ARTICLE 26

How the Seven Musical Scales Relate to the Disdyakis Triacontahedron

by

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Abstract

The disdyakis triacontahedron is unique among the convex polyhedra. Having more faces than any of the Platonic, Archimedean or other Catalan solids, it is the 15th Catalan solid (including enantiomorphs), the 26th member of the family of Archimedean solids and their duals (excluding enantiomorphs) and the 31st polyhedron when Platonic solids included. It is therefore prescribed by the Divine Names YAH (number value 15), YAHWEH (26) and EL (31), as well as by the other Godnames. The musical counterpart of duality is inversion symmetry of the intervals of the seven scales. This analogy implies that the disdyakis triacontahedron and its dual, the truncated icosidodecahedron, are the last and penultimate members of a seven-fold sequence of polyhedra that are counterparts of the musical scales. The disdyakis triacontahedron corresponds to the mathematically most harmonious Pythagorean scale. Confirmation that the analogy is valid is the fact that the seven musical scales show the same 7:11:15 pattern of Pythagorean tone ratios as that displayed by the disdyakis triacontahedron in the sheets of its three types of vertices. The 13 Archimedean solids and their duals correspond to the 13 different intervals and their inverses found in the seven scales. The shape of the disdyakis triacontahedron is created by the 1680 vertices, edges and triangles surrounding its axis of symmetry. They correspond to the 1680 circularly polarized oscillations shaping the helical whorl of the $E_8{\times}E_8$ superstring described by Annie Besant and C.W. Leadbeater. As confirmation, these geometrical elements group into two sets whose numbers are the number values of 'Cholem' and 'Yesodoth,' the words making up the Kabbalistic name for the Mundane Chakra of Malkuth. The disdyakis triacontahedron is therefore the polyhedral representation of this superstring. The number of yods surrounding its axis that lie on boundaries of the tetractyses from which it can be constructed is that of 264 tetractyses. As the inner Tree of Life contains 264 yods when built from tetractyses, this is compelling evidence that the disdyakis triacontahedron is the polyhedral counterpart of the former. The Platonic Lambda quantifies its archetypal properties, which are prescribed by the Godnames of the ten Sephiroth of the Tree of Life.

Table 1. Number values of the ten Sephiroth in the four Worlds.

	SEPHIRAH	GODNAME	ARCHANGEL	ORDER OF ANGELS	MUNDANE CHAKRA
1	Kether (Crown) **620**	EHYEH (I am) **21**	Metatron (Angel of the Presence) **314**	Chaioth ha Qadesh (Holy Living Creatures) **833**	Rashith ha Gilgalim First Swirlings. (Primum Mobile) **636**
2	Chokmah (Wisdom) **73**	YAHWEH, YAH (The Lord) **26**, **15**	Raziel (Herald of the Deity) **248**	Auphanim (Wheels) **187**	Masloth (The Sphere of the Zodiac) **140**
3	Binah (Understanding) **67**	ELOHIM (God in multiplicity) **50**	Tzaphkiel (Contemplation of God) **311**	Aralim (Thrones) **282**	Shabathai Rest. (Saturn) **317**
	Daath (Knowledge) **474**				
4	Chesed (Mercy) **72**	EL (God) **31**	Tzadkiel (Benevolence of God) **62**	Chasmalim (Shining Ones) **428**	Tzadekh Righteousness. (Jupiter) **194**
5	Geburah (Severity) **216**	ELOHA (The Almighty) **36**	Samael (Severity of God) **131**	Seraphim (Fiery Serpents) **630**	Madim Vehement Strength. (Mars) **95**
6	Tiphareth (Beauty) **1081**	YAHWEH ELOHIM (God the Creator) **76**	Michael (Like unto God) **101**	Malachim (Kings) **140**	Shemesh The Solar Light. (Sun) **640**
7	Netzach (Victory) **148**	YAHWEH SABAOTH (Lord of Hosts) **129**	Haniel (Grace of God) **97**	Tarshishim or Elohim **1260**	Nogah Glittering Splendour. (Venus) **64**
8	Hod (Glory) **15**	ELOHIM SABAOTH (God of Hosts) **153**	Raphael (Divine Physician) **311**	Beni Elohim (Sons of God) **112**	Kokab The Stellar Light. (Mercury) **48**
9	Yesod (Foundation) **80**	SHADDAI EL CHAI (Almighty Living God) **49**, **363**	Gabriel (Strong Man of God) **246**	Cherubim (The Strong) **272**	Levanah The Lunar Flame. (Moon) **87**
10	Malkuth (Kingdom) **496**	ADONAI MELEKH (The Lord and King) **65**, **155**	Sandalphon (Manifest Messiah) **280**	Ashim (Souls of Fire) **351**	Cholem Yesodoth The Breaker of the Foundations. The Elements. (Earth) **168**

The Sephiroth exist in the four Worlds of Atziluth, Beriah, Yetzirah and Assiyah. Corresponding to them are the Godnames, Archangels, Order of Angels and Mundane Chakras (their physical manifestation). This table gives their number values obtained by the ancient practice of gematria, wherein a number is assigned to each letter of the alphabet, thereby giving a number value to a word that is the sum of the numbers of its letters.

1. Geometrical composition of the disdyakis triacontahedron

The Tree of Life at the heart of Kabbalah represents Adam Kadmon, or 'Heavenly Man.' What this means is that the Divine archetypes determining the nature of the manifestation in space-time of holistic systems, including humans, find symbolic expression in the sacred geometry of the Tree of Life. Works on Kabbalah traditionally depict the Tree of Life in two dimensions as a set of 16 triangles with their 10 vertices connected by 22 straight lines, with only brief mention that it is actually 3-dimensional.
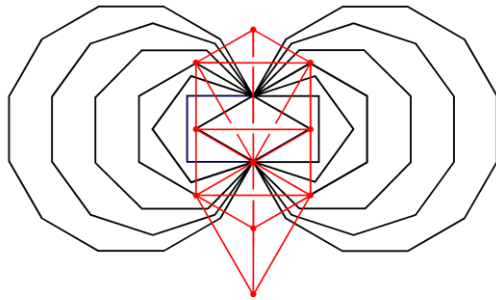

Figure 1. The outer (red) & inner (black) Tree of Life.

However, this is only the outer form of the Tree of Life. It has an inner form as well (Fig. 1), consisting of two similar sets of seven regular polygons enfolded in one another: triangle, square, pentagon, hexagon, octagon, decagon and dodecagon. One set is the mirror image of the other, the mirror lying along the central Pillar of Equilibrium at right angles to the plane containing the 14 polygons. Projected onto the latter, the three Sephiroth on either side of the central pillar coincide with the uppermost and lowest corners of the hexagons and the outer corners of the triangles (indicated by the red dots in Fig. 1). The two endpoints of the 'root edge' shared by both sets of polygons coincide with Daath and Tiphareth. This means that eight of the 70 corners of the polygons are shared with the Tree of Life, leaving **62** unshared corners that represent geometric degrees of freedom that are intrinsic to its inner form alone. **62** is the number value of *Tzadkiel*, the Archangel of Chesed (see Table 1; numbers from this table will be written throughout the article in **boldface**).

It was pointed out in Article 25 that these **62** degrees of freedom correspond to the **62** vertices of the disdyakis triacontahedron — the most visible sign that it is the 3-dimensional realisation of the inner Tree of Life. The two endpoints of the root edge correspond to any pair of diametrically opposite vertices and the 30 unshared corners associated with each half of the inner Tree of Life correspond to the 30 additional vertices forming each half of the polyhedron. The mirror symmetry displayed by the two identical halves of the inner Tree of Life corresponds to the mirror symmetry of the positions of all the vertices. The polyhedron has **62** vertices, 180 edges and 120 triangular faces — a total of 362 geometrical elements. Between any two opposite

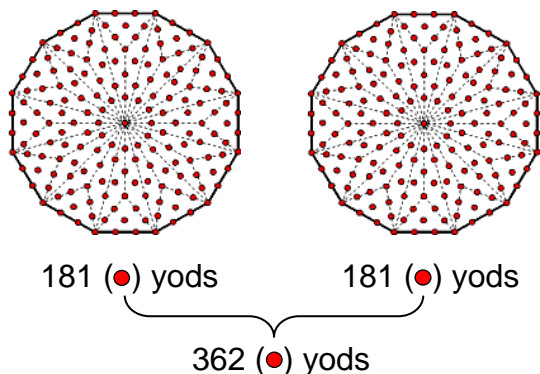

181 (•) yods   181 (•) yods

362 (•) yods

Figure 2. The 181 yods in each dodecagon symbolize the 181 geometrical elements making up each half of the disdyakis triacontahedron. Just as each yod in one dodecagon is the mirror image of a yod in the other dodecagon, so each geometrical element has its inverted counterpart.

vertices are 360 elements, where 360 = **36**×10, showing how the Godname ELOHA assigned to Geburah with number value **36** prescribes this polyhedron. Because of the symmetry of this polyhedron with respect to inversion though its centre, every one of its geometrical elements has its inverted counterpart. It is therefore meaningful to

associate **31** vertices, 90 edges and 60 faces with one 'half' and their inverted mirror images with the other half, i.e., 181 geometric elements constitute each half. This composition is expressed in the pair of dodecagons — the last of the polygons forming the inner Tree of Life — as the 181 yods needed to construct each dodecagon when its sectors are regarded as three tetractyses (Fig. 2). The 60 yods along the sides of the 12 sectors in each dodecagon — that is, the yods *defining their shapes* — symbolize the 60 faces that shape each half of the disdyakis triacontahedron. The 120 yods inside the 12 sectors of each dodecagon denote the 120 vertices and edges making up each half of the polyhedron. The yods at the centres of the two dodecagons denote any two diametrically opposite vertices.

As the disdyakis triacontahedron is the 3-dimensional counterpart of the inner Tree of Life, we should expect to see the number 362 embodied in the latter. Indeed, this is so.

362 =

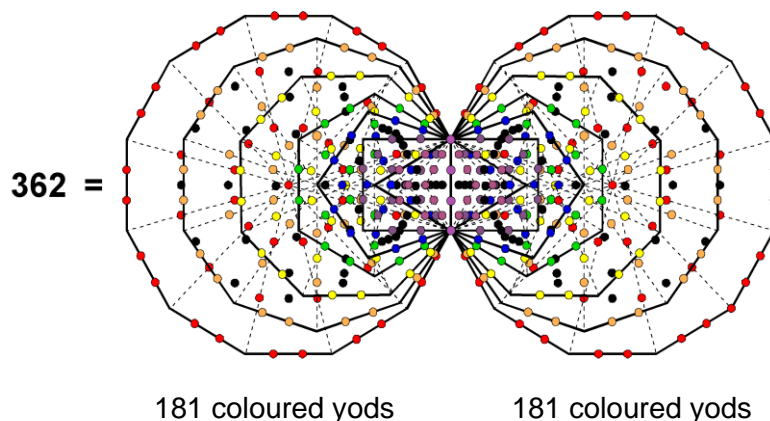181 coloured yods   181 coloured yods

Figure 3. The (7+7) enfolded polygons have 362 yods other than external corners that lie on the 175 edges of their 94 tetractyses. These *shape-determining* degrees of freedom denote the 362 vertices, edges & faces of the disdyakis triacontahedron, which is the 3-dimensional counterpart of these two sets of polygons.

Starting with the two pairs of seven enfolded polygons, the number of yods lying on the 175 edges of their 94 tetractyses is 362 (Fig. 3). The two endpoints of the root edge correspond to any two diametrically opposite vertices of the polyhedron and the 180
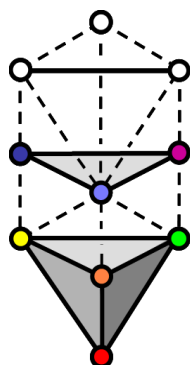

Figure 4. The trunk of the Tree of Life is the sequence of a point, a line, a triangle & a tetrahedron.

yods lining the tetractyses that are associated with each set of polygons correspond to the 180 vertices, edges and triangles forming each half of the polyhedron. Each set of polygons has 47 tetractyses with **87** sides outside the root edge. **87** is the number value of *Levanah* ("The Lunar Flame"), the Mundane Chakra of Yesod. The 94 tetractyses have **80** corners, where **80** is the number value of Yesod.

2. The seven musical scales

The ten Sephiroth of the Tree of Life are traditionally depicted as spheres (circles in its 2-dimensional representation). Geometrically speaking, they are the 10 vertices of 16 triangles with 22 sides (Fig. 4). Just as the tetractys symbolizes the four-fold sequence of point, line (its two endpoints), triangle (its three vertices) and tetrahedron (its four vertices), so the ten Sephiroth mark the same sequence in 3-dimensional space. This is the 'trunk' of the Tree of Life. It comprises 10 vertices, 10 lines, 5 triangles and one tetrahedron (Fig. 5), totalling **26** geometric elements. This illustrates the archetypal quality of the Divine Name YAHWEH with number value **26**: YHVH = 10 + 5 + 6 + 5 = **26**. YAH (YH), its older version with number value **15**, prescribes the tetrahedron — the

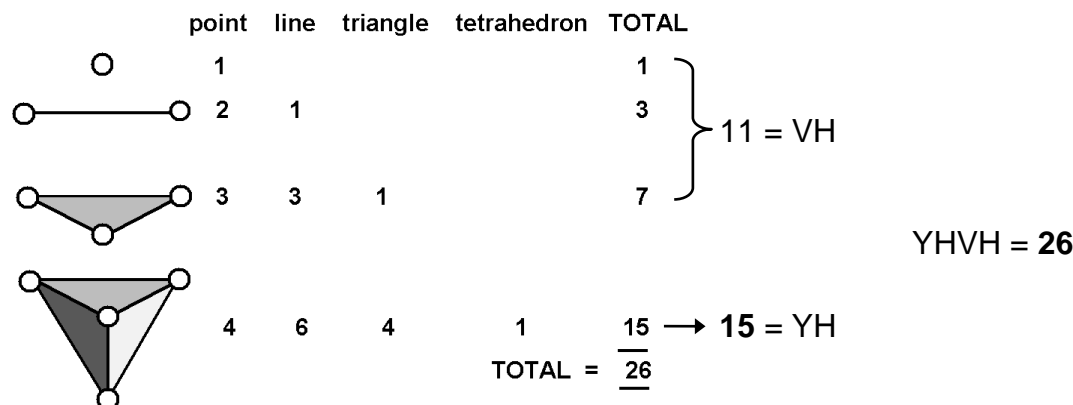|	point	line	triangle	tetrahedron	TOTAL	
○	1				1	
○——○	2	1			3	11 = VH
(triangle)	3	3	1		7	
(tetrahedron)	4	6	4	1	15 → **15** = YH
				TOTAL =	26	

YHVH = **26**

Figure 5. The trunk of the Tree of Life is composed of **26** geometrical elements.

simplest Platonic solid — because it is composed of **15** geometrical elements. Inspection of Fig. 4 shows that there are 12 lines and 11 triangles outside the trunk, i.e., there are (**26**+12+11=**49**) geometrical elements in the Tree of Life. This shows how the
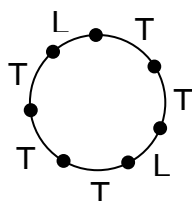

Figure 6. The circle of five tone intervals (T) and two leimmas (L).

Godname EL CHAI assigned to Yesod with number value **49** determines the geometrical composition of the Tree of Life — trunk and branches.

The Pythagorean scale consists of five tone intervals T of 9/8 and two leimmas L of 256/243 (corresponding to, but not identical with, the modern semitone):

T T L T T T L.

By starting successive sequences of intervals on the circumference of the circle displayed in Fig. 6, it is seen that there can be only seven different octave species:

1.	T L T T T L T	Dorian (mode 1)
2.	L T T T L T T	Phrygian (mode 3)
3.	T T T L T T L	Lydian (mode 5)
4.	T T L T T L T	Mixolydian (mode 7)
5.	T L T T L T T	Hypodorian (mode 2)
6.	L T T L T T T	Hypophrygian (mode 4)
7.	T T L T T T L	Hypolydian (mode 6)
	T L T T T L T	Hypomixolydian (mode 8)

The eighth sequence of intervals is the same as mode 1. These are the eight Church musical modes. The three 'hypo' versions of the Dorian, Phrygian and Lydian modes are separated from the latter by a perfect fourth (Fig. 7). The early Catholic Church
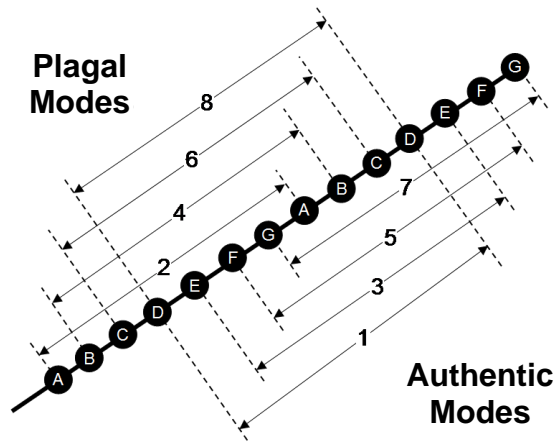

Figure 7. The four Authentic Modes and the four Plagal Modes of Church plainsong.

added an extra mode called the Hypomixolydian (mode 8) to be the counterpart of the Mixolydian. It is the same octave species as the Dorian but has a distinct ethos

Church Musical Modes

S = semitone
T = whole tone

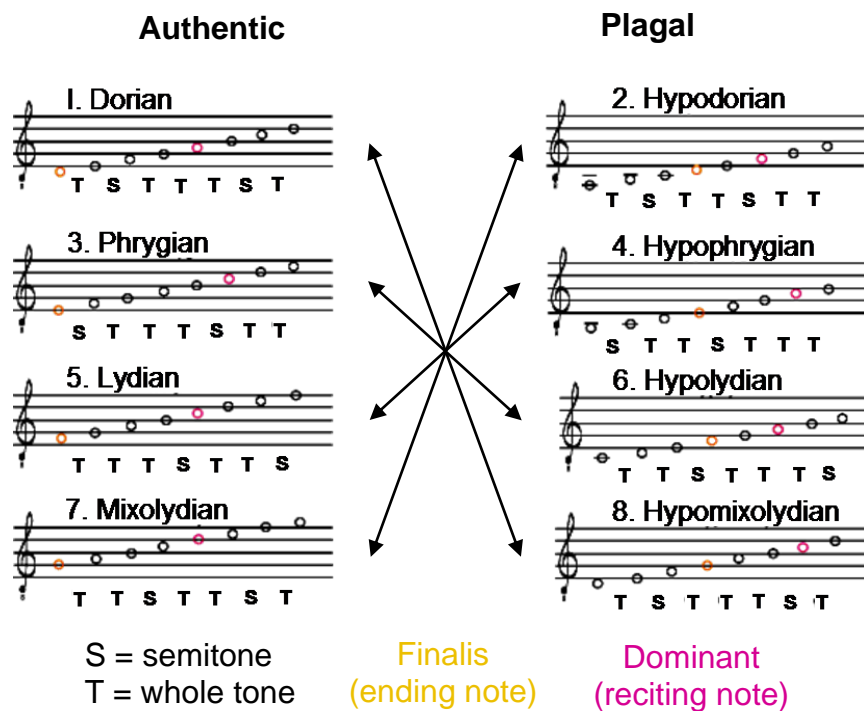Finalis (ending note)

Dominant (reciting note)

Figure 8. The pattern of intervals of each Authentic Mode is the reverse, or mirror image, of a Plagal Mode linked by an arrow.

because its finalis (ending note) and dominant (reciting note) are different.

