we find that two pairs of polygons in a different order display an 84:84 division of yods on the boundaries of tetractyses that has significance for superstrings.

According to Table 6, the first four polygons of vertices in the disdyakis triacontahedron have 184 yods on the edges of their tetractys sectors and the last three polygons have 123 such yods. The seven polygons

whorl   336 turns   84 turns   **168** turns

360°   90°   180°
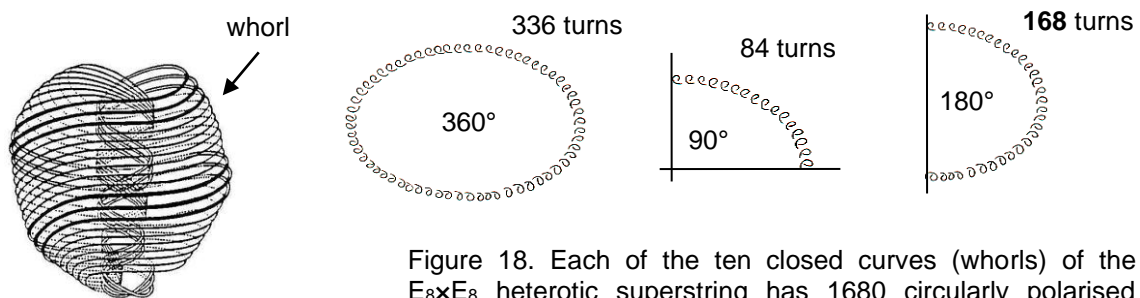
E₈×E₈ heterotic superstring

Figure 18. Each of the ten closed curves (whorls) of the E₈×E₈ heterotic superstring has 1680 circularly polarised waves or oscillations. 840 waves belong to the outer 2½ revolutions and 840 waves belong to the inner 2½ revolutions around its spin axis. There are 84 waves in a 90° twist, **168** waves in an 180° twist and 336 waves in one revolution.

therefore have 307 yods lining their tetractyses. 307 is the **153**rd odd integer after 1. This shows how the Godname ELOHIM SABAOTH of Hod with number value **153** prescribes the seven polygons of vertices.

Table 7 lists the yod populations of the seven polygons when their sectors are turned into three tetractyses:

Table 7. Yod populations of the seven polygons (three tetractyses).

		n	Number of vertices 2n+1	Number of yods 15n+1	Number of hexagonal yods 13n
1		8	16+1	120+1	104
2		10	20+1	150+1	130
	Subtotal =	18	**36**+2	270+2	234
3		6	12+1	90+1	78
4		12	24+1	180+1	156
	Subtotal =	**36**	**72**+4	540+4	468
5		6	12+1	90+1	78
6		10	20+1	150+1	130
7		8	16+1	120+1	104
	Total =	60	120+7	900+7	780

120 yods are vertices surrounding their seven centres, where 120 is the **15**th triangular number. The Godname YAH with number **15** also prescribes the number of yods per sector surrounding their centres. Including the seven centres and the A vertices at the apex and nadir of the disdyakis triacontahedron, there are (120+7+2=**129**) vertices altogether. The Godname YAHWEH SABAOTH with number value **129** therefore prescribes the disdyakis triacontahedron.

The seven polygons have 900 yods surrounding their seven centres. Article 26[11] showed that, when both the internal and external triangles formed by the edges of the disdyakis triacontahedron are divided into three tetractyses, it has 900 triangles. The yod population of the seven polygons is equal to the number of triangles needed to build the disdyakis triacontahedron.
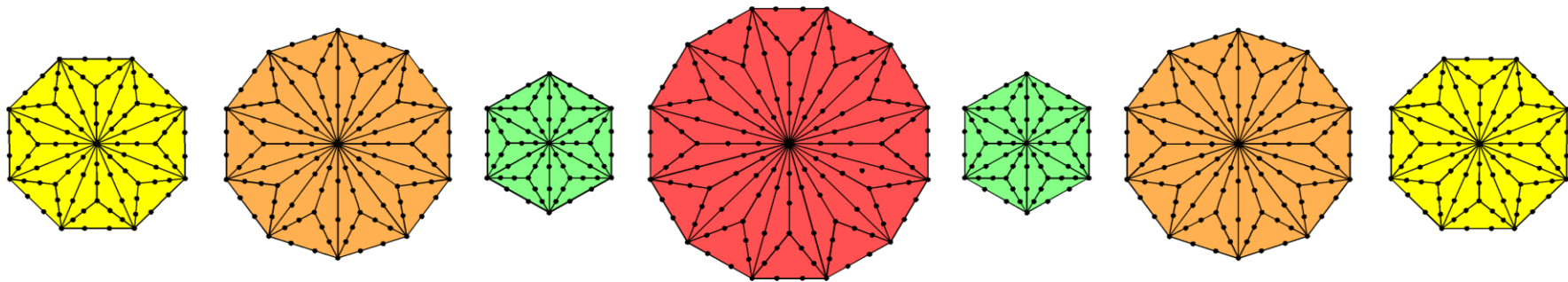
An n-sided polygon has (**15**+1) yods when its sectors are constructed from three tetractyses. Three yods

7 regular polygons of the inner Tree of Life
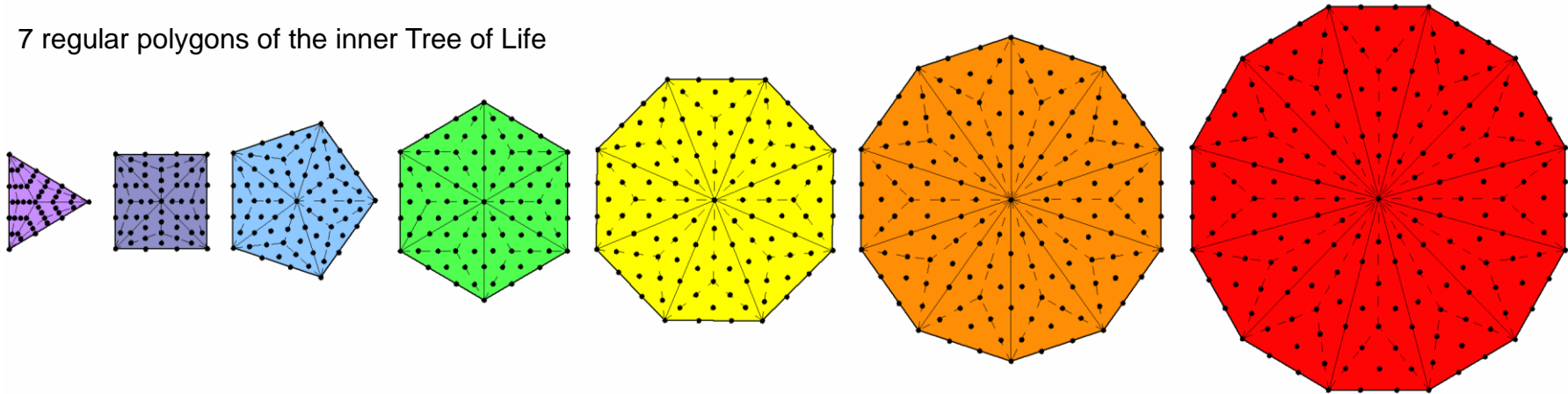
727 yods

7 polygons in the disdyakis triacontahedron
(regular counterparts shown)

727 yods

Figure 19. The 7 polygons of vertices constitute a Tree of Life pattern because they have as many yods on the boundaries of their tetractyses as the seven regular polygons of the inner Tree of Life have yods.

in each sector are at the centres of tetractyses, so that (12n+1) yods lie on the boundaries of the 3n tetractyses. The seven polygons have (12×60+7=727) boundary yods. 727 is the **129**th prime number, showing the remarkable way in which YAHWEH SABAOTH, the Godname of Netzach (the *seventh* Sephirah) with number value **129**, prescribes the *seven* layers of vertices in the disdyakis triacontahedron. 720 boundary yods surround the seven centres. This constitutes a Tree of Life pattern because the seven separate, regular polygons of the inner Tree of Life have 720 yods surrounding their seven centres when their sectors are constructed from three tetractyses. The two sets of polygons are compared in Fig. 19 (for the sake of clarity, the regular counterparts of the polygons in the seven layers are shown). It is yet more proof that the disdyakis triacontahedron is the polyhedral form of the Tree of Life.