Figure 8 shows the four Authentic Modes and the four Plagal Modes (the modern notation S for the semitone is used here instead of the leimma L). The double-headed arrows link an Authentic Mode and a Plagal Mode whose patterns of intervals are mirror images of each other. The pattern of intervals of the Dorian mode:

T L T T T L T

is its own mirror image and so the addition of the Hypomixolydian mode is needed to restore the left-right mirror symmetry between the three Authentic Modes and the three Plagal Modes even though it is not distinct from the Dorian mode in terms of its pattern of tone intervals, differing in the positions of the finalis and the dominant.

Table 2 displays the tone ratios of the seven types of musical scale. Cells containing tone ratios that belong to the Pythagorean scale (C scale):

$$\begin{array}{cccccccc} \text{C} & \text{D} & \text{E} & \text{F} & \text{G} & \text{A} & \text{B} & \text{C'} \end{array}$$

$$\text{Tone ratio} = \quad 1 \quad 9/8 \quad (9/8)^2 \quad 4/3 \quad 3/2 \quad 27/16 \quad 243/128 \quad 2$$

are coloured; white cells contain tone ratios that have non-Pythagorean values. The C scale is unique in having only Pythagorean notes (no white cells).

Table 2

Musical scale	Tone ratio							
F scale	1	9/8	81/64	729/512	3/2	27/16	243/128	2
E scale	1	256/243	32/27	4/3	3/2	128/81	16//9	2
D scale	1	9/8	32/27	4/3	3/2	27/16	16/9	2
C scale	1	9/8	81/64	4/3	3/2	27/16	243/128	2
B scale	1	256/243	32/27	4/3	1024/729	128/81	16/9	2
A scale	1	9/8	32/27	4/3	3/2	128/81	16/9	2
G scale	1	9/8	81/64	4/3	3/2	27/16	16/9	2

(Coloured cells indicate Pythagorean notes and white cells indicate non-Pythagorean notes.)

Table 3 shows the number of non-Pythagorean notes in each scale.

Table 3

Musical scale	Number of non-Pythagorean tone ratios
F scale	1
E scale	4
D scale	2
C scale	0
B scale	5
A scale	3
G scale	1

16

Excluding the tonic with tone ratio 1 and the octave with tone ratio 2, the seven musical scales have **26** Pythagorean tone ratios (coloured cells). Their tonal spectrum is therefore prescribed by YAHWEH (YHVH), the values of the letters of the Godname denoting the numbers of tone intervals of one or two types:

26 = 5(9/8) + 6(3/2) + 3(81/64) + 2(243/128) + 6(4/3) + 4(27/16)

$$\begin{array}{cccc} 5 & 6 & 5 & 10 \\ \text{H} & \text{V} & \text{H} & \text{Y} \end{array}$$

11

15

This prescription can be viewed in an alternative way. Table 2 indicates that the

7

distribution of Pythagorean tone ratios other than 1 in the seven scales is:

Mode 1	D scale	4		
Mode 2	A scale	3		
Mode 3	E scale	2	10 = Y	
Mode 4	B scale	1		15
Mode 5	F scale	5 —— H		
Mode 6	C scale	6 —— V		
Mode 7	G scale	5 —— H	11	

Remarkably, the traditional Church numbering of the seven distinct musical modes generates not only the correct letters values of the Hebrew letters of YHVH but also *the order in which they are pronounced and written*! It is, indeed, the Dorian scale (mode 1) — the only scale whose pattern of tone intervals is its own mirror image — that can be considered the source of all the musical scales. This was confirmed in Article 19 by comparing the eight church modes with the eight trigrams of the Chinese I Ching and by proving that there is only one scheme of correspondence between them that permits mirror symmetry in both.[1] This is where the first trigram — the Heaven trigram — corresponds to the Dorian mode and the eighth trigram — the Earth trigram — corresponds to the Hypomixolydian mode — the eighth mode. Fittingly, in view of its mathematical perfection, the seventh and *last* distinct scale generated from the Dorian scale is the Pythagorean scale, or Hypolydian mode (see the bottom of p. 5).

The **49** notes of the seven musical scales below the octave consist of 33 notes with tone ratios of the Pythagorean musical scale (coloured cells) and 16 notes with non-Pythagorean ratios (white cells):

$$33 = 7(1) + \mathbf{26} = \underline{7} + \underline{11} + \underline{\mathbf{15}},$$

where $\underline{7} = 7(1)$, $\underline{11} = 5(9/8) + 6(3/2)$ and $\underline{\mathbf{15}} = 3(81/64) + 2(243/128) + 6(4/3) + 4(27/16)$. Alternatively, the 33 Pythagorean tone ratios may be thought of as made up of the seven tone ratios of the Pythagorean scale (C scale) and their **26** repetitions in the six other scales:

$$\mathbf{26} = 5(3/2) + 6(1) + 5(4/3) + 4(9/8) + 3(27/16) + 2(81/64) + 1(243/128).$$

$$5 \quad\quad 6 \quad\quad 5 \quad\quad\quad\quad 10$$
$$H \quad\quad V \quad\quad H \quad\quad\quad\quad Y$$
$$11 \quad\quad\quad\quad\quad \mathbf{15}$$

As before, the letter values of YHVH denote numbers of different notes in various sets.

Comparing the geometrical composition of the Tree of Life discussed earlier with the composition of notes in the seven musical scales, we find the following correspondence:

Tree of Life	Seven musical scales
trunk { 11 geometrical elements in point, line & triangle	11 notes of modes 6 & 7 or 11 tonics/ perfect 5ths or 11 major 2nds/perfect 5ths
15 geometrical elements in tetrahedron	**15** notes of modes 1–5 or **15** notes other than either tonics/perfect 5ths or major 2nds/perfect 5ths
outside trunk { 23 geometrical elements	23 tonics/non-Pythagorean notes or 23 notes (7 Pythagorean, 16 non-Pythagorean)

8

The **26** notes of the seven scales other than their tonics (or, alternatively, the **26** repetitions of the notes of the Pythagorean scale) correspond to the **26** geometrical elements composing the *trunk* of the Tree of Life. The 23 tonics or non-Pythagorean notes (or, alternatively, the seven notes of the Pythagorean scale and the 16 non-
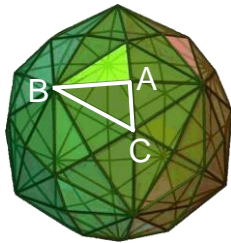

Figure 9. The three types of vertices of the disdyakis triacontahedron.

Pythagorean notes) correspond to the 23 geometrical elements making up the *branches* of the Tree of Life. This parallelism demonstrates the way in which the seven musical scales conform to the archetypal pattern of the Tree of Life.[2] Music based upon these scales is intrinsically sacred. As the Pythagoreans declared, "music is geometry."

3. The 33 sheets of vertices of the disdyakis triacontahedron
The **62** vertices of the disdyakis triacontahedron are made up of the 32 vertices of a rhombic triacontahedron, whose 30 faces are golden rhombi, and 30 raised centres of
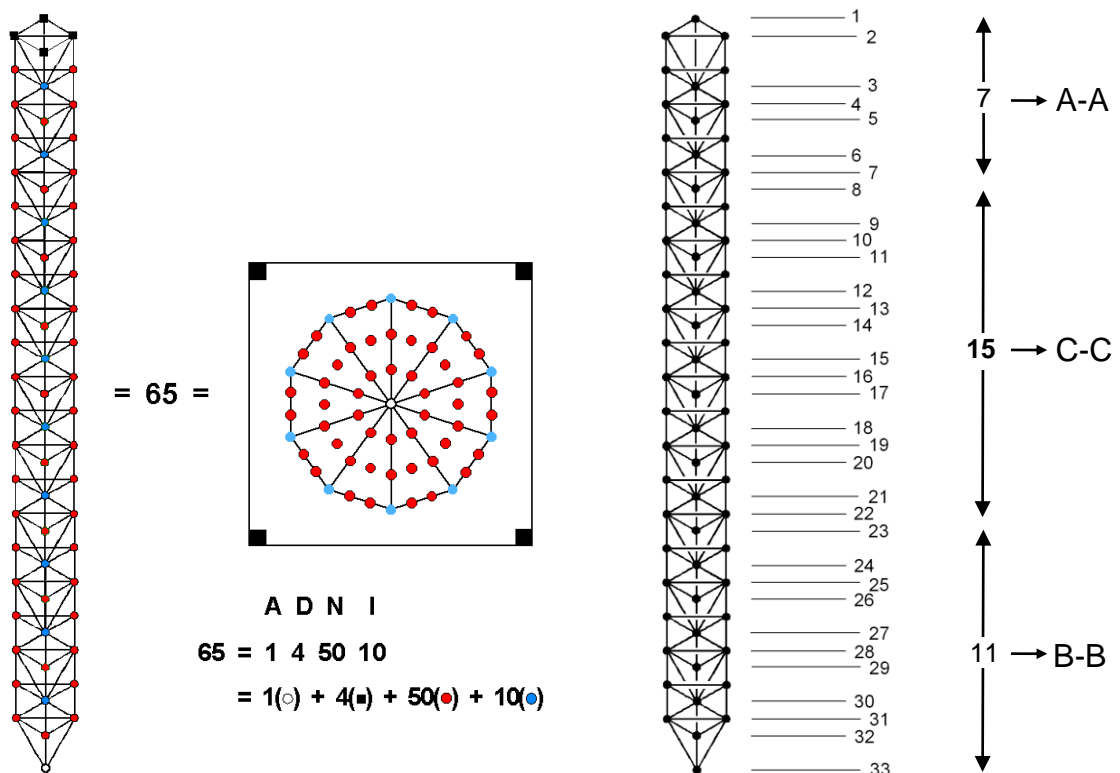

Figure 10. Equivalence of the 10-tree prescribed by the Hebrew Godname ADONAI and its tetractys representation. Ten overlapping Trees of Life have 33 tree levels. The lowest **26** tree levels denote the **26** dimensions of space-time predicted by quantum mechanics for bosonic strings. The lowest 11 tree levels correspond to the 11 sheets of vertices perpendicular to an axis joining two opposite B vertices, the next **15** tree levels correspond to the **15** sheets of vertices perpendicular to a C-C axis and the last seven tree levels correspond to the seven sheets perpendicular to an A-A axis.

the latter. The long diagonals of each golden rhombus are the edges of an icosahedron and its short diagonals are the edges of a dodecahedron. Labelling the vertices of the 120 triangular faces A, B & C (Fig. 9), there are 30 A vertices. 12 B vertices (corners of the icosahedron) & 20 C vertices (corners of the dodecahedron) arranged in seven sheets sandwiched between diametrically opposite A vertices, in 11 sheets perpendicular to a B-B axis and in **15** sheets perpendicular to a C-C axis. The disdyakis triacontahedron has therefore (7+11+**15**=33) generic sheets of vertices. There are more sheets than this because each pair of opposite vertices defines its own set of sheets.

In Articles 22-24,[3] evidence was presented to indicate that the disdyakis triacontahedron is the 3-dimensional realisation of the inner form of the Tree of Life. The 33 tree levels of 10 overlapping Trees of Life correspond to the 33 sheets of vertices (Fig. 10). The **62** vertices of the disdyakis triacontahedron correspond to the **62** of their **65** Sephirothic emanations whose projections onto the plane containing the 14 polygons enfolded in each tree coincide with corners of these polygons, that is, points shared between the inner and outer forms of each tree. The lowest **26** tree levels, which are spanned by **50** Sephirothic emanations, may be physically interpreted[4] as the **26** dimensions of space-time predicted for spinless strings by quantum mechanics (this demonstrates how ELOHIM, the Godname of Binah with number value **50**, prescribes the dimensionality of space-time). They correspond to the **26** sheets of vertices perpendicular to either a B-B or a C-C axis. The highest seven tree levels of ten overlapping Trees of Life correspond to the seven sheets of vertices perpendicular to an A-A axis. The lowest 11 tree levels denote the 11 dimensions of space-time predicted by supergravity theory; they correspond to the 11 sheets of vertices. The next **15** tree levels signify the **15** higher dimensions whose compactification generates superstrings and their unified forces described by O(32) and $E_8 \times E_8$; they correspond to the **15** sheets of vertices. The compactification of 11-dimensional space-time into $S_1 \times S_6 \times M_4$, where S_1 is a 1-dimensional space (line segment or circle), S_6 is the 6-dimensional, compactified space predicted by superstring theory and M_4 is Einsteinian, 4-dimensional space-time, conforms to the letter values of V and H in the well-known Godname YHVH:
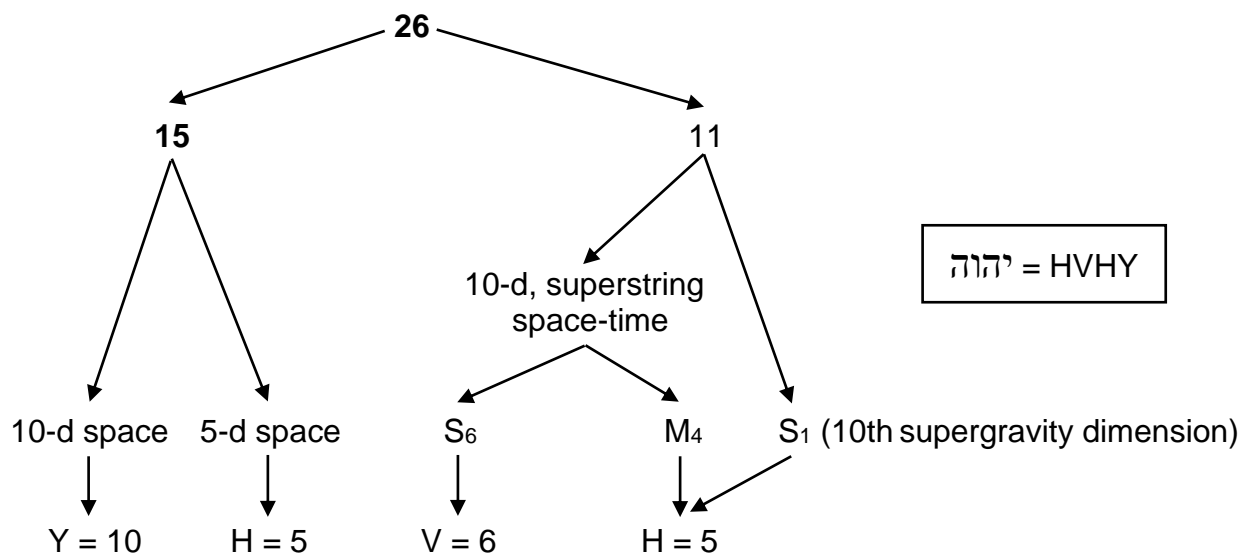
4. The Archimedean & Catalan solids

Let us now compare the 33 generic sheets of vertices in the disdyakis triacontahedron with the 33 notes of the seven musical scales (seven tonics and **26** notes below the octave) that have Pythagorean tone ratios. The following correspondences exist:

A-A sheet of vertices $\longrightarrow$ tonic with tone ratio of 1;
B-B sheet of vertices $\longrightarrow$ major 2nd (9/8) or perfect 5th (3/2);
C-C sheet of vertices $\longrightarrow$ major 3rd (81/64), perfect 4th (4/3), major 6th (27/16) or major 7th (243/128).

If, as discussed earlier, the **26** notes are the repetitions of the Pythagorean notes in the six other scales, an alternative set of correspondences is:

A-A sheet of vertices $\longrightarrow$ Pythagorean note;
B-B sheet of vertices $\longrightarrow$ tonic/perfect 5th;
C-C sheet of vertices $\longrightarrow$ major 2nd/major 3rd/major 6th/major 7th.

If we follow the previously discussed scheme based upon the natural ordering of successive sequences of intervals, starting with the Dorian, the correspondences are:

A-A sheet of vertices $\longrightarrow$ tonic with tone ratio of 1;
B-B sheet of vertices $\longrightarrow$ note of modes 6 and 7;
C-C sheet of vertices $\longrightarrow$ note of modes 1–5.