The first four polygons have (12×**36**+4=436) yods on the edges of tetractyses. Of these, **76** are vertices, where **76** is the number value of YAHWEH ELOHIM, Godname of Tiphareth, leaving 360 (=**36**×10) yods. YAHWEH ELOHIM prescribes the first four polygons in terms of their vertices and the Godname ELOHA with number **36** prescribes those yods that are not vertices.

The first or last three polygons with 24 vertices have (12×24+3=291) yods lying on edges of tetractyses. 288 yods surround their three centres, where $288 = 1^1 + 2^2 + 3^3 + 4^4$, showing how the integers 1, 2, 3 & 4 symbolized by the tetractys express properties of Tree of Life patterns. They have 240 geometrical elements surrounding their centres. As the seven separate polygons of the inner Tree of Life have 288 yods surrounding their centres and 240 hexagonal yods, this indicates that each set of three polygons constitute a Tree of Life pattern, just as the first four polygons do.

Table 7 indicates that the seven polygons need 907 yods to build them out of tetractyses. 907 is the **155**th prime number, where **155** is the number value of ADONAI MELEKH, the Godname of Malkuth. This is a truly sublime property, for it powerfully demonstrates how the Godname of the most material Sephirah — Malkuth — arithmetically prescribes the number of yods needed to construct all seven polygons from tetractyses. ADONAI MELEKH also prescribes the fourth polygon because it has 156 hexagonal yods, where 156 is the **155**th integer after 1. Article 26 showed[12] that the number of yods other than polyhedral vertices needed to construct each of its 120 faces from three tetractyses is 1560 (= 156×10), so that ADONAI MELEKH prescribes the shape of the surface of the disdyakis triacontahedron as well as its interior polygons.

The last three polygons have **363** yods. This is the number value of SHADDAI EL CHAI, the complete Godname of Yesod. It differentiates between the first four layers forming half of the polyhedron and its remaining three layers. The latter have **48** vertices surrounding their centres, that is, they have 51 vertices. 51 is the **50**th integer after 1, showing how ELOHIM, the Godname of Binah with number value **50**, differentiates the first four layers of vertices from the last three layers. The first two polygons have **272** yods. This is the number value of *Cherubim*, the Order of Angels assigned to Yesod.

The seven polygons have **187** yods that lie on internal edges of tetractyses. This is the number value of *Auphanim*, the Order of Angels assigned to Chokmah. The first four polygons have **112** such yods. This is the number value of *Beni Elohim*, the Order of Angels assigned to Hod. **36** of these are vertices, leaving **76** radial yods that are not vertices of the disdyakis triacontahedron. YAHWEH ELOHIM therefore prescribes the four layers of each half of the disdyakis triacontahedron. The number of yods other than polyhedral vertices that lie on the 300 edges of the 180 tetractyses in the first four polygons is 400 = $(2+4+6+8)^2$. This is another beautiful illustration of the Tetrad Principle because 400 is the square of the sum of the first *four* even integers.

6. Encoding of the superstring structural parameter 336

The disdyakis triacontahedron has 60 vertices, 180 edges and 120 triangular faces (360 geometrical elements) surrounding any axis passing through two diametrically opposite vertices. The fourth polygon of vertices forming its equator has 12 vertices and 12 edges, totalling 24 geometrical elements. (360–24=336) elements lie above and below the equator, **168** on either side. As the polyhedron has 180 edges, there are (180–12=**168**) edges above and below its equator, 84 on either side. There are **48** vertices above and below the equator, 24 vertices on either side. Furthermore, there are 60 triangles on each side of the equator. Therefore, the **168** geometrical elements on each side comprise 24 vertices, 84 edges and 60 triangles, that is, 84 edges and (24+60=84) vertices and triangles. Once again, we find the 84:84 division in the superstring structural parameter **168**. It is created by the mirror symmetry of the disdyakis triacontahedron whereby every geometrical element has its mirror image, both 84's referring to the same upper or lower half of the polyhedron. Its inversion symmetry divides the 336 elements into two sets of **168** geometrical elements, one the mirror image of the other.

Figure 20. The correspondence between the first (6+6) enfolded polygons and the I Ching table.

EL CHAI ("Almighty God") with number value **49** prescribes the encoding of the geometry of the disdyakis triacontahedron in the inner Tree of Life because the **49**th prime number is 227, which is the number of yods in the 7 enfolded polygons outside their root edge that are not corners.
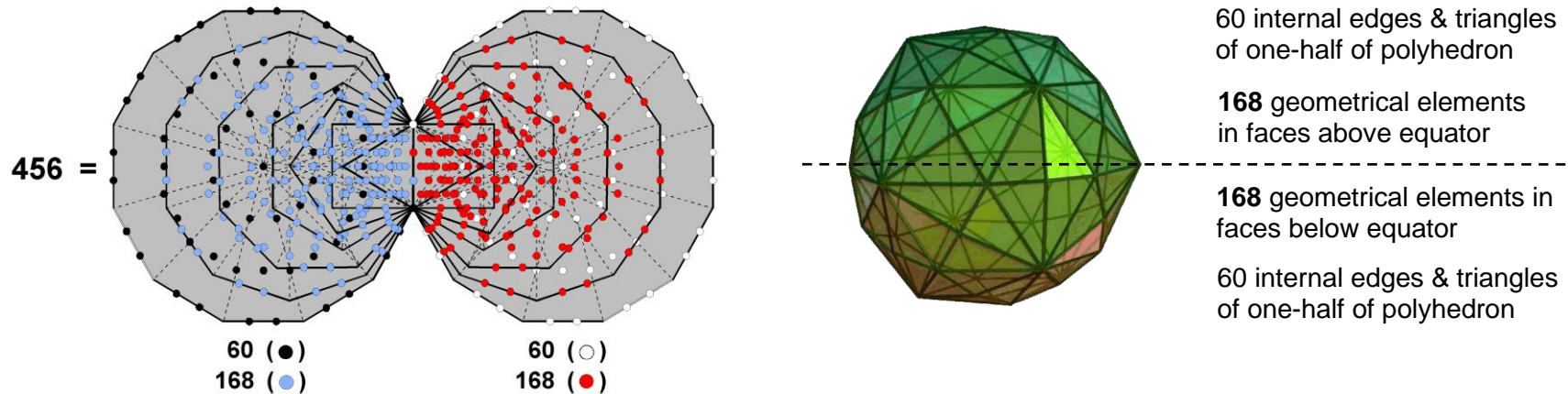

456 =

60 (●)
168 (●)

60 (○)
168 (●)

60 internal edges & triangles of one-half of polyhedron

168 geometrical elements in faces above equator

168 geometrical elements in faces below equator

60 internal edges & triangles of one-half of polyhedron

Figure 21. The 456 yods of the (7+7) polygons that are not their corners outside their root edge symbolize the 456 geometrical elements of the disdyakis triacontahedron not defining its equator. Associated with each set of the first six polygons are **168** yods symbolizing the **168** elements of the uppermost or lowermost three polygons. The remaining 60 yods on each side symbolize the 60 internal elements in each half of the polyhedron.

The number 336 has the beautiful property:

$$336 = 4{\times}84 = 2^2{\times}(1^2 + 3^2 + 5^2 + 7^2) = 2^2 + 6^2 + 10^2 + 14^2.$$

It is an illustration of the Tetrad Principle, which determines parameters of any system designed according to the blueprint of the Tree of Life.