Whatever the scheme of correspondence, we see that the disdyakis triacontahedron is the geometrical counterpart of the seven octave species. This polyhedron is the spatial realisation of the Tree of Life and the seven species of octaves are its musical manifestation.

Article 14[5] showed that the seven types of musical scale conform to the archetypal pattern of the Tree of Life blueprint. The fact that their notes group in the same 11:**15** pattern that both the geometrical elements of the trunk of the Tree of Life and the B-B and C-C sheets of vertices do is further, strong evidence that the disdyakis triacontahedron is the 3-dimensional counterpart of the Tree of Life. **26**-dimensional space-time mapped by the lowest **26** tree levels of the map of Reality called the 'Cosmic Tree of Life,'[6] which consists of 91 overlapping Trees of Life.

The seven octave species that originate with the Dorian mode (D scale) and realise their perfection in the Hypolydian mode (the Pythagorean, or C scale) are the musical counterpart of seven polyhedra (five regular, two semiregular) that, starting from the tetrahedron, and ordered according to increasing number of corners, completes their perfection with the disdyakis triacontahedron:

tetrahedron is dual to tetrahedron $\longrightarrow$ Dorian mode is reverse of Hypomixolydian mode

octahedron is dual to cube $\longrightarrow$ Mixolydian mode is reverse of Hypodorian mode

icosahedron is dual to dodecahedron $\longrightarrow$ Lydian mode is reverse of Hypophrygian mode

        ?   is dual to   ? $\longrightarrow$ Phrygian mode is reverse of Hypolydian mode

Just as the tetrahedron is self-dual, so the Dorian mode has a pattern of intervals that is the mirror image of itself. It is therefore the same as the pattern of the Hypomixolydian mode, the eighth mode, which both completes and recommences the cyclic sequence of intervals that started with the Dorian. Just as the octahedron and cube are dual to each other, so the Mixolydian and Hypodorian modes possess patterns of intervals that are mirror images of each other. Just as the icosahedron and dodecahedron are dual to each other, so the Lydian and Hypophrygian modes are mirror images of each other. The Phrygian musical mode is the reverse of the Hypolydian mode. What, however, is the pair of non-Platonic polyhedra that are dual to each other and are the counterpart of this fourth pair of modes? As the 32 corners of a rhombic triacontahedron consist of the 12 corners of an icosahedron and the 20 corners of a dodecagon, it might seem natural to think that the next member of the seven-fold sequence of polyhedra after the five

Platonic solids is this one and that the disdyakis triacontahedron is the seventh because it is just the rhombic triacontahedron with the centres of its faces raised. However, it is *not* its dual. If the analogy between polyhedral duality and modal symmetry is to be taken seriously so that the alternation between a polyhedron and its dual persists *beyond* the five Platonic solids and applies as well to the last pair of polyhedra making up the seven polyhedra, it means that either one or both of these polyhedra cannot complete the seven-fold sequence of polyhedra that mirrors the seven musical modes. It might be expected that the disdyakis triacontahedron should, indeed, be the seventh polyhedron because of its status as the polyhedral form of the Tree of Life. But a rigorous mathematical discussion cannot make this assumption, however, persuasively it is suggested by the evidence presented in Articles 22–25. Instead, it must explain why, in strictly mathematical terms, this polyhedron *does* possess a unique significance. After all, the icosidodecahedron is an Archimedean solid with only 32 faces. But it, too, acquires 120 triangular faces when the centres of these faces are raised above them
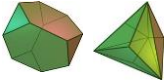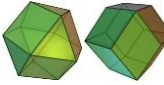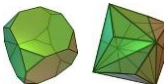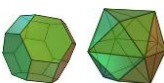
Table 4
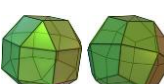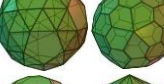
Archimedean solid	Types of faces	Number of triangular faces
cuboctahedron	8 triangles + 6 squares	8×3 + 6×4 = **48**
icosidodecahedron	20 triangles + 12 pentagons	20×3 + 12×5 = 120
truncated tetrahedron	4 triangles + 4 hexagons	4×3 + 4×6 = **36**
truncated cube	8 triangles + 6 octagons	8×3 +6×8 = **72**
truncated octahedron	6 squares + 8 hexagons	6×4 + 8×6 = **72**
truncated dodecahedron	20 triangles +12 decagons	20×3 + 12×10 = 180
truncated icosahedron	12 pentagons + 20 hexagons	12×5 + 20×6 =180
rhombicuboctahedron	8 triangles + 18 squares	8×3 + 18×4 = 96
truncated cuboctahedron	12 squares + 8 hexagons + 6 octagons	12×4 + 12×4 + 12×4 = 144
rhombicosidodecahedron	20 triangles + 30 squares + 12 pentagons	20×3 + 30×4 + 12×5 = 240
truncated icosidodecahedron	30 squares + 20 hexagons + 12 decagons	30×4 + 20×6 + 12×10 = 360
snub cube	32 triangles + 6 squares	32×3 + 6×4 = 120
snub dodecahedron	**80** triangles + 12 pentagons	**80**×3 + 12×5 = 300

(Table 4). Moreover, the transformed polyhedron has **62** vertices (30 old vertices and 32 new ones), which is the same number as the disdyakis triacontahedron. It has 480 hexagonal yods[7] — just as many as the disdyakis triacontahedron. It has (60 + 20×3 + 12×5 = 180) edges — the same number, which means that it, too, has (**62**+180+120=362) vertices, edges and faces, a number that, as Section 1 revealed, is embodied in both the inner Tree of Life and in the last of its regular polygons. Finally, it has (180×2 + **62** = 422) yods along its sides — the same as the disdyakis triacontahedron. It thus shares some of the amazing properties of this polyhedron and their correlations with the Tree of Life that were discussed in Articles 22–25. The only other Archimedean solid with **62** corners and 120 faces when its faces are raised is the snub cube It, too, has 480 hexagonal yods, 362 vertices, edges and triangular faces and 422 yods along its sides. The lack of uniqueness in such properties of the disdyakis triacontahedron means that they, alone, cannot be regarded as evidence that makes it stand out from all other polyhedra. What *does* pick it out is the following: there are 13 Archimedean solids (**15**, including the two solids that possess enantiomorphic

Table 5. Properties of the Archimedean & Catalan solids.

(listed in order of increasing number of vertices)

(listed in order of increasing number of faces)

F	E	C	Archimedean solid		Catalan solid	F	E	C
8	18	12	truncated tetrahedron		triakis tetrahedron	12	18	8
14	24	12	cuboctahedron		rhombic dodecahedron	12	24	14
14	**36**	24	truncated cube		triakis octahedron	24	**36**	14
14	**36**	24	truncated octahedron		tetrakis hexahedron	24	**36**	14
26	48	24	rhombicuboctahedron		deltoidal icositetrahedron	24	**48**	**26**
38	60	24	snub cube		pentagonal icositetrahedron	24	60	38
38	60	24	snub cube (chiral partner)		pentagonal icositetrahedron (chiral partner)	24	60	38
32	60	30	icosidodecahedron		rhombic triacontahedron	30	60	32
26	**72**	**48**	truncated cuboctahedron		disdyakis dodecahedron	**48**	**72**	**26**
32	90	60	truncated icosahedron		triakis icosahedron	60	90	32
32	90	60	truncated dodecahedron		pentakis dodecahedron	60	90	32
62	120	60	rhombicosidodecahedron		deltoidal hexacontahedron	60	120	**62**
92	150	60	snub dodecahedron		pentagonal hexacontahedron	60	150	92
92	150	60	snub dodecahedron (chiral partner)		pentagonal hexacontahedron (chiral partner)	60	150	92
62	180	120	truncated icosidodecahedron		disdyakis triacontahedron	120	180	**62**

C = number of vertices
E = number of edges
F = number of faces

counterparts). This demonstrates how the Godname YAH of Chokmah with number value **15** prescribes the family of solids that have at least two types of regular polygons as their faces. The musical counterpart of this is the fact that the **15** notes of two octaves of the Pythagorean scale are needed to generate the eight Church modes:

```
        C scale              T   T   L   T   T   T   L
        B scale          L   T   T   L   T   T   T
        A scale      T   L   T   T   L   T   T
        G scale      T   T   L   T   T   L   T
        F scale  T   T   T   L   T   T   L
        E scale  L   T   T   T   L   T   T
        D scale  T   L   T   T   T   L   T
        C scale  T   T   L   T   T   T   L   T   T   L   T   T   T   L
                 C   D   E   F   G   A   B   C'  D'  E'  F'  G'  A'  B'  C''
```

The duals of the Archimedean solids are the **15** Catalan solids shown in Table 5. When listed in order of increasing number of corners, starting with the truncated tetrahedron with 12 corners and eight faces, the Archimedean solid with most corners is the truncated icosidodecahedron with 120 corners and **62** faces. When listed in order of increasing number of faces, starting with the triakis tetrahedron with 12 faces and eight

Figure 11. The disdyakis triacontahedron completes the tetractys pattern of the 10-fold generation of this perfect polyhedron from the mathematical point. The tonic (1), octave (2), perfect fifth (3/2) and the seven scales are its musical counterpart.

corners, the Catalan solid with the most faces is the disdyakis triacontahedron with **62** corners and 120 faces. This is the disdyakis triacontahedron! There are 13 Catalan solids apart from the enantiomorphic versions of the pentagonal icositetrahedron and the pentagonal hexacontahedron. The disdyakis triacontahedron is therefore the **26**th and *last* of the two families of Archimedean and Catalan solids. The Divine Name YAHWEH with number value **26** prescribes these two families of solids and determines the disdyakis triacontahedron as the solid with the most faces, whilst its dual, the truncated icosidodecahedron, is the solid with the most corners. These most complex of the Archimedean and Catalan solids are the sixth and seventh members of the seven-fold sequence of polyhedra that mirror the seven musical scales:

tetrahedron is dual to tetrahedron ⟷ Dorian mode is inverse of Hypomixolydian mode
octahedron is dual to cube ⟷ Mixolydian mode is inverse of Hypodorian mode
icosahedron is dual to dodecahedron ⟷ Lydian mode is inverse of Hypophrygian mode

truncated icosidodecahedron is dual to disdyakis triacontahedron ⟷ Phrygian mode is inverse of Hypolydian mode

The disdyakis triacontahedron corresponds to the mathematically perfect Pythagorean scale (Hypolydian mode). It completes the tetractys pattern representing the generation from the mathematical point of the perfect solid (Fig. 11).

The centre and corners of a polyhedron with E edges define E internal triangles. The (6+4×3=18) tetractyses inside and on the surface of a tetrahedron have 70 yods surrounding its centre.[8] In other words, starting from this point, 70 yods are needed to construct the tetrahedron from tetr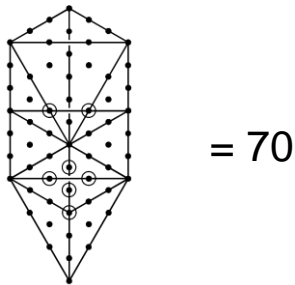actyses. This is the meaning of the 70 yods in the Tree of Life constructed from 16 tetractyses (Fig. 12). As the simplest Platonic solid, it is the first in the archetypal sequence of seven solids leading to the 3-dimensional realisation of the Tree of Life — the disdyakis triacontahedron. As the 'seed' that grows into the flower of the latter, it *must* possess these 70 degrees of freedom. **62** of the yods are hexagonal yods (**48** in the four faces, 14 internal). They symbolize *in potentia* the **62** faces of the fully-grown, polyhedral Tree of Life. If, instead, the internal triangles formed by the vertices and centre are divided into three tetractyses, the tetrahedron has 124 surrounding its centre.[9] In other words, 120 yods are necessary, starting from a tetrahedron, to construct it from the 30 tetractyses. These symbolize *in potentia* the 120 faces of the disdyakis triacontahedron.

$= 70$

Figure 12. The Tree of Life contains 70 yods.

As we saw in Section 1, the disdyakis triacontahedron has 362 geometrical elements. Including its centre, there are **363** elements. This shows how SHADDAI EL CHAI, the
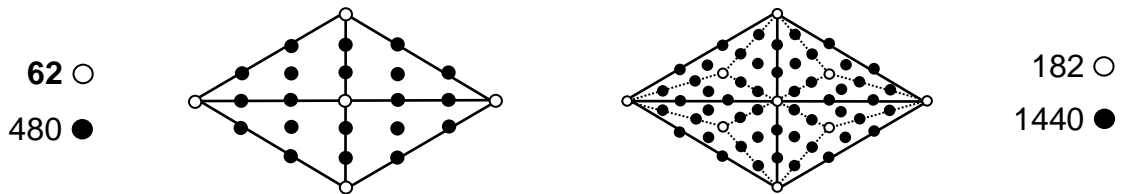
62 ○
480 ●

182 ○
1440 ●

Figure 13. As single tetractyses, the 120 triangular faces of the disdyakis triacontahedron create 1680 geometrical elements around its axis. As three tetractyses, they generate 2400 elements. 720 elements are added by the conversion.

complete Godname of Yesod, prescribes this polyhedron. With each internal triangle a single tetractys, there are 180 internal tetractyses with **62** internal sides ending on the

major whorl

whorl

UPA/superstring

The helical whorl has 1680 coils.
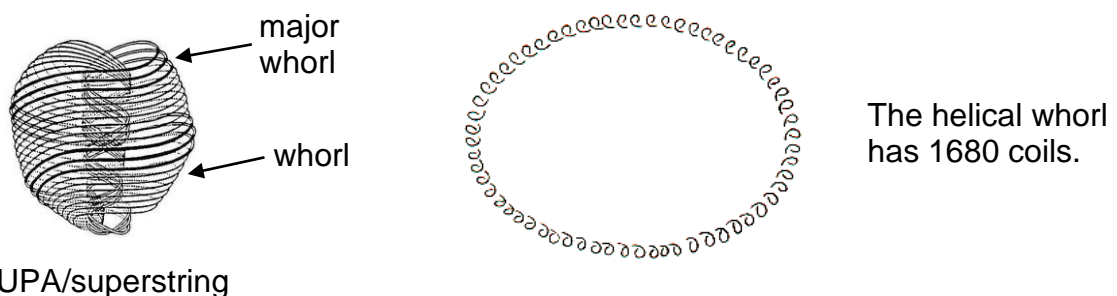
Annie Besant   C.W. Leadbeater

Figure 14. The Theosophists Annie Besant and C.W. Leadbeater observed subatomic particles with the aid of a yogic siddhi called 'anima.' The basic particle of matter (identified by the author as the heterotic superstring constituent of up and down quarks) consists of ten closed curves, or 'whorls.' Each one is a helical coil with 1680 turns.

vertices. The number of geometrical elements inside the polyhedron and on its faces = 362 + 180 + **62** + 1 = 605. Turning each internal triangle into three tetractyses generates three sides, two triangles and one vertex for each one, creating in total (605 + 180×6 = 1685) elements. Converting the triangular faces into three tetractyses adds (6×120=720) elements, totalling 2405. Constructed both internally and externally from tetractyses, the polyhedron has a central axis made up of its centre, two diametrically opposite vertices and the lines joining them to this centre, that is, five geometrical elements. The disdyakis triacontahedron therefore has 2400 (=240×10) elements surrounding its central axis, of which 1680 (=**168**×10) elements comprise it with single tetractyses for its 120 faces (Fig. 13). This property is truly remarkable evidence (indeed, proof) that the disdyakis triacontahedron does, indeed, represent the Tree of Life blueprint. This is because the basic unit of matter described 110 years ago by the Theosophists Annie Besant and C.W. Leadbeater, using a yogic siddhi, consists of ten closed curves, or "whorls," each a helix with 1680 turns (Fig. 14). This fundamental superstring structural parameter is the *number of geometrical elements in the disdyakis triacontahedron that surround any axis joining two diametrically opposite vertices*.

The division:

$$2400 = 240 \; \therefore = \; \textbf{72} \; \therefore + \; \textbf{168} \; \therefore$$

was encountered in many previous articles. Its significance in the context of the superstring gauge symmetry group E_8 is as follows: The roots of the E_8 algebra can be described in terms of eight orthonormal unit vectors $\{u_i\}$. Eight 'zero roots' correspond to points at the centre of the root diagram and 240 'nonzero roots' all have length $\sqrt{2}$. They are given by

$$\pm u_i \pm u_j \qquad\qquad (i, j = 1, 2, \dots 8)$$

and

$$\tfrac{1}{2}(\pm u_1, \pm u_2, \dots \pm u_8) \qquad\qquad \text{(even number of +'s)}$$

Their explicit forms as 8-tuples and their numbers are listed below:

(1, 1, 0, 0, 0, 0, 0, 0) and all permutations. Number = $\begin{bmatrix} 8 \\ 2 \end{bmatrix}$ = 28;

(-1, -1, 0, 0, 0, 0, 0, 0) and all permutations. Number = $\begin{bmatrix} 8 \\ 2 \end{bmatrix}$ = 28;

(1, -1, 0, 0, 0, 0, 0, 0) and all permutations. Number = $2\times\begin{bmatrix} 8 \\ 2 \end{bmatrix}$ = 56;

(-½, -½, ½, ½, ½, ½, ½, ½) and all permutations. Number = $\begin{bmatrix} 8 \\ 2 \end{bmatrix}$ = 28;

(-½, -½, -½, -½, -½, -½, ½, ½) and all permutations. Number = $\begin{bmatrix} 8 \\ 2 \end{bmatrix}$ = 28;

(-½, -½, -½, -½, ½, ½, ½, ½) and all permutations. Number = $\begin{bmatrix} 8 \\ 4 \end{bmatrix}$ = 70;

(½, ½, ½, ½, ½, ½, ½, ½). Number = 1;

(-½, -½, -½, -½, -½, -½, -½, -½). Number = 1.