The 12-sided polygon in the equator has 24 vertices and edges on its boundary and 24 edges and triangles in its sectors (12 edges & 12 triangles). Using the results obtained above, the pattern of elements in the disdyakis triacontahedron is set out below:

	vertices	internal edges	external edges	internal triangles	external triangles	Total
First 3 layers:	24	24	84	24	60	**216**
Equator:	(6+6)	(6+6)	(6+6)	(6+6)	0	**48**
Last 3 layers:	24	24	84	24	60	**216**
Total =	60	60	180	60	120	480

The first three layers have **216** internal and external geometrical elements, where **216** is the number value of Geburah. Similarly for the last three layers. The fourth polygon has **48** elements (24 internal, 24 external) surrounding its centre, totalling **49** elements. This shows how the Godname EL CHAI of Yesod with number value **49** prescribes the central polygon. **48** is the number value of *Kokab*, the Mundane Chakra of Hod. The seven layers have 480 geometrical elements made up of 240 elements and their 240 mirror images. Each set of 240 elements consists of **168** external ones (24 vertices, 60 triangles & 84 edges) belonging to the three layers and **72** elements comprising six vertices and six external edges of the fourth polygon, 30 internal edges and 30 internal triangles. This **72**:**168** division corresponds to the division of non-zero roots of the superstring gauge symmetry group E_8. The **72** elements correspond to the **72** non-zero roots of its subgroup E_6 and the **168** elements correspond to the **168** roots of E_8 that are not also roots of E_6. The geometry of the disdyakis triacontahedron therefore encodes the 480 non-zero roots of the $E_8{\times}E_8$ heterotic superstring, the direct product of the two identical groups reflecting the inversion symmetry of the polyhedron, not the distinction between its exterior and interior, which creates a 120:360 division. One-half of the polyhedron encodes the forces between superstrings of ordinary matter, its mirror image half encodes the forces between superstrings of shadow matter.

The **168**:24:24:**168** pattern of yods in the (6+6) polygons of the inner Tree of Life appears in the I Ching table (Fig. 20). The 24 corners outside the root edge of the first six polygons on one side correspond to

24 (●) yods   24 (●) yods

168 vertices & edges in upper half

24 vertices & edges on equator

168 coloured yods   168 coloured yods

24 internal edges & triangles on equator

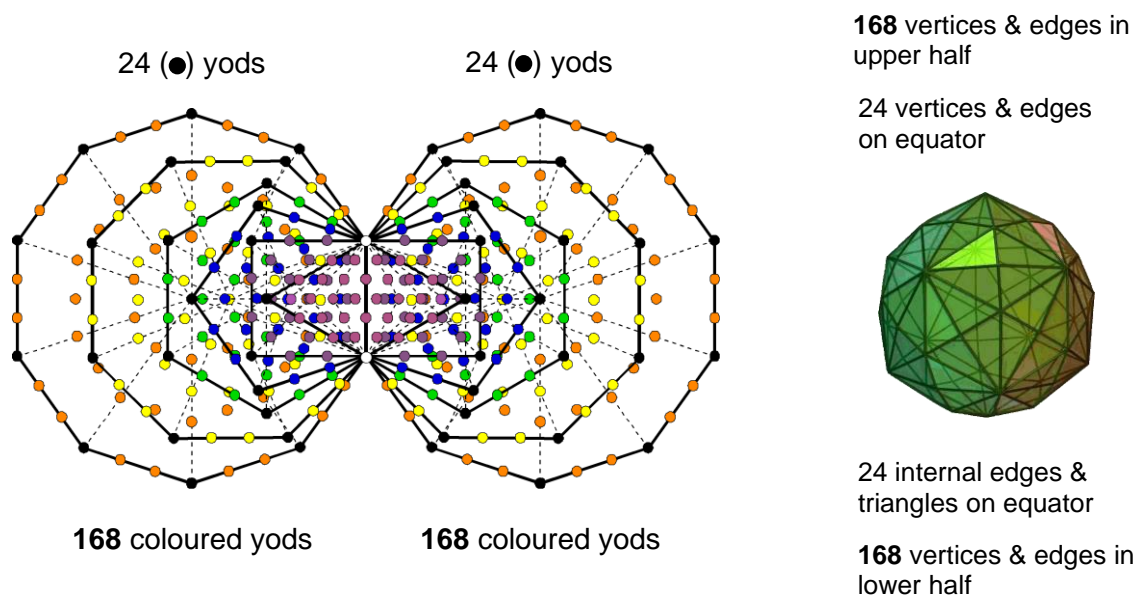**168** vertices & edges in lower half

Figure 22. Correspondence between the first (6+6) polygons and the disdyakis triacontahedron. The two ends of the root edge symbolize two opposite A vertices, the **48** corners of the 12 polygons denote the **48** geometrical elements of the 12-sided polygon in the equator and the **168** coloured yods associated with each set of six polygons denote the **168** geometrical elements in the polyhedral faces on either side of the equator.

8 vertices

10 vertices

6 vertices

12 vertices

6 vertices

10 vertices

8 vertices

A

A

8 (●) vertices
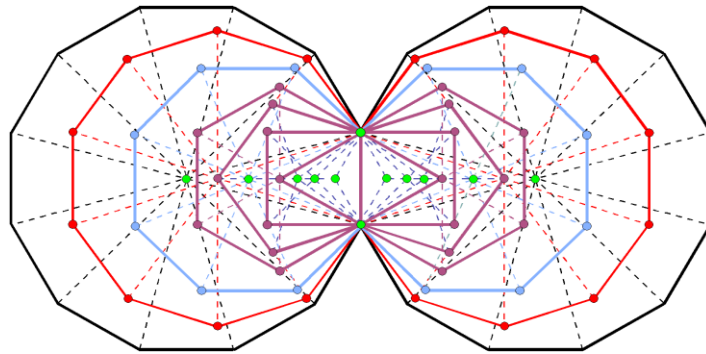
10 (●) vertices

6 (●) vertices

12 (●) vertices
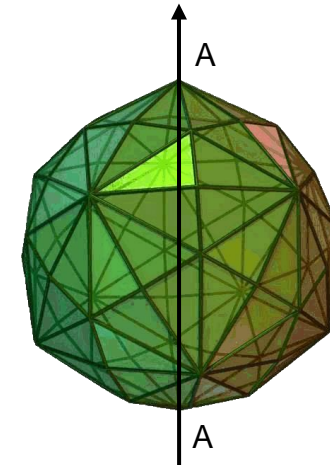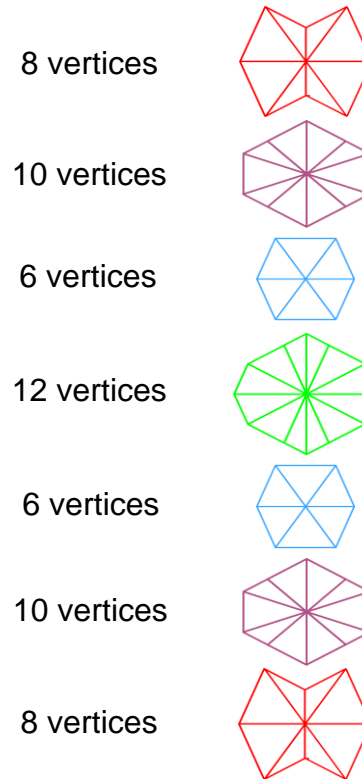
8 (●) vertices

10 (●) vertices

6 (●) vertices

Figure 23. The equivalence of the (24+12+24) vertices of the sectors in the first (6+6) polygons and the (24+12+24) vertices of the seven polygons between opposite A vertices of the disdyakis triacontahedron.

the 24 lines in the upper trigrams of the eight diagonal hexagrams and the 24 corners on the other side correspond to the 24 lines in the lower trigrams. The **168** yods associated with each set of polygons that are not corners correspond to the **168** lines and broken lines in the 28 hexagrams on either side of the diagonal. The fourth polygon in the disdyakis triacontahedron has (6+6) polyhedral edges, (6+6) polyhedral vertices, ((6+6) internal edges and (6+6) internal triangles, that is, eight groups of six geometrical elements. They correspond to the eight diagonal hexagrams, each with six lines/broken lines. The disdyakis triacontahedron is the polyhedral counterpart of the ancient I Ching table of **64** hexagrams, each line or broken line denoting a geometrical element of its faces and internal polygons. The 12 lines and 12 broken lines in the lower trigrams of the diagonal hexagrams correspond to the 12 vertices and 12 edges of the fourth polygon. The **168** lines and broken lines in the 28 off-diagonal hexagrams in the lower half of the table correspond to the **168** geometrical elements in the faces either above or below the equator. The 12 lines and 12 broken lines in the upper trigrams of the diagonal hexagrams correspond to the 12 internal edges and 12 triangular sectors of the 12-sided polygon, whilst the **168** lines and broken lines in the hexagrams above the diagonal correspond to the **168** elements that are the mirror images of the other set. There are (**48+168+168**=384) elements in the faces of the disdyakis triacontahedron and in its central polygon, i.e., lines and broken lines in the I Ching Table. This is the 383rd integer after 1. 383 is the **76**th prime number, showing how YAHWEH ELOHIM, Godname of Tiphareth, prescribes the disdyakis triacontahedron.

The seven polygons in the disdyakis triacontahedron have 60 internal edges and 60 internal triangles (30 edges and 30 triangles per half). Including the **168** external elements in the first three layers and similarly for the last three layers, there are (30+30+**168**=228) elements and their mirror images other than the 12 vertices and 12 edges in the equator of the disdyakis triacontahedron. Including the top and bottom A vertices, there are 229 elements in each half of the disdyakis triacontahedron other than its equator. 229 is the **50**th prime number, showing how ELOHIM with number value **50** prescribes the disdyakis triacontahedron. Figure 21 depicts the inner Tree of Life counterpart of this. There are 228 yods other than external corners of polygons associated with each set of seven polygons (456 yods in total). Outside
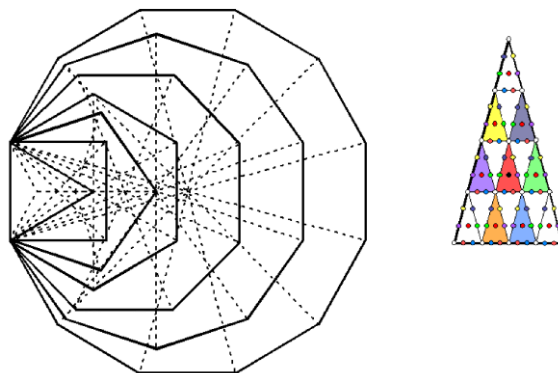

Figure 24. Constructed from 2nd-order tetractyses, the seven regular polygons constituting the inner Tree of Life contains 3360 yods.

the root edge are 227 yods in each set of polygons other than their corners. The Divine Name EL CHAI of Yesod with number value **49** prescribes this set of yods symbolizing the geometrical elements of the seven polygons because 227 is the **49**th prime number and 228 is the 227th integer after 1. Associated with either set of the first six enfolded polygons are **168** yods symbolizing the **168** elements in the three polygons above or below the equator. 60 yods symbolizing the 60 elements inside each half of the disdyakis triacontahedron are associated with each dodecagon. The first six polygons in the Tree of Life and their mirror images, reflected across the root edge, are the counterpart in the disdyakis triacontahedron of the first three layers of polygons and their mirror images, inverted through the centre of the polyhedron.

The (6+6) enfolded polygons constitute a Tree of Life pattern in themselves because they are prescribed by the ten Godnames. The simplest example of this is the fact that they have **50** corners, where **50** is the number value of ELOHIM, Godname of Binah (the complete prescription is given in Article 4).[13] As before, the **168** yods associated with each set of the first six polygons symbolize the **168** geometrical elements in each set of three polygons, whilst the 24 corners on each side of the root edge denote the 24 vertices and edges on the equator and the 24 edges and triangles inside the polyhedron (Fig. 22). These degrees

of freedom mark out the boundary of the new Tree of Life pattern just as the **48** elements create the polygon that completes the sequence of polygons forming half of the disdyakis triacontahedron — a Tree of Life pattern in itself as the remainder of the polyhedron is merely its mirror image

Figure 23 displays the equivalence between the seven layers of polygons with 60 vertices and the first (6+6) regular polygons of the inner Tree of Life. The six vertices of the fourth polygon and their mirror images correspond to the five centres and one end of the root edge on either side of the root edge. The eight vertices of the first polygon correspond to the eight external corners of the decagon. The 10 vertices of the second polygon correspond to the 10 external corners of the first four regular polygons. The six vertices of the third polygon correspond to the six external corners of the octagon. The mirror symmetry of the inner Tree of Life corresponds to the invariance of the disdyakis triacontahedron under inversion.

Figure 18 shows that each curve of the $E_8 \times E_8$ heterotic superstring makes 1680 circularly polarised oscillations as it twists five times around its axis. 336 oscillations occur during one complete revolution, **168** in each half revolution. They correspond to the **168** geometrical elements defining the shape of each set of three polygons. The 24 polyhedral vertices and edges on the equator correspond to the 24 E_8 gauge charges that are spread along each curve.

$E_8 \times E_8$ has 336 non-zero roots that do not belong to the subgroup E_6 in each factor. Their associated gauge charges are sources of 10-dimensional vector gauge fields, which therefore have 3360 components. As discussed in Article 31,[14] there are 28 regular and semi-regular solids defined by the vertices of the disdyakis triacontahedron. When their faces are constructed from tetractyses, they have 3360 hexagonal yods that symbolize the seven Sephiroth of Construction. This amazing property shows
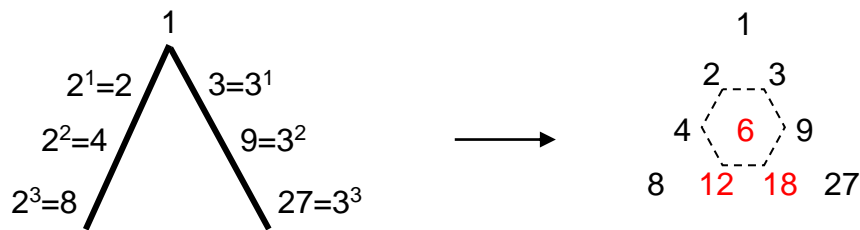

Figure 25. Plato's Lambda and its tetractys interpolation

how the superstring parameter 3360 is present in the geometrical potential of the disdyakis triacontahedron as a number characterizing the forms of all the possible polyhedra within it. It is a Tree of Life parameter because, when the seven enfolded polygons of the inner Tree of Life are constructed not from the tetractys but from its next higher order — the 2nd-order tetractys — it is found to have exactly 3360 yods (Fig. 24). This is the number of circularly polarized oscillations made by all ten curves of the $E_8 \times E_8$ heterotic superstring during one complete revolution. Following through with the analogy with the **168** elements in the 60 faces of the disdyakis triacontahedron above or below its equator, it would be more accurate to associate each set with the inner or outer half of the superstring curves, so that the **168** elements in one set correspond to **168** oscillations in an outer half-revolution and the **168** elements in the other set correspond to the **168** oscillations made as the curve winds in a half revolution in the core of the
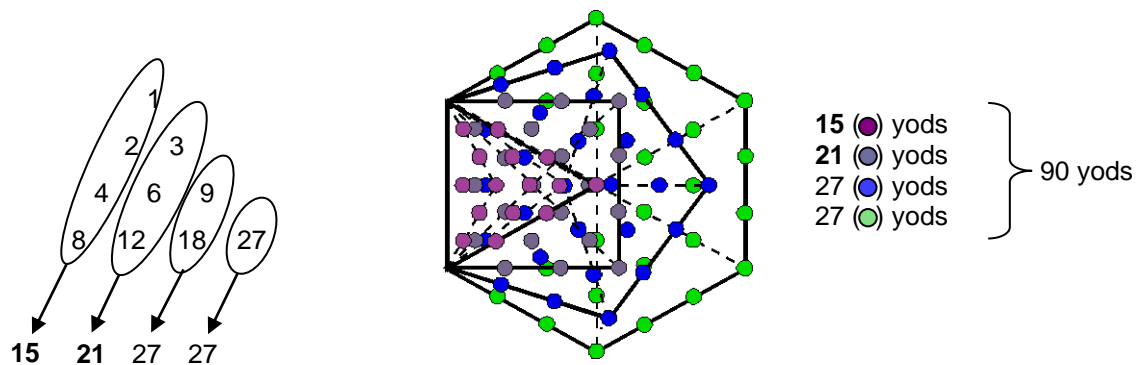

Figure 26. The sums of the four diagonal rows of integers in the Lambda Tetractys are the numbers of yods in the first four polygons of the inner Tree of Life outside their shared root edge.

superstring. It is truly remarkable that the same number **168** should be encoded not only in the faces above or below the equator but also in the fourth polygon forming the equatorial plane as the extra number of yods needed to construct its 12 sectors from three tetractyses!