The 240 non-zero roots of E_8 comprise **168** made up of four sets of 28 and one set of 56, one set of 70 and two single ones. E_6, an exceptional subgroup of E_8, has **72** non-zero roots. The 240 non-zero roots therefore comprise **168** roots that do not belong to E_6 and **72** roots that do. Each root defines its associated group generator and gauge charge that couples to a 10-dimensional gauge field. The 1680 geometrical elements of

the disdyakis triacontahedron surrounding its central axis correspond to the 1680 components of the **168** 10-dimensional gauge fields of the superstring symmetry group E_8 that are not also gauge fields of its subgroup E_6.

With its faces divided into three tetractyses, the surface of the disdyakis triacontahedron is made up of (**62**+120=182) vertices, (180 + 120×3 = 540) edges and (120×3=360) triangles, that is, 1082 geometrical elements. Starting from a point denoting a vertex, *1081 elements are needs to create the shape of the polyhedron*. **1081** is the number value of Tiphareth, the Sephirah at the centre of the Tree of Life. Tiphareth means "beauty." The polyhedron embodies in its geometry the archetype of Divine beauty.

Inside each triangular face divided into three tetractyses are 10 yods. The number of yods other than vertices in the faces = 120×10 + 180×2 = 1560 = 156×10. The yods in 156 tetractyses are needed to construct the surface of the polyhedron. As 156 is the **155**th integer after 1, this shows how ADONAI MELEKH, the complete Godname of Malkuth with number value **155**, prescribes the outer form of the disdyakis triacontahedron, in keeping with the meaning of Malkuth as the outer physical form.

156 is also the sum of the number values of the four types of combinations of the letters Y, H and V in YHVH, the Godname of Chokmah:

$$Y = 10, H = 5, V = 6$$

1. Y + H + V   = **21**
2. YH + HV + YV + HH   = 52
3. YHV + HVH + HYH   = 57
4. YHVH   = **26**
                              TOTAL = 156

156 is the sum of the first 12 even integers that can be arranged in a square:

$$156 = \begin{array}{cccc} 2 & 4 & 6 & 8 \\ 24 & & & 10 \\ 22 & & & 12 \\ 20 & 18 & 16 & 14 \end{array}$$

The number of corners of the 360 tetractyses in the surface of the polyhedron = **62** + 120 = 182. With internal triangles divided into three tetractyses, the number of corners of the 540 internal tetractyses = 180 + 1 = 181. The (360+540=900) tetractyses needed to build the disdyakis triacontahedron have (182+181=**363**) corners. This is how the Godname SHADDAI EL CHAI with number value **363** prescribes its tetractys composition (see Table 1).

There are seven yods on the boundaries of the three tetractyses inside each of the 120 triangular faces. Each of the 180 sides of the polyhedron has two hexagonal yods between their **62** ends. The number of yods on the boundaries of the (120×3=360) tetractyses = **62** + 180×2 + 120×7 = 1262. There are **1260** such yods between any two diametrically opposite vertices. **1260** is the number value of *Tarshishim*, the Order of Angels assigned to Netzach. The number value of the name of the angelic order assigned to the *seventh* Sephirah is the number of yods between two diametrically opposite vertices that shape the 120 faces of the disdyakis triacontahedron, the *seventh* polyhedron in the sequence matching the *seven* musical scales.

1260 is the number of yods in 126 tetractyses. It is remarkable that the number 126 is

the sum of the number values of the four types of combinations of the letters A, H and I in AHIH, the Godname of Kether:

$$A = 1, H = 5, I = 10$$

1. A + H + I   =   16
2. AH + HI + AI + HH   =   42
3. AHI + HIH + AHH   =   47
4. AHIH   =   **21**

TOTAL = 126

This shows how EHYEH prescribes the population of yods between opposite vertices that define the shapes of the 360 tetractyses in the faces of the disdyakis triacontahedron.[10]

The number 126 has the remarkable property that it is the arithmetic mean of the first **26** triangular numbers, where **26** is the number value of YAHWEH:

$$126 = \frac{\sum_{n=1}^{26} T_n}{26}.$$

It is an example of the profound arithmetic connections between Godname numbers. If

840 turns   840 turns   840   840
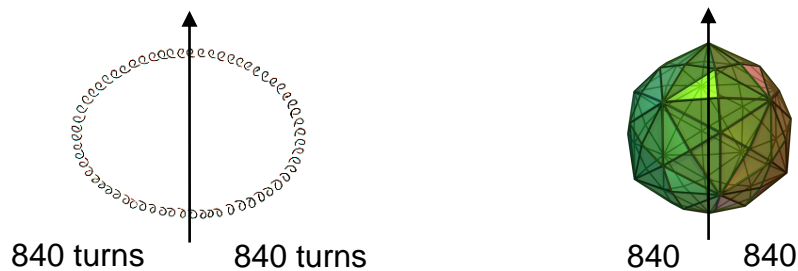
Figure 15. The inner and outer halves of the UPA/superstring have 840 turns in each whorl. They correspond to the two halves of the disdyakis triacontahedron (one the mirror image of the other), each of which has 210 yods between its poles that generate the number 840 when assigned the number 4, the Pythagorean Tetrad.

the Pythagorean Tetrad 4 is assigned to the boundary yods of the disdyakis triacontahedron, the sum generated by the 420 yods between diametrically opposite vertices shaping the 120 faces is 1680. With the latter divided into three tetractyses, the sum generated by the **1260** yods between diametrically opposite vertices shaping the 360 tetractyses is 4×1260 = 5040 = 3×1680. Turned into tetractyses, the 120 faces of the disdyakis triacontahedron embody the number of turns of each helical whorl of the UPA/heterotic superstring (Fig. 15). Turned into three tetractyses, they embody the number of turns of its three so-called 'major whorls' (the thicker whorls).

As

$$5040 = 71^2 - 1 = 3 + 5 + 7 + \dots + 141,$$

and, as the Tree of Life turned into tetractyses contains 70 yods, the number of helical turns in the three major whorls is simply the sum of the 70 odd integers after 1 that can be assigned to the yods in the Tree of Life (Fig. 16). Its Lower Face (shown shaded) has 30 yods, the rest of the Tree of Life having 40 yods. As

$$1680 = 41^2 - 1 = 3 + 5 + 7 + \dots + 81,$$

$$3 \times 1680 = 5040 =$$
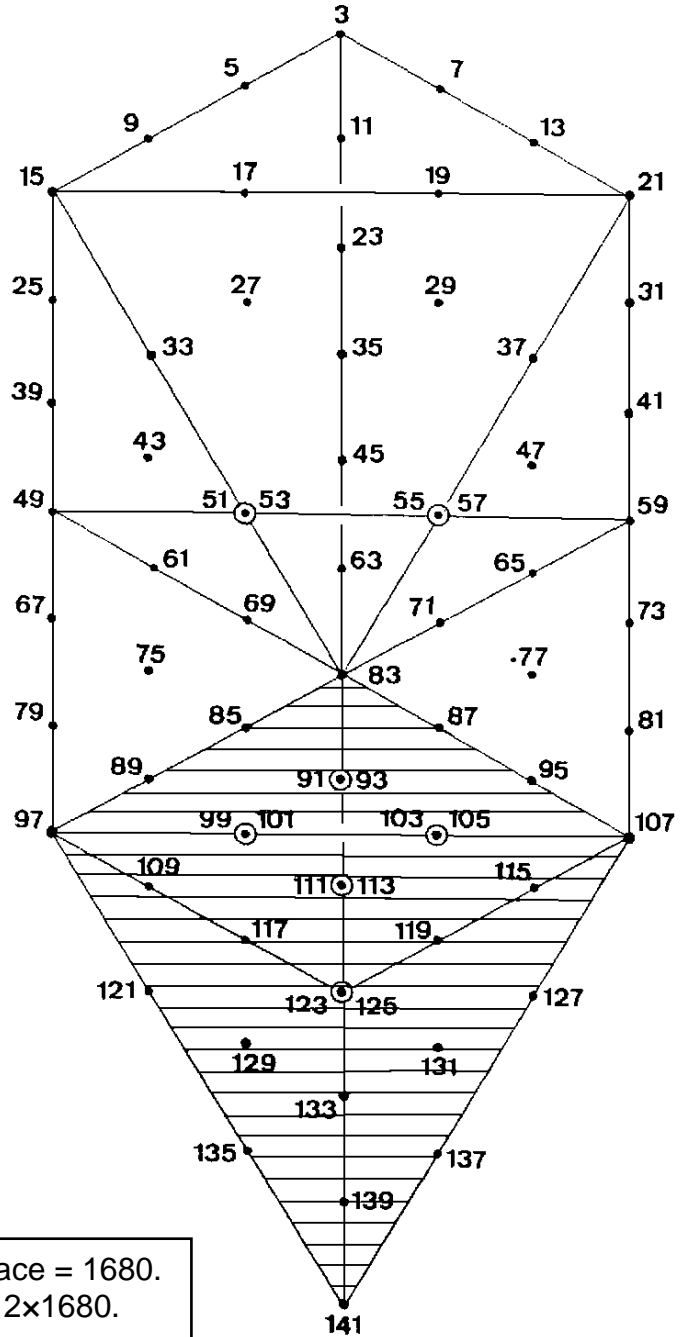
Sum of integers outside Lower Face = 1680.
Sum of integers in Lower Face = 2×1680.
Total = 3×1680

Fig. 16 Assembled from the odd integers after 1, the Tree of Life arithmetically encodes the number (5040) of turns in the three helical, major whorls of the UPA/superstring. This is the counterpart of the disdyakis triacontahedron generating this number through the Pythagorean Tetrad.

the sum of the first 40 odd integers up to 81 *outside* the Lower Face is 1680, and the sum of the next 30 odd integers composing the Lower Face is 2×1680. The Tree of Life encodes arithmetically the number of turns in the three major whorls, which are the string manifestation in 4-dimensional space-time of what Kabbalists call the 'Supernal Triad,' Christians call the 'Holy Trinity' and what is familiar to Hindus as the 'Trimûrti' of Shiva, Vishnu and Brahma. Such is the close relationship between number and the

geometry of the Tree of Life that, when imagined as actually constructed from the odd integers, starting with 3, its Lower Face reproduces the distinction between the first member of the Supernal Triad and the two others in the context of their correspondence to the three major whorls in the subquark state of the E8×E8 heterotic superstring.

The largest odd integer, which coincides with the position of Malkuth at the base of the Tree of Life, is 141 = **76 + 65**, i.e., the sum of the numbers of YAHWEH ELOHIM and ADONAI. It is the 71st odd integer, where 71 = **21 + 50**, i.e., the sum of the numbers of
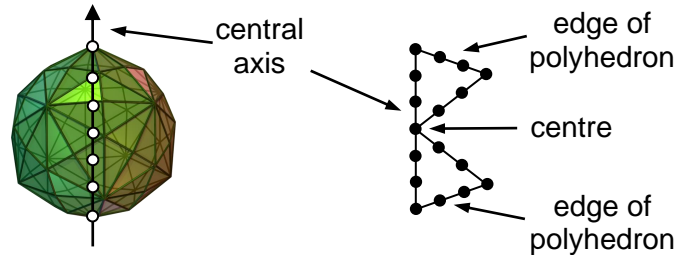
Fig. 17. Seven yods lie on the vertical axis of the disdyakis triacontahedron when constructed from tetractyses because each half of the axis is the edge of an internal tetractys with four yods lying on it.

EHYEH and ELOHIM. The integers are assigned in Fig. 16 sequentially from left to right. Remarkably, the sum of the integers located at the positions of the 10 Sephiroth is

$$
\begin{array}{cc}
\begin{matrix}
3 \\
\mathbf{15}\ \ \mathbf{21} \\
59\ \ \mathbf{49}\ \ 83 \\
97\ \ 107\ \ 125\ \ 141
\end{matrix}
& = 700 =
\begin{matrix}
70 \\
70\ \ 70 \\
70\ \ 70\ \ 70 \\
70\ \ 70\ \ 70\ \ 70,
\end{matrix}
\end{array}
$$

i.e., it is the sum of the number 10 (the Pythagorean Decad) assigned to each of the 70 yods in the Tree of Life! It is an example of the way in which the beautiful, mathematical properties of the Tree of Life clearly point to the designing *Intelligence* behind it.

$$264 = 36 + 228 = $$

36 corners (●)
228 coloured yods
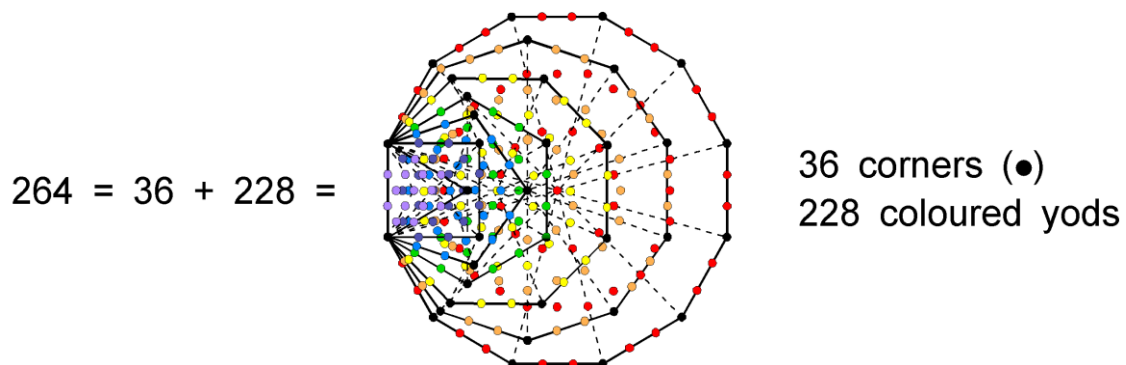
Figure 18. The seven enfolded polygons forming each half of the inner Tree of Life can be constructed from 47 tetractyses with 264 yods. Assigning 10 to each yod generates as their sum the number of yods distributed around the axis of the disdyakis triacontahedron that shape the 900 tetractyses from which it can be built. The **36** corners of the polygons define the 360 corners surrounding the axis and the 228 hexagonal (coloured) yods define the 2280 hexagonal yods around it.

The following, amazing property of the disdyakis triacontahedron provides further proof that it is the polyhedral version of the inner Tree of Life: when its 180 internal triangles are each divided into three tetractyses, there are within each triangle seven yods lying on edges of the (3×180=540) tetractyses. The line joining each polyhedral vertex to the centre of the polyhedron has two hexagonal yods in it. Therefore, the number of yods lying on edges of the internal tetractyses = 2×62 + 7×180 + 1 = 1385. We saw earlier that 1262 yods lie on the boundaries of the 360 tetractyses forming the 120 faces of the polyhedron. The total number of boundary yods inside and on the surface of the polyhedron = 1385 + 1262 = 2647. Four yods lie on the common vertical edge of the 10 tetractyses symmetrically arranged around the axis that are part of internal triangles having sides that converge on the uppermost vertex (Fig. 17). Actually, they are edges of the rhombic faces and raised halves of their longer diagonals. Similarly, four yods lie on the common vertical edge of the 10 tetractyses belonging to internal triangles whose sides converge on the lowest vertex diametrically opposite the highest one. This means that there are seven yods on the vertical axis running through the highest and lowest vertices. The number of yods lying on the boundaries of the (540+360=900) tetractyses that *surround* the vertical axis = 2647 – 7 = 2640 = 264×10. 264 is the number of yods in the seven enfolded polygons forming half of the inner form of the Tree of Life (Fig. 18)! Assigning 10 to each such yod therefore generates as their sum the number of yods lying on the boundaries of the 900 tetractyses that can be assembled into the disdyakis triacontahedron As seen earlier, these tetractyses have **363** corners prescribed by SHADDAI EL CHAI, Godname of Yesod. Three of the corners lie on the central axis (see Fig. 17). 360 corners surround the axis. This number is the sum of the integers 10 that can be assigned to the **36** *corners* of the seven enfolded polygons. There are (264–**36**=228) yods that are not corners. This is the number of yods on the edges of the 900 tetractyses that are *not* corners. A remarkable correspondence emerges between the inner Tree of Life and the disdyakis triacontahedron, whose axis is equivalent to the root edge shared by both sets of polygons. It exists because the disdyakis triacontahedron *is* its polyhedral form.

How do the properties of the dual of the disdyakis triacontahedron compare with it? It is readily shown that the truncated icosidodecahedron has in its **62** faces the same numbers of yods, hexagonal yods, corners and boundary yods. It also has the same number (**363**) of corners of tetractyses making up its faces and interior. However, the latter differs in the numbers of its hexagonal and boundary yods. The number of boundary yods inside and on the surface of the polyhedron is 2763, not the number 2647 for the disdyakis triacontahedron that proved its correspondence with the seven enfolded polygons. Its 900 tetractyses have 1563 corners and edges, a total of 2463 geometrical elements. This compares with the 2405 elements for the disdyakis triacontahedron that implied that 2400 elements surround its central axis. As expected, the properties of its dual fall short of the beautiful correlations with the properties of the seven polygons, showing that the disdyakis triacontahedron — not its dual — should be regarded as the seventh and last member of the set of seven polyhedra because it represents exact correspondence with the Tree of Life.