6. The Platonic Lambda

In his *Timaeus*, Plato described how the Demiurge measured the World Soul, or substance of the spiritual universe, as a strip divided according to the simple proportions of the first three squares of 2 and 3. This is traditionally represented by his 'Lambda,' so-called because of its resemblance to the Greek letter Λ (Fig. 25). These numbers line two sides of a tetractys array of ten numbers from whose relative proportions the scientists and musicians of ancient Greece worked out the frequencies of the notes of the now defunct Pythagorean musical scale.[15] The numbers missing from the Lambda are shown in red in Fig. 25. The sum of the 10 integers is 90 and the sum of the integers 1, 8 and 27 at the corners of the tetractys is **36**. The seven integers at the centre and corners of the hexagon shown in Fig. 25 with dashed edges add up to 54 and the sum of the integers at the corners of the hexagon is **48**.

The archetypal nature of this arrangement of integers was demonstrated in previous articles. It is known today for how it generates the tone ratios of the notes of the Pythagorean musical scale, the ancient father of the modern equal-tempered scale. What is not realised by historians of music and mathematics is that it has connections to the I Ching table,[16] superstrings,[17] the seven musical scales[18] and the nature of the spiritual cosmos.[19] This is because it is the arithmetic representation of the universal Tree of Life pattern underlying these examples of holistic systems. The first four enfolded polygons in the inner form
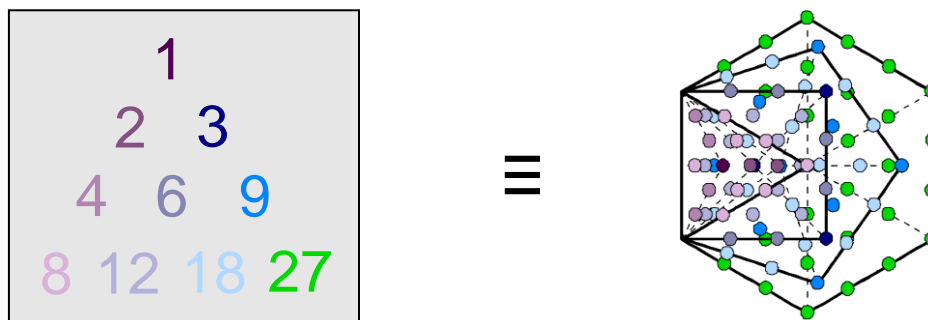

Figure 27. The numbers in the four diagonal rows of the Lambda tetractys denote numbers of yods in the enfolded triangle, square, pentagon & hexagon.

of the Tree of Life have 90 yods outside their shared edge (Fig. 26). This is the sum of the ten numbers of the Lambda Tetractys. This cannot be coincidental because the yod populations of the *individual* polygons are also the sums of diagonal rows of integers in the Lambda! Indeed, the *individual* numbers in each diagonal row can be seen to denote different sets of yods in each polygon (Fig. 27)! They are listed below:

Triangle
1   centre;
2   2 hexagonal yods on horizontal edge of tetractys;
4   4 hexagonal on 2 internal edges;
8   8 yods on edges of triangle & at centres of 3 tetractyses.

Square
3   centre & 2 corners;
6   6 hexagonal yods on sides of square;
12   12 internal hexagonal yods.

Pentagon
9   9 yods at corners & centres of 5 tetractyses;
18   18 hexagonal yods.

Hexagon
27   27 yods.

Article 26[20] showed how the Lambda Tetractys matches the tetractys composition of the disdyakis

triacontahedron. Constructed from tetractyses, its 120 triangular faces divide into (120×3=360) tetractyses. Each of its 180 edges is the side of an internal triangle formed by two vertices and its centre. They are constructed from (180×3=540) tetractyses. The total number of tetractyses forming the disdyakis triacontahedron= 360 + 540 = 900. The following correspondences appear:

Lambda Tetractys		Disdyakis triacontahedron
Sum of 10 integers = 90	⟶	number of tetractys = 900 = 90×10;
Sum of integers at corners = **36**	⟶	number of tetractyses in faces = 360 = **36**×10;
Sum of integers at centre and corners of hexagon = 54	⟶	number of internal tetractyses = 540 = 54×10;
		number of internal tetractyses having edges of
Integer at centre = 6	⟶	rhombic triacontahedron = 60 = 6×10;
Sum of integers at corners of		number of internal tetractyses without edges of
hexagon = **48**	⟶	rhombic triacontahedron = 480 = **48**×10;
Sum of integers surrounding		number of tetractyses without edges of rhombic
centre = 84	⟶	triacontahedron = 840 = 84×10.

The numbers 1^3, 2^3 and 3^3 at the shape-defining corners of the Lambda Tetractys add up to **36**, which is the number of tetractyses whose yod population is the number of tetractyses creating the *shape* of the disdyakis triacontahedron. The seven numbers located at the centre and corners of a hexagon add up to 54 — the number of tetractyses whose yod population is the number of tetractyses *inside* the disdyakis triacontahedron. Notice that the number **36** — the number value of the Godname ELOHA — defines the external shape in both cases and that the number 54 in both cases defines the interior. Moreover, the largest of the seven integers in the hexagon is 18, whilst the number of internal tetractyses creating the 180 edges is 180 = 18×10. Lastly, of the 900 tetractyses in the disdyakis triacontahedron, 60 (=6×10) are tetractyses having edges that are edges of the rhombic triacontahedron, leaving 840 (=84×10) tetractyses other than these. The integer 6 at the centre of the Lambda Tetractys therefore corresponds to the 60 internal tetractyses of the disdyakis triacontahedron sharing edges with the rhombic triacontahedron, whilst the sum (84) of the integers on the boundary of the Lambda Tetractys correspond to the 840 additional tetractyses needed to construct the disdyakis triacontahedron. Six arithmetic properties of the Lambda Tetractys correlate precisely with the geometry of the disdyakis triacontahedron. This cannot be coincidental. Instead, it demonstrates that the Lambda Tetractys and the disdyakis triacontahedron are, respectively, arithmetic and geometric expressions of the *same* universal archetype expressing the holistic nature of God.

Table 7 indicates that the seven polygons formed by the vertices of the disdyakis triacontahedron have 900 yods surrounding their centres, that is, the number of yods in 90 tetractyses. The first four polygons in half the disdyakis triacontahedron have 540 yods surrounding their centres. This is the number of yods in 54 tetractyses. The remaining three polygons have 360 yods surrounding their centres — the number of yods in **36** tetractyses. This 54:**36** division is identical to that displayed by the Lambda Tetractys. 60 yods of the fourth (12-sided) polygon other than its centre lie on its sides and on radial edges of tetractyses — the number of yods in six tetractyses. They correspond to the number 6 at the centre of the Lambda Tetractys. There remain 480 yods surrounding the centres of the first four polygons — the number of yods in **48** tetractyses. They correspond to the sum (**48**) of the integers at the corners of the hexagon in the Lambda Tetractys. There are (900–60=840) yods in the seven polygons not lying on the sides of the fourth polygon and radial edges of tetractyses. They correspond to the sum of the integers surrounding the integer 6 at the centre of the Lambda Tetractys.

As

$$29^2 - 1 = 840 = 3 + 5 + 7 + 9 + \ldots + 57,$$

the second perfect number 28 defines the number 840 because the latter is the sum of the first 28 odd integers after 1. The third perfect number **496** is the dimension of the two possible gauge symmetry groups of superstrings (O(32) and $E_8 \times E_8$). As discussed in Section 5, 840 is also the number of circularly polarised oscillations made during 2½ revolutions in an outer or inner half of each of the ten closed curves making up the heterotic superstring (see Fig. 18). As the seven polygons have 900 yods around their centres, of which 60 are vertices of the disdyakis triacontahedron, 840 yods surround their centres that are not polyhedral vertices. In other words, *840 extra yods are needed to construct the seven polygons from tetractyses.* The appearance of the superstring structural parameter 840 in the polygons formed by the vertices of the disdyakis triacontahedron is astounding and demonstrates the form-creating character of this number. It was encountered earlier in the analysis of the disdyakis triacontahedron, when it was found to have 840 tetractyses not sharing edges with the 60 edges of the rhombic triacontahedron formed by B and C vertices. The seven polygons therefore collectively encode *both* the

form of the superstring *and* its gauge symmetry group E$_8$. As we saw earlier, the central polygon needs **168** extra yods to construct it from tetractyses. It, too, embodies a superstring structural parameter, for each closed curve of the heterotic superstring makes **168** oscillations during half of a complete revolution around its spin axis.

To summarise, the following remarkable correspondences exist between the Lambda Tetractys, the disdyakis triacontahedron and the seven polygons formed by its vertices:

Lambda Tetractys	Disdyakis triacontahedron	7 Layers
Sum of 10 integers = 90	900 (=90×10) tetractyses	900 (=90×10) yods surround 7 centres;
Sum of integers at corners = **36**	360 (=**36**×10) tetractyses in 120 faces	360 (=**36**×10) yods in lowest 3 polygons surround 7 centres;
Sum of integers at centre and corners of hexagon = 54	540 (=54×10) tetractyses inside polyhedron	540 (=54×10) yods in first 4 polygons surround 7 centres;
Integer at centre = 6	60 (=6×10) internal tetractyses with edges of rhombic triacontahedron	60 (=6×10) yods on edges of sectors of 4th polygon surround 7 centres;
Sum of integers at corners of hexagon = **48**	480 (=**48**×10) internal tetractyses do not share edges of rhombic triacontahedron	480 (=**48**×10) other yods in first 4 polygons surround 7 centres;
Sum of integers surrounding centre = 84	840 (84×10) tetractyses do not share edges of rhombic triacontahedron	840 (=84×10) yods surround centres other than those on edges of 4th polygon.

The amazing parallels demonstrate the holistic nature of the disdyakis triacontahedron because the construction of both its external shape and its seven interior polygons from tetractyses conforms to the archetypal Lambda Tetractys array of integers, whose ratios determine the tone ratios of the notes of the Pythagorean musical scale.

The corners of a tetractys symbolize the Supernal Triad and its seven hexagonal yods denote the seven Sephiroth of Construction. The former correspond to the three lowest layers and the latter correspond to the first four layers in half the polyhedron. The yod at the centre of a tetractys symbolizes Malkuth, the lowest Sephirah signifying the material aspect of the Tree of Life, e.g., the physical universe. Its counterpart in the seven layers of the disdyakis triacontahedron is the *form-generating boundary* of the
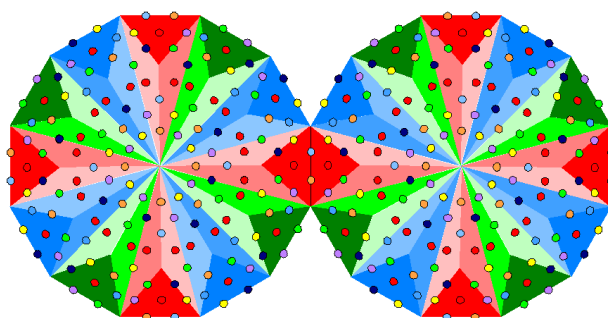
155 yods   **155 yods**

Figure 28. The 12-sided polygon in the middle of the disdyakis triacontahedron has 156 hexagonal yods when its sectors are constructed from three tetractyses. ADONAI MELEKH, Godname of Malkuth with number value 155, prescribes this central polygon because 156 is the **155**th integer after 1. Furthermore, **155** hexagonal yods are associated with each of the two dodecagons belonging to the inner Tree of Life when their sectors are each constructed from three tetractyses.

12 sectors of the polygon in the *central* layer of vertices. This is delineated by 60 yods (**36** yods on the edges of the polygon and 24 yods inside it). Division of these sectors into three tetractyses requires **168** more yods (see Fig. 13). This is the number value of *Cholem Yesodoth*, the Mundane Chakra of Malkuth, confirming the correspondence with Malkuth of the number 6 at the centre of the Lambda Tetractys and the 12-sided polygon as the middle layer of the disdyakis triacontahedron. The 12-sided polygon has 156 hexagonal yods, where 156 is the **155**th integer after 1, showing how ADONAI MELEKH, the Godname of Malkuth, prescribes this polygon. **155** is also the number of hexagonal yods associated with each

dodecagon in the inner Tree of Life when each of its 12 sectors is divided into three tetractyses (Fig. 28). The number 156 is also the sum of the first 12 even integers that can be assigned to the vertices of a 12-sided polygon (Fig. 29). Remarkably, it is the sum of the letter values of all possible combinations of the Divine Name YHVH assigned to Chokmah with number value **26**:

$$Y = 10, H = 5, V = 6$$

Y + H + V	=	**21**
YH + HV + YV + HH	=	52
YHV + HVH + HYH	=	57
YHVH	=	**26**
TOTAL	=	156

This illustrates the mathematical power in a Godname, for the disdyakis triacontahedron is the 26th of the family of 13 Archimedean solids and their duals — the 13 Catalan solids, and so it is chosen by YAHWEH as the mother shape encoding the archetypal pattern of the form of the Tree of Life.

According to Table 7, the number of hexagonal yods in the seven polygons is 780. This is the sum of the first 12 multiples of 10 that can be assigned to its 12 corners:

$$780 = 10 + 20 + 30 + … + 120.$$

The 12-sided dodecagon is the tenth regular polygon and 120 is the sum of the first ten odd integers:

$$120 = 3 + 5 + 7 + … + **21**.$$

The number 10 or Decad was called "All Perfect" by the ancient Pythagoreans. We see that it defines 1. the 120 vertices surrounding the centres of the seven polygons when they are constructed from tetractyses, 2. the 780 hexagonal yods that these tetractyses contain and 3. — through the Lambda
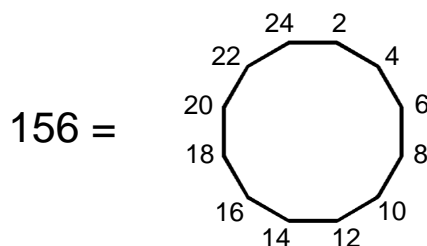

$$156 =$$

Figure 29. The sum of the first 12 even integers assigned to the corners of a 12-sided polygon is 156. This is the number of its hexagonal yods when its sectors are constructed from three tetractyses.

Tetractys, when its numbers are multiplied by 10 — the 900 yods that surround their centres. Truly, 10 is the measure of divine perfection, whose polyhedral manifestation is the disdyakis triacontahedron.

7. Encoding of the superstring parameters 840 &1680

Considered as tetractyses, the 60 sectors of the seven polygons are made up of 360 yods surrounding their seven centres. Four of their 60 edges connect pairs of A vertices (see the 10-sided polygon in Figures 3 & 4). These are not edges of the disdyakis triacontahedron. The seven polygons share 56 sides with it, leaving (180–56=124) unshared edges. Each one has two hexagonal yods. The 124 unshared edges have (124×2=**248**) hexagonal yods. This is the dimension of the superstring gauge symmetry group E_8. There are nine hexagonal yods inside each of the 120 faces of the disdyakis triacontahedron when divided into three tetractyses, i.e., (120×9=1080) hexagonal yods, and 120 vertices, totalling 1200 yods. The polygons have four sides with eight hexagonal yods unshared with the faces of the disdyakis triacontahedron. They have (360–8=352) yods that are either inside them or lying on edges shared with the polyhedron. The faces and internal polygons of the disdyakis triacontahedron have (**248**+1080+120+352=1800) such yods. Of these, (1800–120=1680) are yods that are not vertices inside faces. Other than the vertices of the polyhedron, they are all hexagonal yods. Its mirror symmetry means that every one of the 840 yods in one half of the polyhedron has a yod that is its mirror image. This 840:840 division manifests in the $E_8 \times E_8$ heterotic superstring as the 840 circularly polarized waves in its outer half and the 840 waves in its inner half (see Fig. 18). Divided into three tetractyses, a face of the disdyakis triacontahedron has three internal edges, three triangles and a vertex, making seven geometrical elements. The 120 faces therefore have (7×120=840) elements inside them. The appearance again of the number 840 is further evidence that the disdyakis triacontahedron encodes the oscillatory form of the $E_8 \times E_8$ heterotic superstring.