Of the 264 yods in the seven regular polygons forming each half of the inner Tree of Life, **73** yods belong to the dodecagon. 69 of them lie outside the root edge, leaving 195 yods in the first six polygons. Associating two yods in the shared edge with each set of six polygons, there are 193 yods associated with each set. As the uppermost corner of a hexagon coincides with the lowest corner of the hexagon enfolded in the next higher tree, there are (192n+1) yods associated with the 6n polygons enfolded in n trees. The set of polygons has **26** corners and **31** sides, each with two hexagonal yods, where **26**

is the number value of YAHWEH and **31** is the number value of EL. Hence, there are (**26** + **31**×2 = **87**) yods along their boundaries, where **87** is the number value of *Levanah*, the Mundane Chakra of Yesod. The number of boundary yods associated with each set of six polygons is 85. There are (84n+1) boundary yods associated with
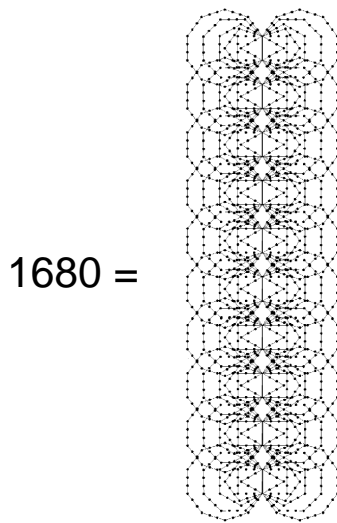

1680 =

Figure 19. The (6+6) polygons enfolded in 10 Trees of Life have as many yods on their boundaries as the disdyakis triacontahedron has geometrical elements surrounding an axis joining diametrically opposite vertices.

the 6n polygons enfolded in n trees. The Godname ADONAI with number value **65** prescribes the lowest 10 trees with **65** Sephiroth. There are 1921 yods associated with the 60 polygons enfolded in them. Of these, 841 yods lie on their boundaries, inside which are 1080 yods. As the uppermost corner of the top hexagon coincides with the lowest corner of the hexagon belonging to the adjacent tree, there are 840 yods intrinsic to each set of polygons, i.e., there are (2×840=1680) boundary yods intrinsic to both

= 620 =
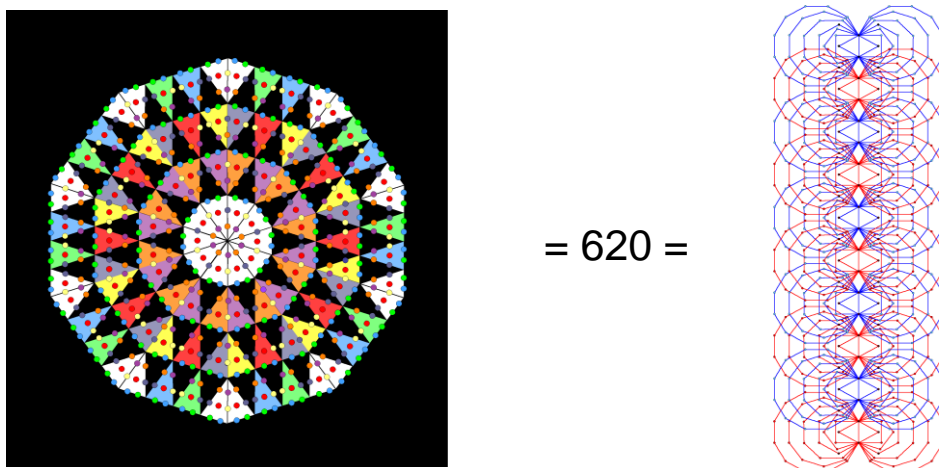
Figure 20. The number value **620** of Kether is the number of corners in the **140** polygons enfolded in 10 Trees of Life that are unshared with their outer form. It is also the number of hexagonal yods in a decagon whose sectors are 2nd-order tetractyses. The number **620** is embodied in the disdyakis triacontahedron with the Decad assigned to its **62** vertices.

sets, which have (2×1080=2160) yods inside their boundaries. We find that the disdyakis triacontahedron has as many geometrical elements surrounding its axis as there are yods forming the shapes of the two sets of the first six polygons enfolded in 10 Trees of Life. The yods symbolize bits of information — geometrical elements in the

disdyakis triacontahedron and circularly polarised oscillations in each whorl of the UPA/superstring. **216** is the number value of Geburah, the second Sephirah of Construction. Its Godname ELOHA prescribes the 360 tetractyses in the 120 faces of the disdyakis triacontahedron because its number value is **36**.

If the number 10, which was regarded by the Pythagoreans as the perfect number, is assigned to every yod in the 360 tetractyses, the sum of the integers assigned to their 182 corners is 1820. This is the number of yods surrounding the centres of the five Platonic solids when they are constructed from tetractyses.[11] It is another remarkable property of the disdyakis triacontahedron that illustrates its archetypal qualities. As 1820 = 70×**26**, it is the sum of the number value **26** of YAHWEH assigned to each of the 70 yods that make up the Tree of Life when its 16 triangles are converted into tetractyses (see Fig. 12). It indicates that the five regular solids constitute a *whole* in themselves.

The sum of the integers 10 assigned to the **62** vertices of the polyhedron is **620**. This is the number value of Kether ("crown"), the first Sephirah of the Tree of Life. It is also the number of corners intrinsic to the 70 polygons enfolded on each side of 10 Trees of Life that are unshared with their Sephiroth (Fig. 20). A symbol of the number 10, the

● = Archimedean or Catalan solid

● = Platonic solid

YHVH = **26** (●)   EL = **31** = **26**(●) + 5(●)
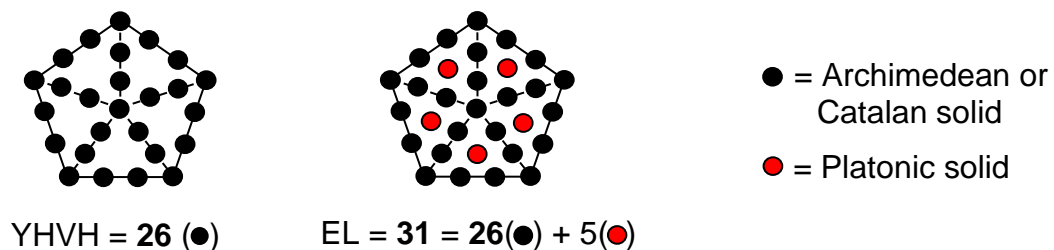
Figure 21. The **26** yods on the boundaries of the five tetractyses in the pentagon symbolize the **26** Archimedean and Catalan solids. The five yods at their centres symbolize the five Platonic solids.

decagon divided into 2nd-order tetractyses has 720 yods surrounding its centre. **620** of them are hexagonal yods. The **62** vertices of the disdyakis triacontahedron correspond to the **62** hexagonal yods per sector of the decagon.

As the dual of the disdyakis triacontahedron, the truncated icosidodecahedron has the same number (1440) of hexagonal yods in its **62** faces.[12] The five Platonic solids have (720=**72**×10) hexagonal yods in their **50** faces.[13] The seven solids have 3600 (=**36**×10×10) hexagonal yods in their 232 faces, showing how the Godname ELOHA of Geburah with number value **36** prescribes the hexagonal yod population of this archetypal set of seven solids. The first six solids have (720+1440=2160) hexagonal
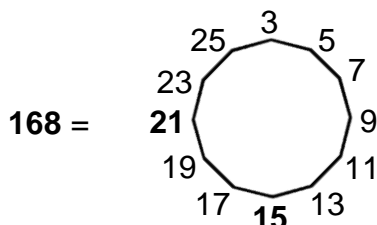
Figure 22. Assigning the 12 odd integers after 1 to the corners of a dodecagon generates the number **168**.

yods, which is the number of yods in **216** tetractyses. The number value of Geburah is **216** and the number value of Chesed, the first Sephirah, is **72**. It cannot be coincidence that successive Sephirah have such significant values.

Adding the five Platonic solids to the **26** Archimedean and Catalan solids creates a new family of **31** solids. The Divine Name EL ("God") assigned to Chesed therefore

prescribes how many regular and quasi-regular solids there are. Constructed from tetractyses, the pentagon has **31** yods (Fig. 21). The yods on the sides of the tetractyses symbolize the Archimedean and Catalan solids and the five yods at their centres represent the five Platonic solids.

As

$$\textbf{168} = 13^2 - 1 = 3 + 5 + 7 + 9 + 11 + 13 + \textbf{15} + 17 + 19 + \textbf{21} + 23 + 25,$$

this number is the sum of the odd integers after 1 that can be assigned to the 12 corners of the dodecagon, which is the seventh type of polygon appearing in the inner Tree of Life and the tenth polygon per se (Fig. 22). The number **168** is also the number of yods generated in the dodecagon when its sectors are each divided into three

168 (●) =
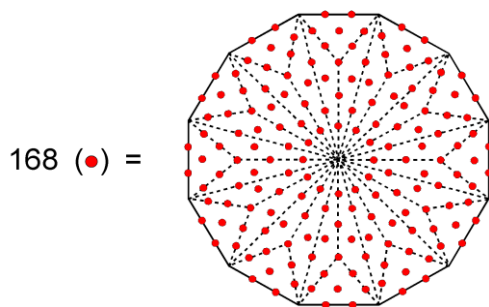
Figure 23. **168** yods other than corners surround the centre of a dodecagon whose sectors are each divided into three tetractyses.

tetractyses (Fig. 23). The total number of yods surrounding its centre is 180, which is the number of sides of the disdyakis triacontahedron. This is how the last polygon in the inner Tree of Life embodies information about the properties of its polyhedral version.

168 is the number value of *Cholem Yesodoth*, the Mundane Chakra of Malkuth. As the structural parameter of superstrings — the physical manifestation or Malkuth aspect of the Tree of Life in the subatomic world — this number is embodied in the disdyakis triacontahedron (apart from the Tree of Life factor of 10) as the number of geometrical elements surrounding its axis. Just as the seven solids correspond to the seven musical
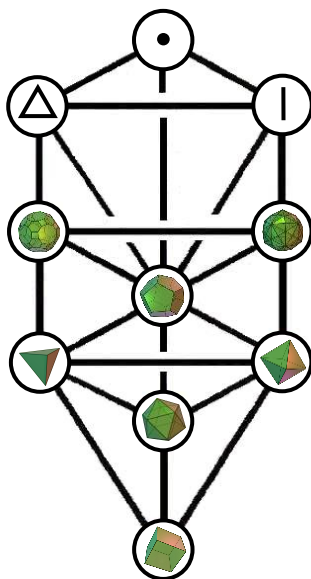
Figure 24. The five lowest Sephiroth of Construction correspond to the Elements of Earth (Malkuth, cube), Water (Yesod, icosahedron), Fire (Hod, Tetrahedron), Air (Netzach, octahedron) and Aether (Tiphareth, dodecahedron) known to exoteric versions of ancient philosophy. However, esoteric traditions speak of two higher Elements, which correspond to Chesed and Geburah. The disdyakis triacontahedron and its dual, the truncated icosidodecahedron, should be assigned, respectively, to these Sephiroth.

scales, so they correspond to the seven Sephiroth of Construction. Indeed, it is the latter that is responsible in the first place for *all* seven-fold patterns in nature. Which solid is associated with which Sephirah? The solids comprise five Platonic solids and

two Archimedean or Catalan solids, whilst the seven Sephiroth of Construction consist of Chesed and Geburah, which are unshared by adjacent overlapping trees, and the five Sephiroth that form the Lower Face of the Tree of Life and which are shared by adjacent trees. In view of this, it is natural to make the following association:

Chesed   ↔   disdyakis triacontahedron
Geburah   ↔   truncated icosidodecahedron
Tiphareth   ↔   dodecahedron
Netzach   ↔   icosahedron
Hod   ↔   cube
Yesod   ↔   octahedron
Malkuth   ↔   tetrahedron

The ancient Greeks believed that the Platonic solids are the shapes of particles of the five elements of Earth, Water, Air, Fire and Aether. The reasons they gave for associating each element with its particular solid may now appear naïve. However, the association of the elements with the five lowest Sephiroth of Construction certainly is not so. Fig. 24 depicts the assignment of the seven solids to the Sephiroth according to the ancient association of the Platonic solids with the Elements. It shows the cube corresponding to Malkuth because it was regarded as the shape of particles of Earth, the element corresponding to Malkuth. It is inconsistent with the tetractys array of solids depicted in Fig. 11, which shows the disdyakis triacontahedron at the *centre* of the tetractys — the yod that symbolizes the Malkuth level or aspect of any holistic system. However, this discrepancy is not serious, because it merely reflects the fact that the association between Elements and Platonic solids made by the ancient Greeks was based upon wrong and naïve considerations, namely, the relation between the sensory experience of these polyhedra and the sensory quality of water, fire, earth, etc. They were also based upon incomplete considerations because in esoteric formulations of mystical traditions there are seven Elements, not five, the latter forming part of their exoteric versions. The order of the seven solids should be based on monotonic increase or decrease of the number of their corners or faces, i.e., with the dimension of their symmetry groups (Table 6):

Table 6

| Polyhedron	Dual Polyhedron	Number of Symmetries	Symmetry Group
tetrahedron	tetrahedron	24 (12)	T_d (T)
cube	octahedron	**48** (24)	O_h (O)
octahedron	cube		
icosahedron	dodecahedron	120 (60)	I_h (I)
dodecahedron	icosahedron		
truncated icosidodecahedron	disdyakis triacontahedron	120 (60)	I_h (I)
disdyakis triacontahedron	truncated icosidodecahedron		

There are three polyhedra with up to **48** symmetries and four polyhedra with 120 symmetries. This compares in the Tree of Life with the triad of Chesed, Geburah & Tiphareth and the quaternary of Netzach, Hod, Yesod & Malkuth. As the simplest regular polyhedron, the tetrahedron starts the mathematical sequence that ends with the disdyakis triacontahedron. That makes it sensible to associate the simplest solid with Chesed and the most complex solid with Malkuth. It also seems appropriate, given

that the number embodied in this solid is both the number value of the Mundane Chakra of Malkuth and the structural parameter of the heterotic superstring – the physical manifestation or Malkuth aspect of the Tree of Life. On the other hand, the analogy between the 2:5 pattern displayed by the Chesed-Geburah pair and the five lowest Sephiroth and the 2:5 pattern of the Catalan solid and its dual and the five Platonic solids is a strong argument for associating the disdyakis triacontahedron with Chesed rather than Malkuth. If polyhedra are to be associated with Elements at all, then this solid and its dual *must* be associated with, respectively, Chesed and Geburah and with their corresponding sixth and seventh Elements. This argument has to be taken seriously because the seven tone intervals in each of the seven musical scales are the musical manifestation of the universal septenary principle, and the same 5:2 pattern exists in each of them as five tone intervals and two leimmas.

The five Platonic solids correspond to the five whole tones and the Archimedean and Catalan solids correspond to the two leimmas:
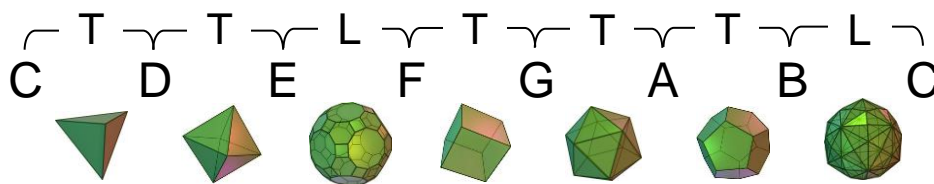
However, this association is only a formal one (although the *sixth* polyhedron, the dual of the disdyakis triacontahedron is *third* in the row above). As now explained, there is a more fundamental correspondence. The product of an interval of a perfect fourth (4/3) and an interval of a perfect fifth (3/2) is the octave: 4/3×3/2 = 2. This is the same note as the tonic but increased in pitch by a factor of 2. As intervals, the perfect fourth and perfect fifth are in a sense inverses of each other. If a note is a perfect fourth above a given note, it is also a perfect fifth below the same note in the next higher octave. Replacing corners of a polyhedron by faces and then replacing faces by corners just leaves the original polyhedron. Forming the dual of a polyhedron is like raising the pitch of a note by a perfect fourth or fifth. An interval of a perfect fourth is 'inverse' to an interval of a perfect fifth in a way that is analogous to how one polyhedron may be dual to another. The octave can be divided into either four perfect fourths or four perfect fifths:
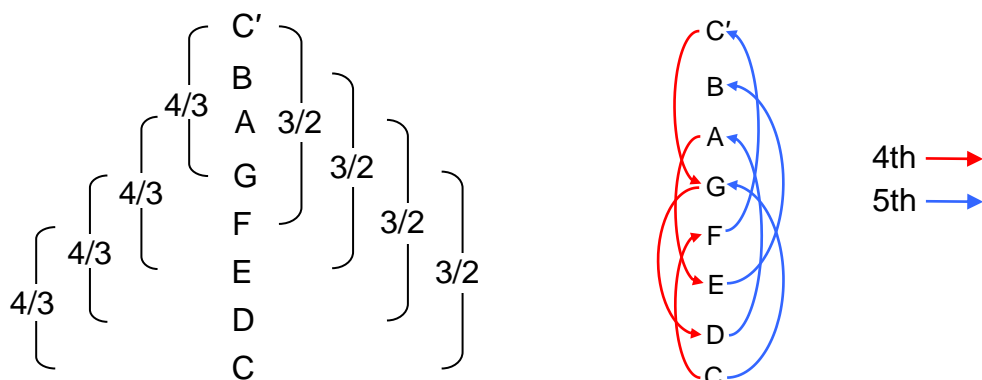
In fact, all eight notes of the octave can be generated by *continuous* leaps of four perfects fourths and three perfect fifths, starting from the tonic C. The seven intervals creating the eight notes consist of one upward jump of a fourth (C-F), three upward jumps of a fifth (D-A, E-B, F-C') and three downward jumps of a fourth (G-D, A-E, C'-G). This 1:3:3 pattern is the same as for the seven musical scales, which consist of three pairs of scales whose intervals are in reverse order, like a reflection in a mirror, and the

Dorian, which is its own inverse or mirror image:[14]

Dorian (D scale)	TSTTTST		TSTTTST	Hypomixolydian (D scale)
Phrygian (E scale)	STTTSTT		TTSTTTS	Hypolydian (C scale)
Lydian (F scale)	TTTSTTS		STTSTTT	Hypophrygian (B scale)
Mixolydian (G scale)	TTSTTST		TSTTSTT	Hypodorian (A scale)

increasing pitch ⟶   mirror   increasing pitch ⟶

It is also the same for the seven polyhedra, which consist of three pairs of duals and one (the tetrahedron), which is self-dual:

tetrahedron ⟶ tetrahedron
octahedron ⟶ cube
icosahedron ⟶ dodecahedron
icosidodecahedron ⟶ disdyakis triacontahedron