A polygon with n sides has (**15**n+1) yods, that is, 14n yods other than its internal corners of tetractyses and centre. The first three polygons with 24 edges have (14×24=336) such yods, as have the last three polygons. The fourth polygon with 12 sides has (14×12=**168**) yods other than internal corners and centres. All seven polygons have (336+**168**+336=840) such yods (60 vertices of polygons and 780 hexagonal yods). Once again, we see this structural parameter of the heterotic superstring encoded in
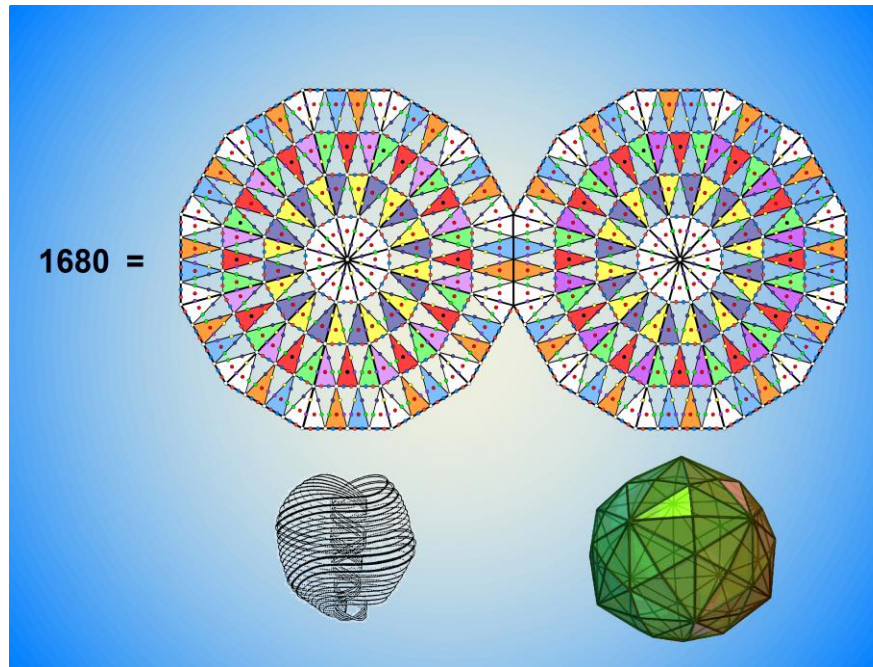

Figure 30. The superstring structural parameter 1680 is encoded in the pair of dodecagons as the number of yods that are outside their shared edge and which surround the centres of their 24 sectors turned into 2nd order tetractyses.

the seven polygons. The 240 vertices and edges of the disdyakis triacontahedron create 840 geometrical elements making up the interior of the faces of the disdyakis triacontahedron and 840 yods other than its vertices making up its seven layers of polygons. Figure 30 shows the counterpart in the inner Tree of Life of this representation of the inner and outer halves of the heterotic superstring. When the 24 sectors of the pair of dodecagons are changed into 2nd-order tetractyses, there are 1680 yods outside their root edge surrounding their centres, 840 yods in each polygon. The two dodecagons correspond to the two halves of the disdyakis triacontahedron. The (12+12=24) sectors signify the 24 gauge bosons of E_8 whose charges are spread along each curve as 1680 circularly polarised waves. They correspond in the disdyakis triacontahedron to the 12 vertices and 12 edges on its equator.

The 840 yods in the seven polygons include the eight hexagonal yods of the four A-A edges of the 10-sided polygon on the second level that are not shared with the polyhedron and which are excluded from the 1680 hexagonal yods and polyhedral vertices calculated above. The seven polygons therefore have (840–8=832) yods made up of 60 polyhedral vertices and 772 hexagonal yods other than the unshared hexagonal yods on the four edges, that is, **833** yods when the topmost A vertex is included. **833** is the number value of *Chaioth ha Qadesh* ("Holy Living Creatures"), the Order of Angels assigned to Kether. Starting from a point, **833** more yods are needed to construct from tetractyses the seven sheets
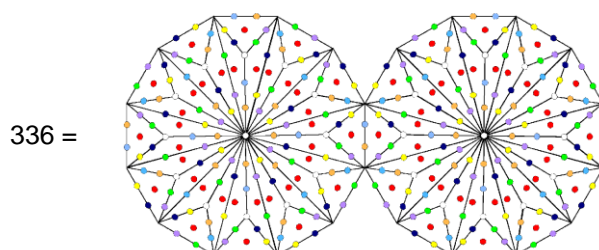

336 =

Figure 31. The last pair of polygons in the inner Tree of Life has 336 yods other than their corners. They symbolize the 336 oscillations made during one revolution of each of the 10 closed curves of the $E_8 \times E_8$ heterotic superstring.

of vertices and the lowest A vertex. Kether is the seed point from which the Tree of Life grows.

The 56 edges of the polygons shared with the disdyakis triacontahedron define 56 tetractyses with (6×56=336) yods surrounding their centres. **168** yods are in the 28 tetractyses in each half of the polyhedron. 84 yods lie on their edges and 84 yods are inside them. The **168**:**168** pattern is the same as that displayed by the edges of the disdyakis triacontahedron above and below its equator. The four shared edges not included in the counting have 24 yods, just as the 12 edges in the 12-sided polygon in the equator have 24 hexagonal yods. In fact, the pair of dodecagons in the inner Tree of Life has 336 yods other than their corners, **168** in each one (Fig. 31).

The 56 sectors of the polygons whose edges are shared with the disdyakis triacontahedron have 840 yods surrounding their centres when they are divided into three tetractyses. This is the same as the number of yods other than corners in all 60 sectors of the seven polygons. In the former case, we have 56×**15** = 840, where **15** is the number of yods per sector; in the latter, we have 60×14=840, where 14 is the number of yods per sector other than corners. Amazingly, both cases encode the same superstring parameter 840. However, as each sector has 10 geometrical elements (two vertices, five edges and three triangles), the 56 sectors have (56×10=560) elements. Each set of 28 sectors has **280** elements. **280** is the number value of *Sandalphon*, Archangel of Malkuth. This is a remarkable conjunction in the same context of the number values of an Archangel and Mundane Chakra of the same Sephirah, for we have just seen that the 28 sectors have **168** yods surrounding the centres of their polygons and **168** is the number value of the Mundane Chakra of Malkuth.

The 56 shared edges have (2×56=112) hexagonal yods. 112 is the number value of *Beni Elohim*, the Order of Angels assigned to Hod. There are also 112 hexagonal yods on the 56 radial edges of the 56 sectors. Including the seven centres, 119 yods line the sectors. The disdyakis triacontahedron has 180 edges and **62** vertices, so that (180×2+**62**=422) yods lie on its edges. (119+422=541) yods line the edges of the polyhedron and the 56 sectors of its polygons. Two of these are the top and bottom A vertices. (541–2=539) yods other than these vertices therefore line the (180+56=236) edges. Amazingly, 539 is the sum of the Godname numbers of the seven Sephiroth of Construction:

$$31 + 36 + 76 + 129 + 153 + 49 + 65 = 539.$$

This demonstrates the constructive power of the Divine Names.

The faces of the disdyakis triacontahedron have the top and bottom A vertices, 124 edges and 120 triangles that it does not share with the seven polygons, that is, **246** geometrical elements. This is the number value of *Gabriel*, the Archangel of Yesod.