The pattern reflects the fact that the seven Sephiroth of Construction consist of two triads: Chesed-Geburah-Tiphareth and Netzach-Hod-Yesod, and one (Malkuth), whilst the tetractys has seven hexagonal yods, six of which are arranged in two equilateral triangles at the corners of a hexagon with the seventh at its centre (Fig. 25). Indeed, it
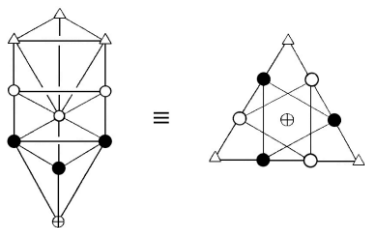
Figure 25. As equivalent symbols of the 10-fold nature of Divine Unity, the Tree of Life and tetractys possess the same 3:3:1 pattern of Sephiroth of Construction and their corresponding hexagonal yods.

was proved in Article 14[15] that the six notes above the tonic form only two chords of three notes whose tone ratios are in the same relative proportions. These two natural triads are DFA and EGB. Corresponding members are separated by a tone interval of
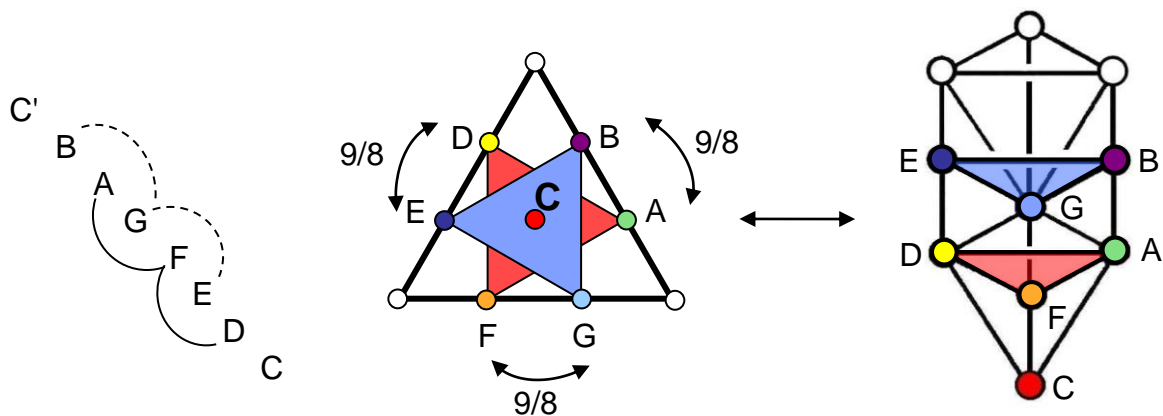
Figure 26. The pairs of notes (E, D), (G, F) and (B, A) have the same relative tone interval of 9/8. Alternate notes D, F and A have tone ratios in the same proportions to alternate notes E, G and B.

9/8 (Fig. 26). This means that the eight notes form four pairs:

C   C'
D   E
F   G
A   B

The perfect fourth F corresponds to the icosahedron, the fourth Platonic solid, the perfect fifth G corresponds to the dodecahedron, the perfect fifth, the major sixth A corresponds to the icosidodecahedron and the major seventh B corresponds to the disdyakis triacontahedron The tonic C and the octave C' correspond to the self-dual tetrahedron.

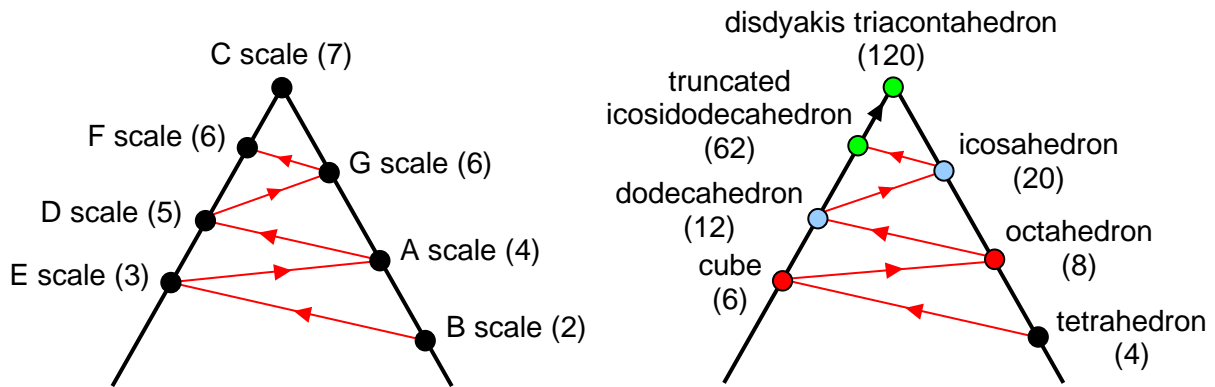Using Table 3, the numbers of Pythagorean tone ratios in the seven scales can be represented in Fig. 27 as converging to the number 7 of the C scale. It also depicts how

Figure 27. As the number of Pythagorean notes increase, the scales converge to the C scale. As the number of faces increase. the polyhedra converge to the disdyakis triacontahedron (coloured points denote dual solids).

the numbers of faces in the seven solids increase to that of the disdyakis triacontahedron. Notice that dual polyhedra (indicated by the same coloured dots) lie on opposite, sloping lines.

As pointed out earlier, the eight notes of the Pythagorean scale are separated by four perfect fourths and four perfect fifths. The prime division of the octave into a perfect fifth
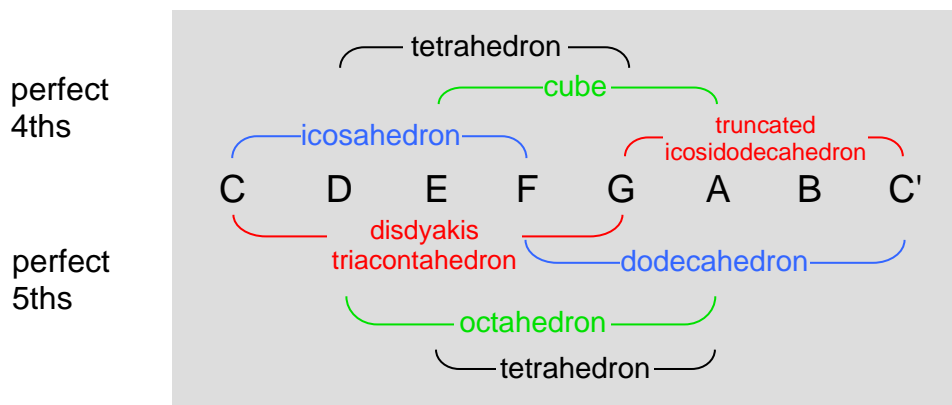
Figure 28. The four perfect fifths and the four perfect fourths of the Pythagorean scale correspond to the two dual pairs of Platonic solids, the disdyakis triacontahedron and its dual and the self-dual tetrahedron.

G, leaving an interval of a perfect fourth, corresponds to the disdyakis triacontahedron being dual to the truncated icosidodecahedron. The division of the octave into a perfect fourth followed by a perfect fifth corresponds to the icosahedron being dual to the dodecahedron. The perfect fifth between note D and note A followed by a drop down by

a fourth corresponds to the duality of the octahedron and the cube. The perfect fourth between D and G corresponds to the tetrahedron. The two possible divisions of the complete octave — 3/2×4/3 and 4/3×3/2 — correspond to the disdyakis triacontahedron and its dual and to the dodecahedron and its dual, whilst divisions within the octave correspond to the tetrahedron, cube and octahedron (Fig. 28).

Inspection of Table 2 reveals that the seven octave species have notes with 13 different tone ratios other than 1, which, together with their 13 inversions, totals **26**. In ascending and descending magnitude of pitch, the two sets of intervals are:

256/243 9/8 **32/27** 81/64 4/3 **1024/729** **729/512** 3/2 **128/81** 27/16 **16/9** 243/128 2

243/256 8/9 **27/32** 64/81 3/4 **729/1024** **512/729** 2/3 **81/128** 16/27 **9/16** 128/243 1/2

This shows how the Divine Name YAHWEH with number value **26** prescribes the number of different rising and falling intervals in the seven scales, i.e., their tonal range. These 13 different, rising intervals and their falling interval counterparts correspond to the 13 Archimedean solid and their duals — the Catalan solids.

The numbers of these intervals vary from scale to scale, with most repeated at least once. Let us consider the basic set of different intervals in each scale. Article 16 calculated that the seven basic sets have 53 Pythagorean intervals (including 1) and 37 non-Pythagorean intervals.[16] There are therefore a minimum of 90 intervals. The significance of this number in the context of the disdyakis triacontahedron will appear shortly.

5. Plato's Lambda & its connection to the disdyakis triacontahedron

n his Timaeus, Plato described how the Demiurge measured the World Soul, or substance of the spiritual universe as a strip divid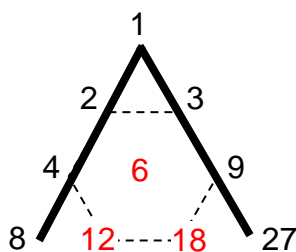ed according to the simple proportions of the first three squares of 2 and 3. This is traditionally represented by his 'Lambda,' so-called because of its resemblance to the Greek letter Λ (Fig. 29). These numbers line but two sides of a tetractys array of ten numbers from whose relative proportions the scientists and musicians of ancient Greece worked out the frequencies of the notes of the now defunct Pythagorean musical scale.[17] The numbers missing from the Lambda are shown in red in Fig. 29. The sum of the 10 integers is 90 and the sum of the integers 1, 8 and 27 at the corners of the tetractys is **36**. The seven integers at the centre and corners of the hexagon shown in Fig. 29 with dashed edges add up to 54.

Figure 29. The Lambda.

Comparing these properties of the Lambda with the number of intervals of the seven musical scales given at the end of the last section, we find that it has the remarkable property of defining not only the tone ratios of the Pythagorean scale as the ratios of its numbers but also how many of these ratios are present as a basic set in the seven types of musical scales! This is no coincidence, because the archetypal nature of the Lambda and its connection with the Tree of Life,[18] the I Ching table,[19] superstrings,[20] the musical scales[21] and the nature of the spiritual cosmos[22] has been demonstrated in previous articles. For example, in conformity with the primacy of the number 4, the first *four* polygons in the inner Tree of Life have 90 yods outside their shared edge (Fig. 30). This cannot be coincidence, because their *individual* yod populations are also the sums of diagonal rows of integers in the Lambda. It should therefore come as no surprise that the Lambda is relevant to the disdyakis triacontahedron, as now explained. Constructed

from tetractyses, its 120 triangular faces divide into (120×3=360) tetractyses. Each of its 180 edges is the side of an internal triangle formed by two vertices and its centre. They are constructed from (180×3=540) tetractyses. The total number of tetractyses forming the disdyakis triacontahedron = 360 + 540 = 900. The following correspondences appear:

Lambda tetractys   **disdyakis triacontahedron**

Sum of 10 integers = 90   ⟶   number of tetractys = 900 = 90×10;
Sum of integers at corners = **36** ⟶ number of tetractyses in faces = 360 = **36**×10;
Sum of integers at centre &
corners of hexagon = 54   ⟶   number of tetractyses inside solid = 540 = 54×10.

The numbers 1^3, 2^3 and 3^3 at the shape-defining corners of the Lambda tetractys add up to **36**, which is the number of tetractyses whose yod population is the number of tetractyses creating the *shape* of the disdyakis triacontahedron. The seven numbers
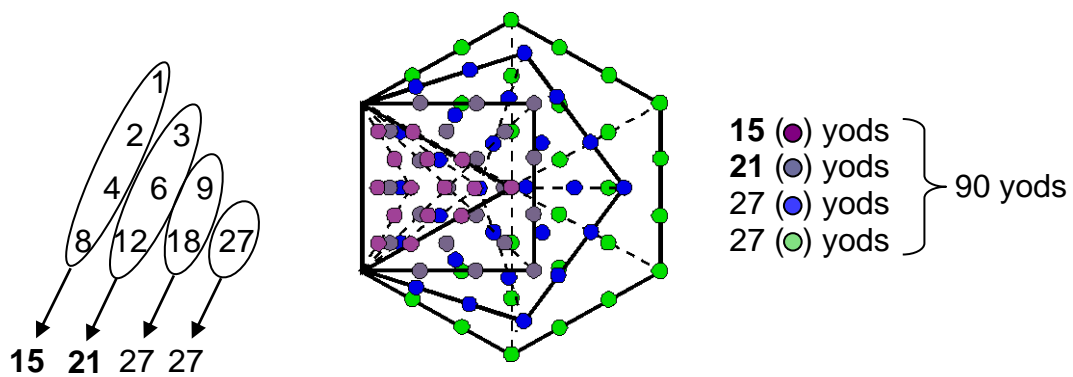
Figure 30. The sums of the diagonal rows of integers in the Lambda are the numbers of yods in the first four polygons of the inner Tree of Life.

inside the Lambda tetractys add up to 54 — the number of tetractyses whose yod population is the number of tetractyses *inside* the disdyakis triacontahedron. Notice the similarity between the function and the character of the two sets of integers. Moreover, the largest of the seven integers in the hexagon is 18, whilst the number of internal tetractyses creating the 180 edges is 180 = 18×10. Four arithmetic properties of the Lambda correlate with the geometry of the disdyakis triacontahedron. This cannot be coincidental.

The integers 1, 2, 3 and 4 symbolized by the four rows of dots in the tetractys express the number of tetractyses generating the disdyakis triacontahedron. As:

$$30 = 1^2 + 2^2 + 3^2 + 4^2,$$

$$900 = 30^2 = (1^2+2^2+3^2+4^2).^2$$

Each of the 120 faces, when divided into three triangles, has three edges meeting at a vertex inside it. The number of geometrical elements making up the insides of the faces = 120×(1+3+3) = 120×7 = 840 = 84×10, where

$$84 = 1^2 + 3^2 + 5^2 + 7^2.$$

The number of golden rhombic-shaped sets of four faces in the disdyakis triacontahedron = 30 = $1^2 + 2^2 + 3^2 + 4^2$.

The number of triangular faces = 120 = 4×30 = $2^2(1^2+2^2+3^2+4^2)$

$$= 2^2 + 4^2 + 6^2 + 8^2.$$

As **31** = $2^0 + 2^1 + 2^2 + 2^3 + 2^4$, the number of vertices = **62** = 2×**31**

$$= 2(2^0+2^1+2^2+2^3+2^4) = 2^1 + 2^2 + 2^3 + 2^4 + 2^5.$$

The number of vertices surrounding the axis of the polyhedron = **62** – 2 = 60

$$= 2^2 + 2^3 + 2^4 + 2^5.$$

We found earlier that the number of hexagonal yods needed to construct the faces of this polyhedron = 1440 = 144×10, where

$$
144 = \begin{matrix}
1^0 & 2^0 & 3^0 & 4^0 \\
1^1 & 2^1 & 3^1 & 4^1 \\
1^2 & 2^2 & 3^2 & 4^2 \\
1^3 & 2^3 & 3^3 & 4^3
\end{matrix}.
$$

The minimum number of triangles needed to construct both the interior and exterior = 120 + 180 = 300 = $(1^2+2^2+3^2+4^2)(1+2+3+4)$. The number of triangles = 660, so that each half of the polyhedron has 330 triangles, where 330 = (1!+2!+3!+4!)(1+2+3+4). We

Figure 31. The number value of *Cholem Yesodoth*, the Mundane Chakra of Malkuth.

also found that the number of geometrical elements surrounding the axis of this polyhedron that make up the 900 tetractyses = 2400 = $(1^3+2^3+3^3+4^3)1×2×3×4$. All these expressions illustrate how the Tetrad Principle[23] determines properties of archetypal objects that possess sacred geometry.
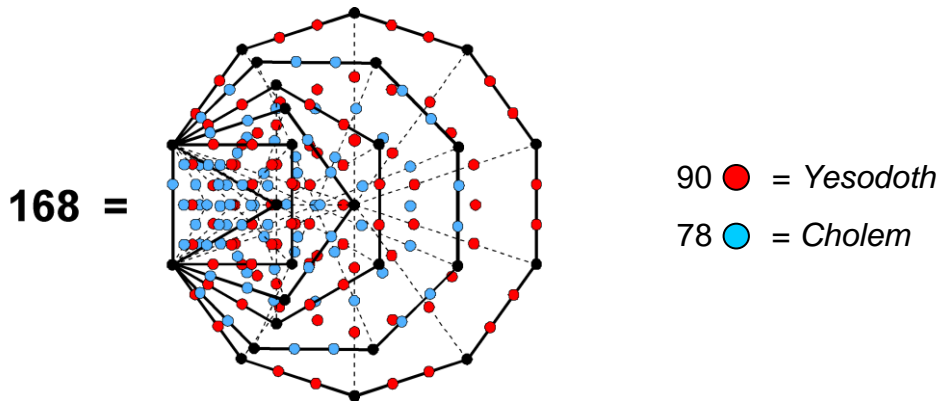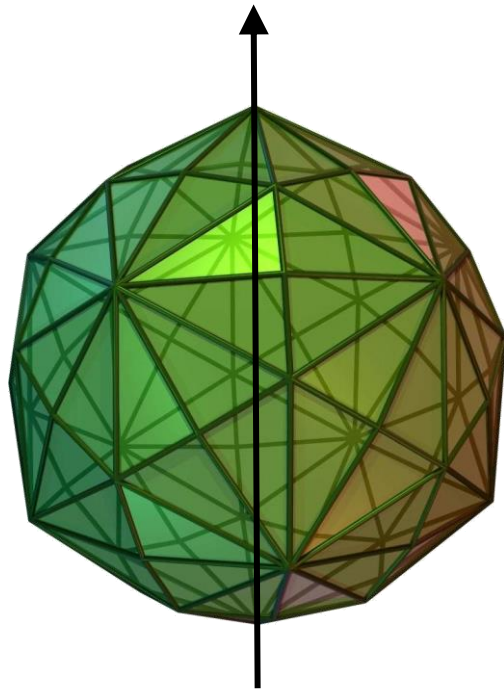

Figure 32. Associated with each set of the first six enfolded polygons are **168** hexagonal yods. 78 of them are in the triangle, pentagon and octagon and 90 hexagonal yods are in the square, hexagon and decagon.
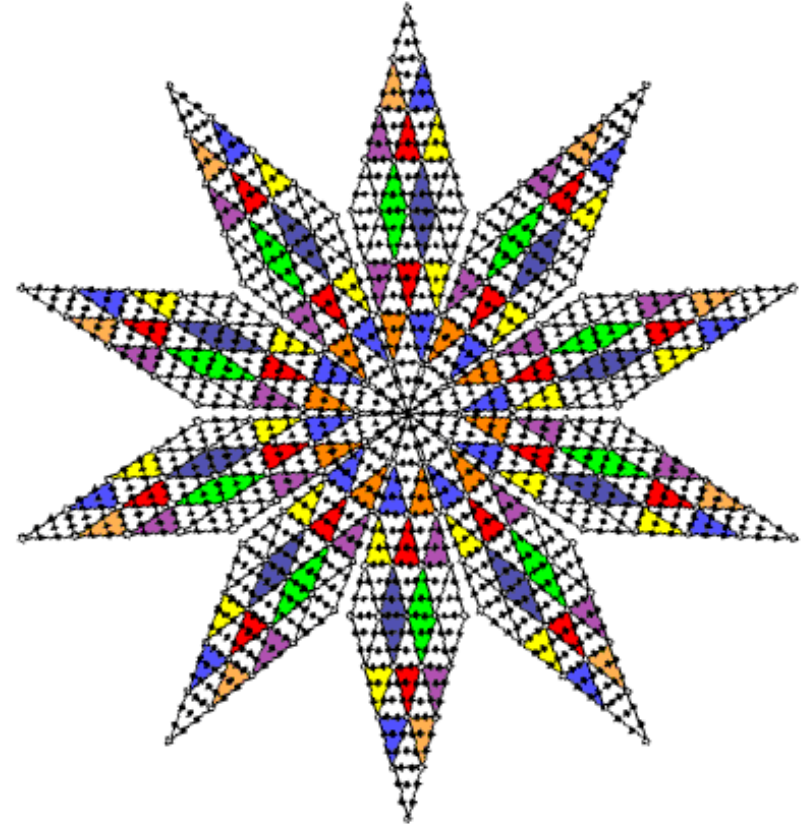
= 1680 =

Figure 33. The faces and interior of the disdyakis triacontahedron have as many corners, sides & triangles surrounding its vertical axis as ten parallelograms constructed from tetractyses have yods surrounding their centre.

6. The superstring structural parameter 1680

Surrounding the axis of the disdyakis triacontahedron are 1680 corners, edges and triangles created when its surface is formed from 180 internal triangles with the centre of the solid as corners and which are then divided into three tetractyses. This is easily seen from the table below:

	Corners	Edges	Triangles
surface:	**62**	180	120
interior:	180	**62** + 3×180 = 602	3×180 = 540
Total =	242	782	660
minus:	−2 (two poles)	−2 (two edges)	
Grand Total =	240 +	780 +	660 = 1680.

There are 780 (=78×10) edges and 900 (=90×10) corners and faces. The significance of this is as follows: the ten Sephiroth of the Tree of Life each manifest as Godnames in Atziluth (the archetypal, or divine, world), Archangels in Beriah (World of Creation), Angels (Yetzirah, or Formative World) and Mundane Chakra (Physical World). According to Kabbalah, the Mundane Chakra of a Sephirah is assigned an astronomical object as its material manifestation. The planet Earth is assigned to Malkuth. It symbolizes the physical aspect of the Tree of Life. Its Mundane Chakra is *Cholem*
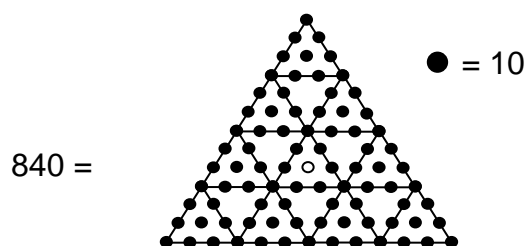
840 =   ● = 10

Figure 34. Weighted with the Decad, the 84 yods surrounding the centre of a tetractys array of tetractyses sum to 840. This is the number of yods in the disdyakis triacontahedron that surround its axis.

Yesodoth, meaning "breaker of the foundations." Through gematria, wherein the 22 letters of the Hebrew alphabet are assigned integers, words acquire number values that are equal to the sum of their letter values. The word 'Cholem' has the number value 78 and the word 'Yesodoth' has the value 90, giving the Hebrew name of the Mundane Chakra a value of **168** (Fig. 31). Remarkably, these numbers define (apart from the Pythagorean factor of 10), respectively, the number of edges and the number of vertices and triangles of the tetractyses needed to construct the disdyakis triacontahedron! Even if, implausibly, the total number 1680 had happened by coincidence to match the number value of the Hebrew name of the Mundane Chakra, it is improbable in the extreme that the geometrical composition of the polyhedron could be quantified by three numbers that happen to be the values of the two words in the name and their sum.

The numbers 90 and 78 appear in the first six polygons of the inner Tree of Life, which themselves constitute a whole Tree of Life pattern, being prescribed by the Godname numbers of all the Sephiroth.[24] Both sets of polygons have 336 hexagonal yods, so that **168** such yods are associated with each set (Fig. 32). The triangle, pentagon and octagon have 78 yods and the square, hexagon and decagon has 90 yods. Once again, it cannot, plausibly, be coincidental that some combination of polygons contains **168** hexagonal yods. Even if this were so, it would still be extremely unlikely that a subset of them would as well contain either 78 or 90 yods just by chance.

As
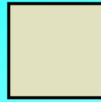
TETRAD PRINCIPLE

The 4th member of the sequence of squares embodies the superstring parameter 168
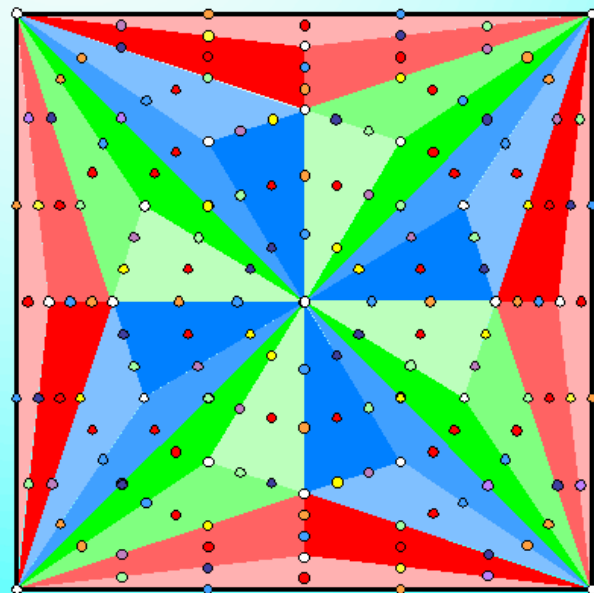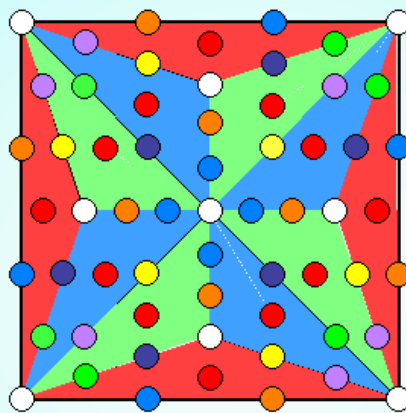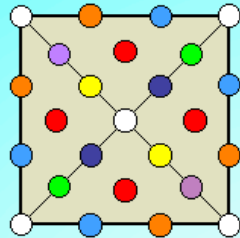
168 yods surround the centre of the square

Figure 35. The fourth stage of construction of a square from tetractyses generates the superstring structural parameter 168 as the number of yods surrounding its centre.

$$13^2 - 1 = \mathbf{168}$$

and, as a tetractys array of ten tetractyses has 13 yods along each edge, a parallelogram formed by two such arrays lying back-to-back has **168** yods below its apex. Ten parallelograms arranged as two pentagrams have 1680 yods surrounding their shared apices (Fig. 33). As the number of yods in a tetractys array = 1 + 2 + 3+…+ 13 = 91, there are 90 yods below its vertex, where 90 is the number value of Yesodoth. The remainder of each arm of the ten-pointed star has (91–13=78) yods, where 78 is the number value of *Cholem*. Each arm is a geometrical representation of the number value of *Cholem Yesodoth*. The inner half of the star has 900 yods, which is the number of vertices and triangles in the interior and faces of the disdyakis triacontahedron surrounding its axis of symmetry. The outer half has 780 yods, which is the number of
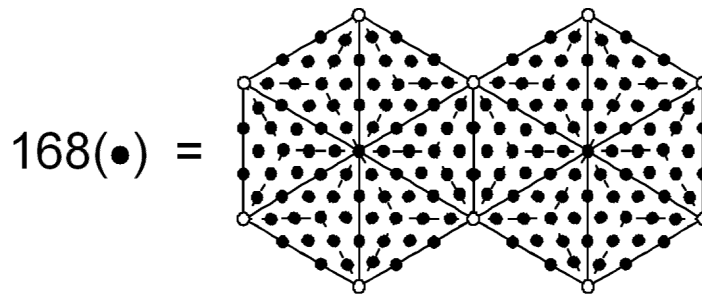

Figure 36. The two adjoined hexagons with their sectors divided into three tetractyses contain **168** yods other than their corners.

edges of the triangles. Each yod denotes one of the 1680 geometrical elements that surround any axis of symmetry that joins two diametrically opposite vertices of the polyhedron.

With the 120 triangular faces as single tetractyses, the number of yods in the 120 tetractyses on the surface = **62** + 2×180 + 120 = 542. With the internal triangles as single tetractyses, the number of internal yods = 2×**62** + 180 + 1 = 305. The total number of yods in its (120+180=300) tetractyses = 847. As there are seven yods along the axis, the number of yods surrounding the centre of the polyhedron = 840. The inner and outer halves of each whorl of the UPA/superstring each consist of 840 circularly polarised oscillations made in 2½ revolutions around its spin axis (see Fig. 14). This superstring structural parameter is therefore embodied in the disdyakis triacontahedron as the number of yods surrounding its axis of symmetry. The Tetrad Principle determines this number because a tetractys array of 10 tetractyses (Fig. 34) has 85 yods, where

$$85 = 4^0 + 4^1 + 4^2 + 4^3,$$

so that 84 yods surround its centre, where

$$84 = 1^2 + 3^2 + 5^2 + 7^2.$$

The Tetrad Principle also determines the structural parameter **168** (the number of oscillations in half a revolution of a whorl of the UPA/superstring) because, starting from a square symbolizing the Tetrad, the *fourth* stage of its construction from tetractyses requires **36** of them with **168** yods surrounding its centre (Fig. 35). Moreover, the *fourth* regular polygon is the hexagon. With its sectors constructed from three tetractyses, it contains 91 yods, i.e., 85 yods other than corners. The two joined hexagons in the inner form of the Tree of Life have **168** yods other than corners (Fig. 36). The superstring

840 vertices,
edges & triangles

≡
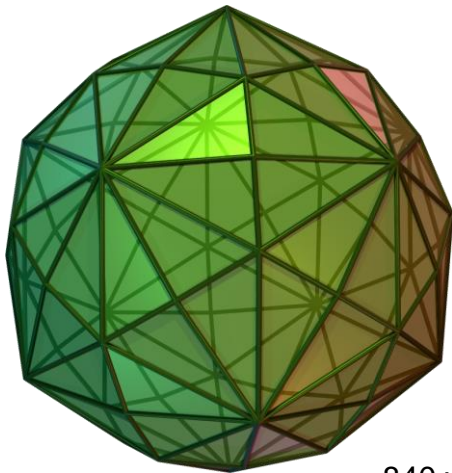
840 vertices,
edges & triangles

31 vertices and their
31 inverted images
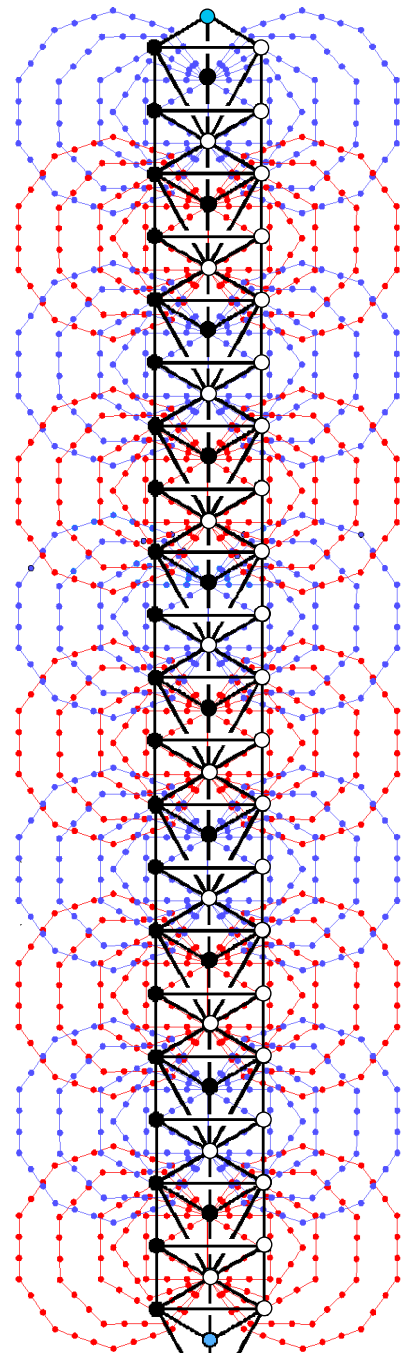
840 (●)   840 (●)

31 ● shared corners
31 ○ shared corners

Figure 37. The disdyakis triacontahedron is the polyhedral counterpart of the inner Tree of Life. Its 62 vertices correspond to the **62** corners of the 120 polygons enfolded in 10 Trees of Life that are shared with the latter. The 1680 geometrical elements surrounding its axis of symmetry correspond to the 1680 yods on the boundaries of the 120 polygons, which are the counterpart of its 120 faces.

structural parameter is the number of extra yods needed to fill up the **36** tetractyses in a pair of hexagons joined along one side. The Godname ELOHA with number value **36** prescribes the structural parameter **168**. The number of hexagonal yods in two separate hexagons is 156, which is the **155**th integer after 1. This shows how ADONAI MELKEH, the full Godname of Malkuth with number value **155**, prescribes the number characterizing the oscillatory form of the $E_8 \times E_8$ heterotic superstring.

7. The disdyakis triacontahedron as the polyhedral Tree of Life

The inner Tree of Life has 70 corners, eight of which coincide with Daath and the seven Sephiroth other than Malkuth, Yesod and Kether (or, rather, the projection of them onto the plane containing the polygons). **62** corners are intrinsic to the inner Tree of Life. These degrees of freedom correspond to the **62** vertices of the disdyakis
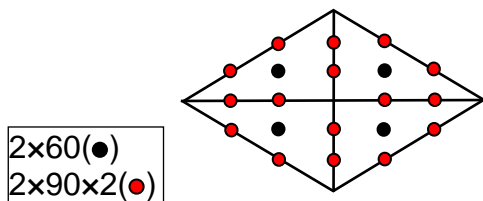

Figure 38. Transformed into tetractyses, 60 faces of the disdyakis triacontahedron have 240 hexagonal yods. Their 60 inverted counterparts also have 240 hexagonal yods.

triacontahedron. The next level of differentiation of the outer Tree of Life is 10 Trees of Life (Fig. 37). Each tree has its own inner form consisting of 14 polygons. The 140 polygons enfolded in 10 trees have (**62**×10=**620**) corners unshared with their **62** Sephiroth. This is why the number value of Kether, the first Sephirah, *has* to be **620**. The **62** corners that the 120 polygons enfolded in 10 trees share with the latter correspond to the **62** vertices of the disdyakis triacontahedron. There are **21** Sephiroth on each side pillar and 10 Tiphareths and 10 Yesods on the central pillar. The **62**
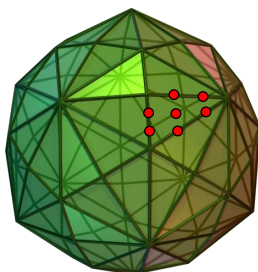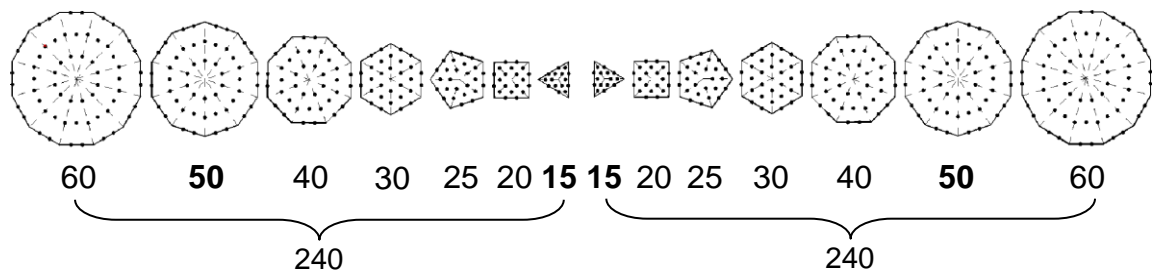

Figure 39. Each set of seven separate polygons, one the mirror image of the other, has 240 hexagonal yods. Each yod denotes a non-zero root of $E_8 \times E_8'$, the gauge symmetry group of heterotic superstrings. Their counterparts in the disdyakis triacontahedron are the 240 hexagonal yods (●) in each of the two sets of 30 faces, one the inverted image of the other.