There are 127 yods on the internal edges of all 60 sectors of the polygons. There are (422+127=549) yods lining both polyhedral edges and polygonal sectors. Excluding the centres of the three polygons on each side of the equatorial plane, there are **543** boundary yods. This is the number value of EHYEH ASHER EHYER ("I am that I am"), the complete Godname assigned to Kether.

With its faces constructed from three tetractyses, the disdyakis triacontahedron has seven yods lying on edges of tetractyses inside each face. There are (120×7=840) such yods. Together with its **62** vertices
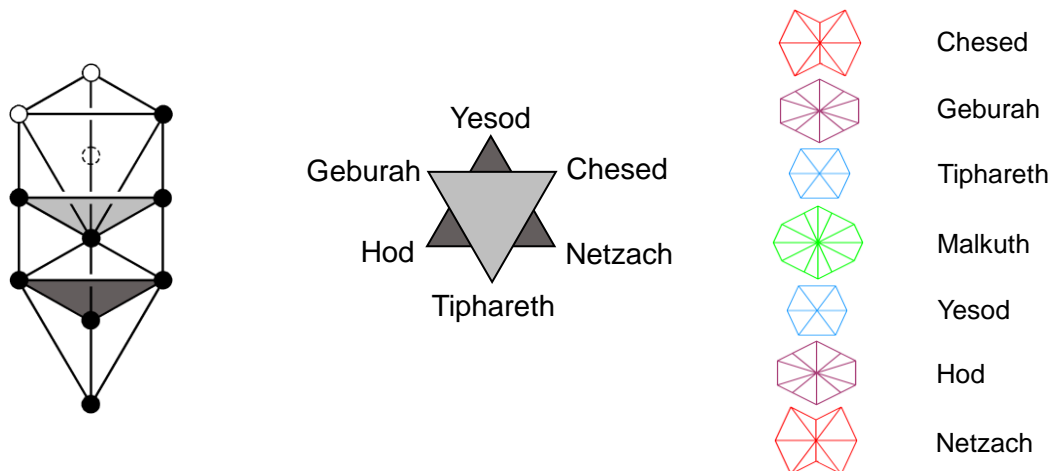

Figure 32. The seven layers of polygons between opposite A vertices correspond to the seven Sephiroth of Construction. The two sets of three layers on either side of the central layer (Malkuth) correspond to the two triads of Sephiroth of Construction above Malkuth (denoted by the differently shaded triangles).

and 360 hexagonal yods lying on its edges, the polyhedron has (840+**62**+360= 1262) boundary yods, that is, **1260** yods other than its two opposite A vertices. This is the number value of *Tarshishim*, the Order of Angels assigned to Netzach.

Of the 541 yods lining the edges of the polyhedron and the 56 sectors with edges shared with it, **67** are vertices of the polygons. (541–**67**=**474**) yods other than vertices lines these edges, creating their shapes. **474** is the number value of Daath ("knowledge"), the non-Sephirah. The number 539 is the sum of 474 and **65**, the number value of ADONAI, Godname of Malkuth. It measures "knowledge of the Lord".

8. The two sets of three polygons

The seven Sephiroth of Construction of the Tree of Life are grouped above Malkuth into two triads: Chesed–Geburah–Tiphareth and Netzach–Hod–Yesod (Fig. 32). In terms of human psychology, they represent, respectively, the transpersonal levels of human consciousness and the incarnated personality (loosely, "Spirit" and "Soul," respectively). The two sets of three polygons above and below the equatorial plane of the disdyakis triacontahedron correspond to this 3:3 pattern, with the Malkuth aspect represented by the 12-sided polygon, the last of the enfolded polygons of the inner Tree of Life, which we saw earlier embodies the number value of *Cholem Yesodoth*, Mundane Chakra of Malkuth (see Fig. 13). A polygon and its mirror image are the counterpart of two Sephiroth of Construction on the same pillar.

9. The Godnames prescribe the seven layers of polygons

Below are listed the ways in which the Godnames of the ten Sephiroth mathematically define properties of the seven layers of vertices in the disdyakis triacontahedron between two diametrically opposite A vertices:

Kether   EHYEH: **21**   disdyakis triacontahedron has **21** layers in 3 perpendicular sets of 7;

        21 A and B vertices in first 4 polygons and their **21** mirror images;

        41 vertices of sectors in first 4 polygons (including top A vertex). 41 = **21**st odd integer after 1;

        5 lowest polygons with **168** geometrical elements have 42 sectors. 42 = **21**st even integer;

        210 (=**21**×10) hexagonal yods in last 5 polygons;

        73 yods in 4th polygon. **73** = **21**st prime number;

EHYEH ASHER EHYEH: **543**   **543** yods other than centres of 1st three polygons on each side of equator lie on the edges of the disdyakis triacontahedron and sectors of 7 polygons.

Chokmah   YAH: **15**   **15** Golden Rectangles in 3 perpendicular sets of 5;

        15 sets of seven layers created by **15** opposite pairs of A vertices;

        15 A vertices in half of disdyakis triacontahedron and their **15** mirror images;

        47 vertices of sectors of lowest 5 polygons. 47 = **15**th prime number;

        120 vertices of 180 tetractyses in 7 polygons surround their centres. 120 = 1 + 2 + 3 + … + **15**;

YAHWEH: **26**   **26** A and B vertices (or A and C vertices) in first 4 polygons;

        51 vertices in first (or last) 3 polygons. 51 = **26**th odd integer.

Binah   ELOHIM: **50**   229 geometrical elements in each half of polyhedron (including top or bottom A vertex). 229 = **50**th prime number;

        51 vertices in first (or last) 3 polygons. 51 = **50**th integer after 1.

Chesed   EL: **31**   **31** vertices in each half of polyhedron;

        127 vertices of sectors of 7 polygons. 127 = **31**st prime number.

Geburah   ELOHA: **36**   360 (=**36**×10) geometrical elements in faces surround axis;

        36 vertices of first 4 polygons;

        36 yods along perimeter of 4th polygon;

        360 (=**36**×10) geometrical elements surround centres of first 4 polygons with **36** vertices/sectors;

        360 (=**36**×10) yods surround centres of 7 polygons;

        360 (=**36**×10) hexagonal yods on boundaries of 108 tetractyses in first 4 polygons.

Tiphareth   YAHWEH ELOHIM: **76**   **76** vertices of tetractyses in first 4 polygons;

<table>
<tr><td></td><td></td><td>384 elements in faces of disdyakis triacontahedron and in central, 12-sided polygon. 384 = 383rd integer after 1. 383 = **76**th prime number.</td></tr>
<tr><td>Netzach</td><td>YAHWEH SABAOTH: **129**</td><td>**129** vertices in 180 tetractyses of 7 polygons, including the top and bottom A vertices;
727 yods on boundaries of 180 tetractyses in 7 polygons. 727 = **129**th prime number.</td></tr>
<tr><td>Hod</td><td>ELOHIM SABAOTH: **153**</td><td>307 yods on boundary of 60 sectors of 7 polygons.   307 = **153**rd integer after 1.</td></tr>
<tr><td>Yesod</td><td>EL CHAI: **49**</td><td>**49** geometrical elements in 4th polygon;
49 yods in first (or last) polygon.</td></tr>
<tr><td></td><td>SHADDAI EL CHAI: **363**</td><td>**363** yods in first (or last) 3 polygons.</td></tr>
<tr><td>Malkuth</td><td>ADONAI: **65**</td><td>**129** vertices in 180 tetractyses of 7 polygons. **129** = **65**th odd integer;</td></tr>
<tr><td></td><td>ADONAI MELEKH: **155**</td><td>907 yods in seven polygons. 907 = **155**th prime number.</td></tr>
</table>

The last property is perhaps the most remarkable of these demonstrations of the power of Godnames to define properties of a Tree of Life system such as the disdyakis triacontahedron. Very aptly, the number value of the Godname of the Sephirah Malkuth signifying the outer form of the Tree of Life defines a prime number that is the very number of yods creating the *complete form* of the seven polygons!

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