Sephiroth therefore split into two sets of **31** associated with the active and passive pillars. They correspond to the **31** vertices and their **31** inverted images. The association of these two numbers is why the Godname EL of Chesed has the number value **31** and the number value of its Archangel *Tzadkiel* is **62**.

The two sets of the first six polygons have **50** corners. They are prescribed as a new Tree of Life pattern because **50** is the number value of ELOHIM, Godname of Binah.

240 =

⊙ = 2

720   **720**

720 + 240 + 720 = 1680

Figure 40. There are 1680 yods in the pair of seven polygons surrounding their centres and in the lowest Tree of Life apart from its Sephiroth. Each yod denotes a geometrical element creating the shape of the disdyakis triacontahedron and a circularly polarized oscillation in a whorl of the $E_8 \times E_8$ heterotic superstring.

There are 12n such polygons with (**48**n+2) corners enfolded in n trees. The 120 polygons enfolded in 10 trees are the polygonal counterpart of the 120 faces of the disdyakis triacontahedron. They have 482 corners. Two of these — the uppermost corners of the pair of hexagons — coincide with the lowest corners of the pair of hexagons enfolded in the eleventh Tree of Life. 480 corners are intrinsic to the 120 polygons. 240 such corners lie on each side of the central pillar. Fig. 38 shows that there are 240 hexagonal yods in 60 faces of the disdyakis triacontahedron and 240 hexagonal yods in their 60 inverted images. These 480 degrees of freedom generated by turning the 120 triangles into tetractyses are the counterpart of the 480 corners of the 120 enfolded polygons. They are also the counterpart of the (240+240=480) hexagonal yods in the (7+7) separate polygons (Fig. 39) They symbolize the 480 non-

Increasing pitch

The spherical geometry of the 42 notes between the tonic and the octave of the seven musical scales. Each point on a great circle denoting a note is diametrically opposite another point on the same circle. The notes of the D scale lie along the vertical axis of the sphere.

- 🔴 F scale
- 🟠 B scale
- 🟡 G scale
- 🟢 D scale
- 🔵 C scale
- 🟣 A scale
- 🩷 E scale
- ⚪ Tonic
- ⚫ Octave

Each of the 420 yods in the 120 tetractyses making up the faces of the disdyakis triacontahedron is diametrically opposite another yod.
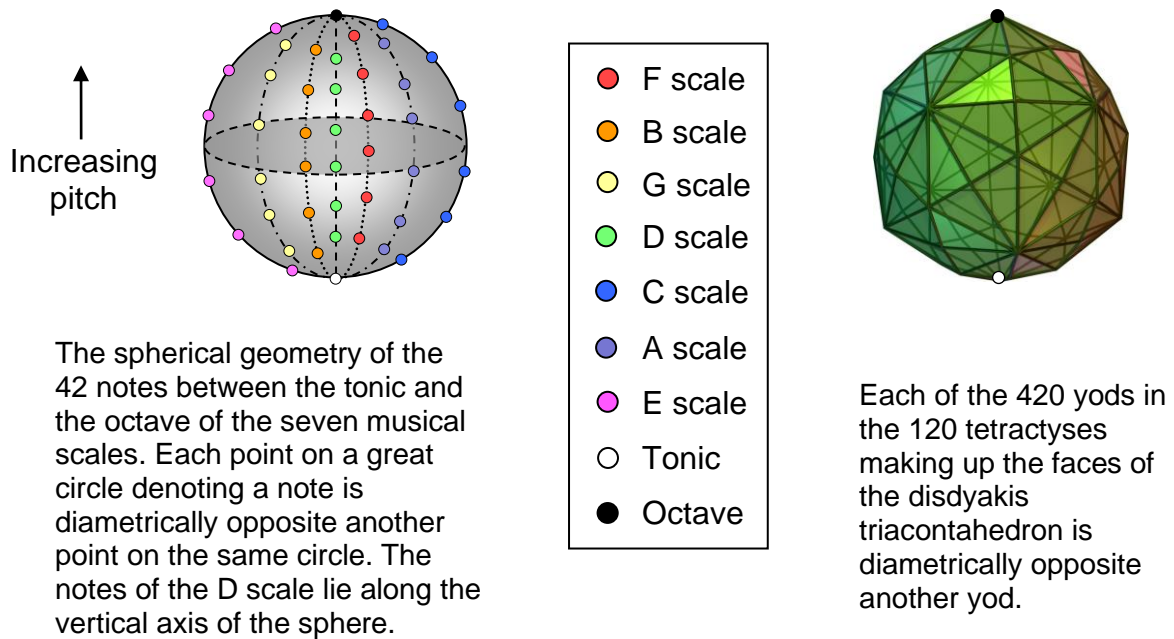
Figure 41. The seven musical scales have 42 notes between the tonic and the octave. Between diametrically opposite vertices (●) and (○), the disdyakis triacontahedron has 420 (=42×10) yods on the edges of its 120 tetractys faces.

zero roots of the heterotic superstring gauge group $E_8 \times E_8'$. The distinction between the 240 hexagonal yods and their inverted images shows itself in the direct product of E_8 and E_8' and in the two sets of polygons enfolded in the 10 trees, which are mirror images of each other. Each hexagonal yod symbolizes a non-zero root of E_8 or E_8' and therefore a gauge boson transmitting the unified superstring force. Hexagonal yods in a tetractys symbolize Sephiroth of Construction. The variety of superstring forces is linked to the differentiation between these cosmic, spiritual potencies.

840 yods lie on the 30 sides of the first six polygons on each side of the central pillar (see Fig. 37). The 1680 boundary yods shaping the two sets of six polygons correspond to the 1680 geometrical elements surrounding the axis of symmetry of the disdyakis triacontahedron. Each yod symbolizes a bit of information needed to characterize the form of the polyhedral version of the Tree of Life, just as the 1680 circularly polarized oscillations of each whorl of the UPA/superstring create its helical shape.

Let us now examine the Tree of Life and the separate polygons. Divided into three tetractyses, the 19 triangles of the lowest tree in any set of overlapping trees acquire

240 yods[25] (Fig. 40). These correspond to the 240 vertices of the 660 tetractyses making up the disdyakis triacontahedron that surround its axis of symmetry. The seven separate regular polygons have **48** corners. With their sectors constructed from three tetractyses, they contain 720 yods surrounding their centres.[26] These correspond to the 720 edges and triangles surrounding the axis that make up the 330 tetractyses in one half of the polyhedron. The similar set of seven polygons on the other side of the lowest tree also contains 720 such yods. They correspond to the 720 edges and triangles in the inverted half of the polyhedron. This exact parallelism between the disdyakis triacontahedron and the outer and inner Tree of Life is powerful evidence that the former is their polyhedral form.

Between the tonic and the octave of a musical scale are six notes. The seven scales have (6×7=42) notes between their tonic and octave, making 44 notes in total if the latter are counted only once. They comprise 28 notes with Pythagorean tone ratios and 16 notes with non-Pythagorean tone ratios. Arranged on a half-circle according to the magnitude of their tone ratios (Fig. 41), the six notes between the tonic and octave of each scale occupy positions that are diametrically opposite notes of the scale that is its mirror image, i.e., there are **21** notes and their mirror images. The counterpart of this inversion symmetry in the disdyakis triacontahedron are the 210 (=**21**×10) yods in one half lying along its edges between diametrically opposite vertices that have their mirror images in the other half of the polyhedron. Actually, only the yods in **21** tetractyses are needed to create the shape of the polyhedron because the rest of it is simply their mirror reflections. This shows how the Godname EHYEH ("I am") of Kether with number value **21** prescribes the disdyakis triacontahedron. Both the seven musical scales and this polyhedron represent holistic systems, one of sound, the other of geometry. The universal, divine archetypes find expression in many contexts, but their mathematical character is always similar, despite the differences in their cultural morphology.

The Godnames of the 10 Sephiroth prescribe the disdyakis triacontahedron in the following ways:

Kether: EHYEH = **21**. There are 420 (=42×10) yods on the edges of the polyhedron between diametrically opposite vertices. 210 (=**21**×10) yods belong to one half of the polyhedron and 210 yods belong to its other, inverted half;
EHYEH ASHER EHYEH = **543**. 542 yods in faces surround the centre of the polyhedron;

Chokmah: YAH = **15**. The 30 golden rhombic faces of the rhombic triacontahedron underlie the disdyakis triacontahedron. 30 = **15**th even integer. Including the chiral versions of the pentagonal icositetrahedron and the pentagonal hexacontahedron, there are **15** Catalan solids, among which the disdyakis triacontahedron has the most faces;
YAHWEH = **26**. The disdyakis triacontahedron is the **26**th and last of the two families of Archimedean and Catalan solids because it has the most faces;

Binah: ELOHIM = **50**. There are 900 tetractyses when both faces and interior triangles are divided into three tetractyses. $90 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2$ = sum of five squares. 900 = 90×10 = sum of **50** squares;

Chesed: EL = **31**. Of the **31** Archimedean, Catalan and Platonic solids, the disdyakis triacontahedron has the most faces. It has **62** vertices, **31** in one half and **31** in its mirror image half;

Geburah: ELOHA: = **36**. Number of corners of 300 tetractyses surrounding an axis that

joins two diametrically opposite vertices = 360 = **36×10** = number of vertices, edges and triangles in 120 faces surrounding this axis; 37 vertices in one half of the polyhedron, where 37 is the **36**th integer after 1.

Tiphareth: YAHWEH ELOHIM = **76**. The **31** Platonic, Archimedean and Catalan solids can be symbolized by the **31** yods of a pentagon with its sectors turned into tetractyses. When the sectors are divided into three tetractyses, the pentagon contains **76** yods. YAHWEH ELOHIM prescribes the pentagon determining the disdyakis triacontahedron as the **31**st polyhedron;

Netzach: YAHWEH SABAOTH = **129**. Number of internal yods and yods on edges of the disdyakis triacontahedron = 727 = **129**th prime number;

Hod: ELOHIM SABAOTH = **153**. Number of internal yods = 2×**62** + 180×1 + 1 = 305 = **153**rd odd integer;

Yesod: SHADDAI EL CHAI = **363**. With faces and internal triangles divided into three tetractyses, the number of their vertices = **62** + 120×1 + 180×1 + 1 = **363**;

Malkuth: ADONAI MELEKH = **155**. 1560 (=156×10) yods other than vertices in faces divided into three tetractyses. 156 = **155**th integer after 1.

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[1]   Phillips, Stephen M. Article 19: "I Ching and the Eight-fold Way," http://www.smphillips.mysite.com/article19.pdf, pp. 15, 17.

[2] For further discussion of the Tree of Life character of the musical modes, see: Article 14: "Why the Greek Musical Modes are Sacred," by Stephen M. Phillips, http://www.smphillips.mysite.com/article14.pdf, pp. 17, 18.

[3] Phillips, Stephen M. Article 22: "The Disdyakis Triacontahedron the 3-dimensional Counterpart of the Inner Tree of Life," Article 23: "The 'Polyhedral Tree of Life," and Article 24: "More Evidence for the Disdyakis Triacontahedron As the 3-dimensional Realisation of the Inner Tree of Life and Its Manifestation in the $E_8 \times E_8$ Heterotic Superstring," at: http://smphillips.mysite.com/html/articles.html.

[4] Phillips, Stephen M. Article 2: "The Physical Plane and Its Relation to the UPA/Superstring and Space-time," http://www.smphillips.mysite.com/article02.pdf.

[5] Ref. 2.

[6] Phillips, Stephen M. Article 5: "The Superstring as Microcosm of the Spiritual Macrocosm," http://www.smphillips.mysite.com/article05.pdf.

[7] Divided into three tetractyses, each of the 20 triangular faces has 9 hexagonal yods inside it. Divided into five tetractyses, each of the 12 pentagonal faces has **15** internal, hexagonal yods. Total number of internal hexagonal yods = 20×9 + 12×**15** = 360. Each of the 60 sides has two hexagonal yods. Total number of hexagonal yods in the icosidodecahedron = 360 + 60×2 = 480.

[8] Proof: the tetrahedron has six edges, each with two hexagonal yods. Number of yods on edges = 4 + 6×2 = 16. It has four faces, each with 10 yods inside its edges. Number of yods on faces = 16 + 4×10 = 56. Each edge is the side of an internal tetractys. Two hexagonal yods lie on the edge joining each corner to the centre of the tetractys. One yod is at the centre of each tetractys. Number of yods inside tetrahedron = 4×2 + 1 + 6 = **15**. Total number of yods = **15** + 56 = 71. 70 yods surround the centre.

[9] Proof: dividing each internal triangle into three tetractyses instead of one tetractys adds 9 yods inside each one. Number of yods = 71 + 6×9 = 125. 120 yods other than corners surround the centre of the tetrahedron.

[10] The last two members of the seven enfolded, regular polygons forming the inner Tree of Life — the decagon and dodecagon — have 126 yods outside their shared edge. As all seven polygons have 264 yods, there are 138 yods in the first five polygons. Excluding diametrically opposite vertices, there are 1260 (=126×10) yods on the boundaries of the 360 tetractyses forming the faces of the disdyakis triacontahedron. Excluding yods along the axis of symmetry joining this pair of vertices, there are 1380 (=138×10) yods on the boundaries of the 540 tetractyses inside the disdyakis triacontahedron. This 5:2

41

differentiation of polygons corresponds to the distinction between the yods on the sides of the internal and external tetractyses making up the disdyakis triacontahedron. The musical pattern of five tone intervals and two leimmas in each of the seven musical scales has its geometrical counterpart in the first five polygons generating the boundary yods inside the disdyakis triacontahedron and the last two polygons generating the boundary yods on its faces.

[11] Phillips, Stephen M. Article 3: "The Sacred Geometry of the Platonic Solids," http://www.smphillips.mysite.com/article03.pdf, pp. 11, 20.

[12] Proof: The truncated icosidodecahedron has 30 square faces, 20 hexagonal faces and 12 decagonal faces. The square has 12 hexagonal yods inside it, the hexagon has 18 internal hexagonal yods and the decagon has 30 internal hexagonal yods. The number of internal hexagonal yods in the polyhedron = 30×12 + 20×18 + 12×30 = 1080. The polyhedron has 180 sides, each with two hexagonal yods. The total number of hexagonal yods = 1080 + 180×2 = 1440.

[13] Ref. 11, pp. 10, 20.

[14] Suppose that each rising interval is replaced by its falling counterpart, i.e., $T \rightarrow T^{-1}$ and $L \rightarrow L^{-1}$. An ascending scale, e.g., TLTTLTT, then becomes $T^{-1}L^{-1}T^{-1}T^{-1}L^{-1}T^{-1}T^{-1}$, which is the descending version of the ascending scale TTLTTLT. Inverting each interval of a scale creates another scale whose pattern of intervals is its mirror image. Only the D scale is its mirror image: TLT**TTLT** = **TLTT**TLT.

[15] Ref. 2, pp. 4, 5.

[16] Phillips, Stephen M. Article 16: "The Tone Intervals of the Seven Octave Species and Their Correspondence with Octonion Algebra and Superstrings," http://www.smphillips.mysite.com/article16.pdf, p. 11.

[17] Phillips, Stephen M. Article 11: "Plato's Lambda — Its Meaning, Generalisation and Connection to the Tree of Life," http://www.smphillips.mysite.com/article11.pdf.

[18] Ibid, pp. 7, 16.

[19] Phillips, Stephen M. Article 20: "Algebraic, Arithmetic and Geometric Interpretations of the I Ching Table," http://www.smphillips.mysite.com/article20.pdf, pp. 18–22.

[20] Ref. 17, p. 7.

[21] Ref. 17, pp. 2–3, and Article 12, "New Pythagorean Aspects of Music and Their Connection to Superstrings," Stephen M. Phillips, http://www.smphillips.mysite.com/article12.pdf, pp. 2–4.

[22] Ref. 17, pp. 3–7.

[23] Phillips, Stephen M. Article 1: "The Pythagorean Nature of Superstring and Bosonic String Theories," http://www.smphillips.mysite.com/article01.pdf, p. 5.

[24] Phillips, Stephen M. Article 4: "The Godnames Prescribe the Inner Tree of Life," http://www.smphillips.mysite.com/article04.pdf, p. 4.

[25] Proof: The lowest tree has 19 triangles with 11 vertices and 25 edges. Divided into three tetractyses, each triangle has 10 internal yods. Number of yods in lowest tree = 11 +25×2 + 19×10 = 251. The 11 Sephiroth are at the vertices of the triangles. Number of yods other than Sephiroth = 251 – 11 = 240.

[26] Proof: Divided into three tetractyses, each sector of a regular polygon has 10 internal yods. The seven polygons have **48** sectors. Each of the **48** sides of the polygons has two hexagonal yods between its ends. Each side joining a corner to the centre of a polygon has two hexagonal yods. Number of yods in seven separate polygons = **48**×10 + **48**×2 + **48**×2 + **48** + 7 = 727. Number of yods surrounding the centres = 727 – 7 = 720.

"The Universe is a thought of the Deity. Since this ideal thought-form has overflowed into actuality, and the world born thereof has realized the plan of its creator, it is the calling of all thinking beings to rediscover in this existent whole the original design."

F. Schiller, *Theosophie des Julius, The World of Thinking Beings.*